SYSTEMATIC STUDY OF SUB-BARRIER FUSION ENHANCEMENT IN HEAVY-ION REACTIONS

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Abstract: A systematic study of the heavy-ion fusion-cross-section enhancement at sub-barrier energies is carried out. The asymptotic energy shift introduced in a previous paper as a measure of this enhancement is considered from a theoretical point of view. It is claimed that this energy shift is composed of two terms: one of them is related to the bulk properties of nuclear matter, and the other corresponds to deviations depending on the specific nuclear structure of the collision partners. We show that the former can be approximately described by the neck-formation model for fusion and the latter is frequently a consequence of static deformation or vibrational excitation of the projectile and/or the target. A comparison of the neck-formation effects with those arising from neutron flow suggests that there is a connection between these two mechanisms.

1. Introduction

Sub-barrier fusion reactions have attracted considerable attention in the last decade \(^1\). The main reason for this interest was the possibility of creating super-heavy elements through the fusion of heavy ions. As a consequence of this research unexpected characteristics of these reactions were uncovered. The most outstanding of these were the surprisingly large values of the measured cross sections which in some cases exceeded the standard theoretical predictions by several orders of magnitude. The work of Balantekin, Koonin and Negele \(^2\) has clearly shown that tunneling under the one-dimensional fusion barrier is incompatible with the available data, implying that additional degrees of freedom must be considered. Theoretical interpretations for this sub-barrier fusion enhancement have been given in terms of deformation \(^3\) and vibration effects \(^4\), neutron transfer \(^5\) and formation of a neck between the collision partners \(^6\). A more fundamental treatment considering the coupling between the elastic and several reaction channels was pursued by various authors \(^7\).

The search for the reaction mechanisms involved in this phenomenon requires a systematic study including many systems along the periodic table. For this purpose we introduced an operational measure of the enhancement in the sub-barrier fusion cross section through a quantity which we called asymptotic energy shift \(^8\), and studied its variation with the system size. This study showed that the asymptotic energy shift has a growing trend with system size, and that the neck-formation model provides a good description of this average behavior. On the other hand, a few
systems exhibited enhancements much above this mean and they should be considered separately. Along this line, Iwamoto and Takigawa have recently studied the physical origin of the large deviation presented by the $^{74}\text{Ge}+^{74}\text{Ge}$ system.

In the present work we carry out a systematic study of the mechanisms leading to these anomalously large enhancements. In sect. 2 we present a detailed discussion of the asymptotic energy shift for a variety of heavy-ion systems. In sect. 3 we calculate the separate contributions of neck formation, static deformation and vibrational motion to the asymptotic energy shift, and compare the theoretical predictions to the empirical values extracted from the data. Also in this section we attempt to correlate the neck-formation process with the neutron flow mechanism recently considered by Stelson. Finally, in sect. 4 we draw the main conclusions from the present work.

2. The asymptotic energy shift as a measure of the enhancement of $\sigma_F$

As discussed in detail by Vaz, Alexander and Satchler, the fusion cross section for light heavy-ion collisions near and above the Coulomb barrier can be described by one-dimensional models with standard theoretical potentials, like that of Krappe-Nix-Sierk (KNS), provided that small adjustments of a single potential parameter are allowed for. A different situation is encountered when one studies sub-barrier fusion of heavy ions. In this case the standard one-dimensional potentials fitted as to describe the fusion cross section above the Coulomb barrier $V_B$ fail in reproducing it at lower energies $E \ll V_B$. A typical situation is depicted in fig. 1, for the $^{58}\text{Ni}+^{58}\text{Ni}$ system. The one-dimensional calculation fits the data for $E \approx 100$ MeV. As the collision energy decreases, the calculation progressively deviates from the data. We introduce the energy shift $\epsilon$, illustrated in fig. 1, by the relation

$$\sigma_F^{\text{exp}}(E) = \sigma_F^{(0)}(E + \epsilon),$$

where $\sigma_F^{(0)}$ is the one-dimensional model fusion cross section. As $\epsilon$ is the energy shift needed in the one-dimensional calculation to make it agree with experiment, it depends on the particular data point considered. In fig. 2 we give $\epsilon$ as a function of $\sigma_F^{\text{exp}}$ for several systems. For this purpose we have used the KNS potential with the radius parameter fitted as to reproduce the data above the Coulomb barrier. We observe that for $\sigma_F^{\text{exp}} < 1$ mb (or $E \ll V_B$) $\epsilon$ tends to a roughly constant value $\Delta E$. This means that at low energies the fusion cross section calculated in a one-dimensional model appears to have approximately the same exponential energy dependence as the data, but shifted by a constant value $\Delta E$.

The discussion above suggests that the asymptotic energy shift $\Delta E$ can be used as an operational measure of the sub-barrier fusion-cross-section enhancement. This quantitative measure of the enhancement is very useful when one carries out a systematic study of this phenomenon for many projectile-target combinations, as we do in the present work.
To determine the asymptotic shift $\Delta E$ we use the following prescription:

(i) The one-dimensional calculation of $\sigma_F^{(0)}$ is approximated by Wong’s formula:

$$\sigma_F^{(0)} \approx R_B^2 \frac{h\omega}{2E} \ln \left\{ 1 + \exp \left[ \frac{2\pi}{h\omega} (E - V_B) \right] \right\},$$

where $R_B$, $h\omega$ and $V_B$ are treated as adjustable parameters.

(ii) We assume that at low energies the logarithm of the cross section calculated with the one-dimensional model has the same slope as the corresponding experimental data. We determine, therefore, the parameter $h\omega$ appearing in eq. (2) from the experimental fusion cross section at low energies. We take, for that purpose, data points such that $\sigma_F^{exp} < 10^{-1}$ mb.

(iii) The remaining parameters in eq. (2), $R_B$ and $V_B$, are fitted as to reproduce the experimental cross section above the Coulomb barrier. For this purpose we use data points in the range $100 \leq \sigma_F^{exp} \leq 500$ mb. We do not include experimental points above 500 mb because the approximation of an $l$-independent barrier radius, used in the derivation of Wong’s formula, becomes inappropriate. Points below 100 mb are excluded since they could be affected by the sub-barrier fusion mechanisms we want to investigate in this work.

(iv) Frequently the selection of the data points for the fit described in (iii) and their large error bars allow for too much freedom in the determination of $R_B$ and $V_B$. We try to remedy this situation by imposing that the nuclear potential have an exponential behavior in the barrier region. This leads to the constraint

$$V_B = \frac{Z_1 Z_2 e^2}{R_B} \left[ 1 - \left( 2 + \frac{\mu \omega^2 R_B^1}{Z_1 Z_2 e^2} \right)^{-1} \right],$$

where $Z_1$ and $Z_2$ are the nuclear charges and $\mu$ is the reduced mass of the two nuclei.
Fig. 2. The function $\varepsilon(\sigma_F^{exp})$ for several systems.
where $\mu$ is the reduced mass and $Z_1, Z_2$ are the atomic numbers of projectile and target, respectively.

As an illustration we show in fig. 1 the application of this procedure to the $^{58}\text{Ni} + ^{58}\text{Ni}$ case. We find $R_B = 8.8$ fm, $h \omega = 2.4$ MeV and $V_B = 99.0$ MeV. The one-dimensional cross section is represented by the solid line. From the figure we read the asymptotic energy shift value $\Delta E = 4.0$ MeV.

To study the dependence of the fusion enhancement on the size of the system we show in fig. 3 $\Delta E$ values for several projectile-target combinations. The results are plotted against the parameter $\zeta$ as in ref. $^8$)

$$\zeta = \left( \frac{Z_1^2}{A_{\text{eff}}} \right) = \frac{4Z_1Z_2}{A_1^{1/3}A_2^{1/3}(A_1^{1/3} + A_2^{1/3})}.$$ (4)

We notice the growing trend of the fusion enhancement $\Delta E$ with the size of the system, in the range considered. For the heaviest systems within this range (both open and solid diamonds in fig. 3) the compound nucleus formed in the reaction decays through channels other than light-particle evaporation. In such cases the measured evaporation residues cross section underestimates that for compound nucleus formation, which is the one given by Wong's formula. In consequence step (iii) of our prescription to determine $\Delta E$ could not be applied.

To deal with this situation let us consider the overall trend of the Coulomb barrier height with system size. For this purpose it is convenient to extract geometrical

![Systematic study of the asymptotic energy shift $\Delta E$. The data were extracted from a) ref. $^{15}$; b) ref. $^{16}$; c,d) ref. $^{12}$; e,f) ref. $^{13}$; g) ref. $^{17}$ and h) ref. $^{18}$.)](image)
factors writing

\[ V_B = v_0 \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}}, \]  

and then study the behavior of \( v_0 \) as a function of \( \zeta \). The result is shown in fig. 4. We have assumed that the dependence of \( v_0 \) on \( \zeta \) is given by the straight line shown in the figure and in this way determined the parameter \( V_B \) for systems where the step (iii) could not be applied. From this value of \( V_B \) and that of \( \hbar \omega \) extracted from the low energy data, we obtain \( R_B \) with the help of the constraint of eq. (3).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Systematic study of the barrier-height parameter \( v_0 \). For details see the text.}
\end{figure}

It is also interesting to notice that for the systems \(^{36}\text{S} + ^{58}\text{Ni}\) and \(^{74}\text{Ge} + ^{74}\text{Ge}\), indicated in fig. 4, the extracted barrier is appreciably higher than the average trend. As will be discussed in the next section, the enhancements found for these systems exceed the theoretical predictions.

3. Study of the asymptotic energy shift \( \Delta E \)

We consider two aspects of the fusion enhancement \( \Delta E \) depicted in fig. 3: its average trend and the fluctuations around this average. We have already shown in ref. \(^8\) that the neck-formation model reproduces well the average trend of the data. A summary of this model is presented subsection 3.1. We then contend that the significant deviations from the average behavior found in this figure arise from strong coupling between the entrance and some specific channels. The nature of these channels depends on the details of the nuclear structure of projectile and/or
target. In the remaining subsections we show that the most pronounced deviations from the average can be traced back to coupling to inelastic collective states.

3.1. THE NECK-FORMATION MODEL OF FUSION REACTIONS

The neck-formation model of fusion reactions has been extensively studied in ref. 5). In this section we summarize the main features of this model, and discuss its predictions for the asymptotic shift $\Delta E$ defined in sect. 2.

The fusion mechanism was shown to be dominated by two characteristics of the neck-formation process:

(i) The disappearance of the potential barrier along the neck degree of freedom when the nuclei reach a critical radial separation $R_n$;

(ii) The very small values of the mass tensor element associated to the neck variable in the early stages of the collision.

Owing to these effects the system becomes unstable with respect to neck formation at separations $r < R_n$. Since $R_n$ is larger than the radius $R_B$ of the Coulomb barrier, this instability appears at a collision energy $E = V_n < V_B$, where $V_n$ and $V_B$ are the values of the total potential (Coulomb + nuclear) at the radial separations $R_n$ and $R_B$, respectively. The existence of this instability means that there is no barrier for the system to move along the neck degree of freedom towards more compact configurations. The consideration of a neck degree of freedom leads, therefore, to an effective barrier lowering

$$\Delta B = V_B - V_n.$$ (6)

It was shown in ref. 5) that at very low energies the fusion cross section predicted by the neck-formation model is very close to that calculated with the one-dimensional model at an energy $E + \Delta B$. The enhancement of the fusion cross section arising from neck formation leads, therefore, to an asymptotic energy shift

$$\Delta E \approx \Delta B.$$ (7)

In fig. 5 we plot the empirical values of $\Delta E$ (displayed in fig. 3) versus the $\Delta B$ values calculated with the neck-formation model for the corresponding systems. It is clear that the neck-formation model describes reasonably the average behavior of the asymptotic energy shift. We notice, however, large isolated deviations, especially for the $^{74}\text{Ge} + ^{74}\text{Ge}$ and $^{40}\text{Ar} + ^{154}\text{Sm}$ systems, which will be discussed in the next subsections. We should also remember that, as mentioned in the previous section, the Coulomb barrier heights for the heaviest systems (those presenting $\Delta B > 7.5$ MeV) were estimated through extrapolation of the average behavior of the potential barriers for lighter systems shown in fig. 4. Because of the uncertainties that arise in such procedure we decided to exclude these systems from further discussion.
We should also remark that the fusion cross section calculated with this model is, in fact, a cross section for neck formation\(^9\). Since in most of the systems represented in fig. 3 the saddle point of the two-dimensional potential energy lies at a separation larger than \(R_n\), it is reasonable to assume that neck formation leads to fusion. However, for very heavy systems (\(\zeta \gtrsim 35\)) the saddle point moves to more compact configurations. In such cases neck formation gives rise not only to fusion but also to deep-inelastic and fast-fission processes. The study of fusion enhancement for such systems should therefore include the details of the dynamic evolution of the system following neck formation.

### 3.2. Static Deformation and the Role of Rotational States in Fusion Enhancement

In fig. 6 we show the calculated \(\Delta B\) values and the corresponding experimental enhancements \(\Delta E\) for a variety of systems, grouped according to the atomic number of the target. While for some systems these two quantities lie close to each other, others show marked dependence on the target mass number. The most systematic of these deviations is that found in the case of the Sm isotopes, and may be immediately correlated with the increasing deformation of such nuclei. In this subsection we discuss the influence of the static deformation on the fusion cross section and calculate its effect on the sub-barrier fusion enhancement.
Fig. 6. Asymptotic energy shift ($\Delta E$) compared to the effective barrier lowering due to the neck formation ($\Delta R$) for several systems grouped according to their mass ranges.

In heavy-ion collisions involving deformed nuclei the entrance channel couples strongly to rotational states and the coupling also affects the fusion cross section. In such cases the angular orientation of the system does not change appreciably during the collision, and the fusion cross section can be calculated using the equivalent-spheres model $^{20}$. In this model the potential barrier is orientation dependent. The prescription to evaluate the fusion cross section is first to calculate it for a given orientation $\Omega$, and then to take the average

$$\sigma_F = \frac{1}{4\pi} \int \sigma_F(\Omega) \, d\Omega. \quad (8)$$

The integrand of eq. (8) can be approximated by Wong's formula provided that we have, for each orientation, the barrier parameters $R_B$, $V_B$ and $h\omega$. The barrier radius is

$$R_B(\Omega) = R_B + \Delta R(\Omega), \quad (9)$$

where $R_B$ is the average barrier radius. Assuming pure quadrupole deformation and approximating the barrier radius variation by the change of the surface radius, $\Delta R(\Omega)$ is given by

$$\Delta R(\Omega) = \beta R_0 Y_{20}(\Omega). \quad (10)$$
In the above equation $R_0$ is the average radius of the deformed nucleus and $\beta$ is the experimental value of the quadrupole deformation parameter.

The orientation dependent barrier height can be obtained from eq. (3). To first order in $\beta$, and neglecting variations of the barrier curvature with orientation, it can be written

$$V_B(\Omega) = V_B \left[1 - g \frac{\Delta R(\Omega)}{R_B}\right],$$

where $V_B$ is the average barrier height and $g$ is the factor

$$g = 1 - \frac{3\mu \omega^2 R_B^3}{V_B(2 + \mu \omega^2 R_B^3/Z_1 Z_2 e^2)}.$$ (12)

Using the potential barrier of eq. (11) in Wong's formula, and substituting this orientation dependent cross section in eq. (8), we can obtain the average cross section.

We are interested in values of $\sigma_F$ at very low energies ($E \ll V_B$) for which the enhancement can be measured by an asymptotic energy shift. In this limit the average fusion cross section can be approximated by the simple formula

$$\sigma_F(E) \approx \sigma_F^{(0)}(E + \Delta E_{\text{rot}}),$$ (13)

where $\sigma_F^{(0)}$ is the fusion cross section in the absence of deformation and $\Delta E_{\text{rot}}$ is the contribution from rotational channels to the asymptotic energy shift, given by

$$\Delta E_{\text{rot}} = \frac{\hbar \omega}{2 \pi} \left[\frac{3}{2} \gamma^2 + \ln(D(\gamma)/\gamma)\right],$$ (14)

where $D(\gamma)$ is the Dawson function (21) and $\gamma^2$ is

$$\gamma^2 = \frac{3}{8V} \frac{R_0}{R_B} \frac{V_B}{\hbar \omega} g \beta.$$ (15)

For appreciable deformations ($\beta \approx 0.1$) the argument of the Dawson function is large and we can use its asymptotic expression. In this case we get

$$\Delta E_{\text{rot}} = \frac{\hbar \omega}{2 \pi} \left[\frac{3}{2} \gamma^2 - \ln(2\gamma^2)\right].$$ (16)

We have thus shown that rotational coupling produces an asymptotic energy shift $\Delta E_{\text{rot}}$. This shift is clearly not contained in the neck-formation mechanism described in the previous subsection, which considered only spherical nuclei. For fusion reactions involving highly deformed nuclei the value of $\Delta E$ extracted from the data should, therefore, be compared to the sum $\Delta B + \Delta E_{\text{rot}}$. This is the case for the fusion of $^{40}$Ar with the highly deformed $^{154}$Sm ($\beta = 0.28$, sec ref. 22)), represented by a solid triangle at $\zeta = 27.74$ in fig. 3. For this system $\Delta E$ deviates strongly from $\Delta B$. This discrepancy can be explained in terms of the coupling to rotational channels discussed above. In order to illustrate this point we show in fig. 7a the values of
\[ \Delta E \text{ in comparison to the sum } \Delta B + \Delta E_{\text{rot}} \text{ for the fusion of } ^{40}\text{Ar} \text{ with three Sm isotopes. We notice that the sum of the two contributions to the asymptotic energy shift agrees very well with the values extracted from the data in all cases. In this case the effect of the deviation from the average trend is fully explained by the static deformation of the Sm isotopes. In the following subsection we show how the dynamic deformations that arise in vibrational motion may help explaining the fluctuations found for other systems.} \]

### 3.3. THE ROLE OF VIBRATIONAL STATES IN FUSION ENHANCEMENT

The role of vibrational states in the enhancement of sub-barrier fusion has been investigated by several authors \(^\text{4,7}\). The earliest attempts to study such effects were based on the sudden approximation, which assumes that the shape of the vibrational nucleus remains unchanged during the collision. The barrier parameters are dependent on the nuclear shape and the probability that the barrier radius is shifted from its average value by \( \Delta R \) is given by

\[
P_{\text{vib}}(\Delta R) = \frac{1}{\sqrt{2\pi s^2}} \exp \left[ -\frac{(\Delta R)^2}{2s^2} \right], \tag{17}
\]

with variance given by \(^\text{4)}\)

\[
s^2 = \sum_{\lambda} (\beta_\lambda R_0)^2 / 4\pi. \tag{18}
\]
In eq. (18) $R_0$ is the average nuclear radius and $\beta_\lambda$ the vibration amplitude for the $\lambda$-mode. The sum is restricted to slow modes as discussed by Braun-Munzinger and Berkowitz. The fusion cross section is given by the integral

$$\sigma_F = \int_{-\infty}^{\infty} \sigma_F(\Delta R) P_{\text{vib}}(\Delta R) \, d(\Delta R),$$  \hspace{1cm} (19)

where $\sigma_F(\Delta R)$ is the fusion cross section obtained by changing the radius of the barrier by $\Delta R$. Using Wong’s formula for $\sigma_F(\Delta R)$ and taking the low energy limit, $\sigma_F$ can be put in the form of eq. (13) with the replacement of $\Delta E_{\text{rot}}$ by the asymptotic energy shift due to zero-point motion, $\Delta E_{\text{ZPM}}$

$$\Delta E_{\text{ZPM}} = \pi g^2 \left( \frac{s^2 V_B^2}{R_B^2 \hbar \omega} \right).$$  \hspace{1cm} (20)

Eq. (20) can be trivially extended to the situation where both projectile and target have vibrational spectra. In this case the variance is given by

$$s^2 = s_1^2 + s_2^2,$$  \hspace{1cm} (21)

where the indices 1 and 2 refer to each of the collision partners.

We have evaluated $\Delta E_{\text{ZPM}}$ for the fusion of $^{74}\text{Ge} + ^{74}\text{Ge}$. In this case the coupling to vibrational motion is very important since the quadrupole vibration amplitude is quite large ($\beta_2 = 0.28$, see ref. 24)). We find $\Delta E_{\text{ZPM}} = 23$ MeV, which by itself exceeds the empirical value $\Delta E = 14$ MeV (indicated by an open circle at $\zeta = 27.68$ on fig. 3). The situation is even worse since this is a nuclear structure effect which should be added to the background enhancement for a comparison with $\Delta E$. We believe that the reason for this discrepancy lies in the use of the sudden approximation which for vibrations is much poorer than for rotations. It should be mentioned that some earlier calculations of zero-point motion effects on sub-barrier fusion using the sudden approximation were in reasonable agreement with experiment. This agreement was, however, accidental, since it resulted from an inappropriate truncation in the $\Delta R$ integration of eq. (19), as discussed in detail in ref. 13).

Here we resort to the coupled channels treatment of Dasso and Landowne. These authors expand the radial wave function in a set of eigenchannel states $|\chi_j(r)\rangle$, defined, at each separation $r$, by the matrix equation

$$W(r)|\chi_j(r)\rangle = \lambda_j(r)|\chi_j(r)\rangle.$$  \hspace{1cm} (22)

Above, $W(r)$ is the coupling operator given by the difference between the full hamiltonian and its projection on the space of the elastic channel. If we work in a truncated functional space containing up to two-phonon states, eq. (22) reduces to a three-dimensional matrix equation and the fusion cross section is given by the approximate expression

$$\sigma_F(E) = \sum_{j=1}^{3} P_j \sigma_j(E).$$  \hspace{1cm} (23)
In eq. (23) $\sigma_j$ is the fusion cross section calculated with the effective one-dimensional barrier $V(r) + \lambda_j(r)$ and $P_j = |\langle \chi_j(R_B) | 0 \rangle|^2$ is the squared projection of the entrance channel state onto the $j$th eigenchannel wave function. As in ref. 27), we assume that $|\chi_j(r)\rangle$ has a slow $r$-dependence in the barrier region and use $|\chi_j(R_B)\rangle$ to determine $P_j$. This approach differs from the constant coupling approximation of Dasso et al. 27) in that the latter neglects the $r$-dependence of both $|\chi_j(r)\rangle$ and $\lambda_j(r)$.

In the present study, we are concerned with energies much below the Coulomb barrier so that the sum in eq. (23) is dominated by the term with the lowest eigenvalue $\lambda_1$ and the cross section $\sigma_1$ can be approximated by the low energy limit of Wong's formula. Assuming that the addition of $\lambda_1(r)$ reduces the barrier height by an amount $\delta$ and produces a small shift in barrier radius, without appreciably changing the curvature $\hbar \omega$, the fusion cross section can be put in the form of eq. (13), with the replacement of $\Delta E_{\text{rot}}$ by $\Delta E_{\text{vib}}$

$$\Delta E_{\text{vib}} \approx \delta + \frac{\hbar \omega}{2 \pi} \ln P_1 .$$

(24)

We have calculated the $\Delta E_{\text{vib}}$ values for fusion reactions involving $^{74}$Ge, namely: $^{58}$Ni + $^{74}$Ge, $^{64}$Ni + $^{74}$Ge and $^{74}$Ge + $^{74}$Ge. We have found the values 2.7, 2.7 and 5.0 MeV, respectively. It should be mentioned that the asymptotic shift 5.0 MeV found in the $^{74}$Ge + $^{74}$Ge case is in agreement with that obtained by Esbensen and Landowne 28), solving the coupled channels equations. In fig. 7b we compare the asymptotic shifts $\Delta E$ of fig. 3 to $\Delta B + \Delta E_{\text{vib}}$. We observe that there is good agreement in the first two cases, while for the $^{74}$Ge + $^{74}$Ge system the sum falls short of the observed value. We believe that this discrepancy can be traced back to the singular behavior of the fusion data at energies above the barrier. As observed in fig. 4, the barrier height extracted from this fusion data deviates strongly from the average trend. In fact, an estimate of $\Delta E$ which uses the average trend of the barrier height, represented by the straight line of fig. 4, yields the value $\Delta E \approx 9.5$ MeV which agrees very well with the sum $\Delta B + \Delta E_{\text{vib}}$ shown in fig. 7b. In this regard it is important to mention the recent work of Iwamoto and Takigawa 9) which discusses the sub-barrier fusion enhancement for the $^{74}$Ge + $^{74}$Ge system in a more fundamental approach. These authors use a microscopic description of the neck degree of freedom based on a two-centered shell model 29) to calculate the potential barrier height as a function of the nuclear deformation. They find that the softness of $^{74}$Ge against deformation plays a fundamental role in the $^{74}$Ge + $^{74}$Ge fusion. In this case the shell and quadrupole deformation energies cooperate in reducing the fusion barrier by 7.4 MeV. When added to $\Delta B$ this barrier lowering leads to a good approximation to the empirical $\Delta E$ value. In conclusion, the large experimental fusion enhancement evidenced by this system is attributed to the effect of a specific structural property of the $^{74}$Ge nucleus. The situation is therefore quite similar to that encountered with the other systems showing major deviations from the average behavior which were discussed before.
3.4. NEUTRON TRANSFER AND FLOW IN FUSION ENHANCEMENT

Inspection of fig. 6 shows other systems exhibiting appreciable departures from the predictions of the neck-formation model. One of such systems is the $^{36}$S + $^{58}$Ni. In this case, the large value of $\Delta E$ may result from the anomalous value of the parameter $V_b$ extracted from the fusion data above the Coulomb barrier, as discussed in the previous section. Of the remaining systems the most noticeable deviation occurs for $^{58}$Ni + $^{64}$Ni. It is not reasonable to attribute this deviation to structure effects like those considered in the previous subsections, since the symmetric systems $^{58}$Ni + $^{58}$Ni and $^{64}$Ni + $^{64}$Ni show much smaller discrepancies. In this case Broglia et al. have shown that the transfer of one or more neutrons has an important effect on the fusion cross section enhancement, specially for reactions with a positive $Q$-value.

We could in principle follow the procedure adopted in the previous subsections and calculate the additional contribution to the asymptotic energy shift that arises from coupling to transfer channels. However, there are large uncertainties in the neutron-transfer form factors involved, such that the resulting energy shifts $\Delta E_i$ cannot be reliably calculated. In most cases the uncertainty in $\Delta E_i$ (fig. 12 in ref. 36) is even larger than the deviation.

It is interesting anyhow, to relate the neck-formation mechanism to the microscopic flow of nucleons between the collision partners. The model of Iwamoto and Harada deals explicitly with this point, since it describes neck formation through the microscopic nucleon motion during the collision. A more recent work along this line of thought is that of Stelson. This author claims that the neutron flow between the collision partners is the main enhancement mechanism, and that coupling to collective states plays only a secondary role. In his work, Stelson assumes a flat distribution of barrier heights and calculates an average cross section. This distribution has as lower limit the nucleus-nucleus potential at a critical separation $R_L$, where the valence neutrons find no barrier to flow freely between projectile and target. This neutron current marks the onset of the fusion process. As this separation is larger than the barrier radius $R_B$ the fusion threshold is lowered by the amount

$$\delta_t = V(R_B) - V(R_L).$$

The critical distance $R_L$ is defined in ref. 10 as that for which the sum of the neutron shell-model potentials has a maximum value on the line connecting the nuclear centers, $V(n)_{\text{max}}$, that lies a few MeV deeper than the average binding energy of the valence neutrons.

In what follows we study the general trend with system size of the barrier lowering due to the appearance of the neutron flow. For this we took a constant value of $V(n)_{\text{max}} = -15$ MeV, which is the value taken by Stelson for the $^{58}$Ni + $^{58}$Ni system. We have assumed a uniform barrier-height distribution from $V_B - \delta_t$ to $V_B + \delta_t$ and taken the low energy limit of Wong's formula. This last assumption is justified since we are, as before, interested in the low-energy limit of the fusion cross section.
which defines the asymptotic energy shift. We find that the enhancement due to the neutron flow is given by

$$\Delta E_t = \frac{\hbar \omega}{2\pi} \ln \left[ \frac{\hbar \omega}{2\pi \delta_i} \sinh \left( \frac{2\pi \delta_i}{\hbar \omega} \right) \right],$$

(26)

where $\delta_i$ is the half width of the barrier-height distribution, given by the difference between the Coulomb barrier $V_b$ and the nucleus-nucleus potential at the separation where the neutron potential reaches the value $V(n)_{\text{max}} = -15 \text{ MeV}$. In fig. 8 the calculated $\Delta E_t$ values are plotted against the system size parameter $\zeta$, in comparison with the neck-formation energy shift $\Delta B$. We note that both energy shifts show the same monotonically growing trend with the system size. However, there are large differences in the predictions for the heaviest systems considered. This discrepancy could be due to our choice of the parameter $V(n)_{\text{max}}$ or the radius parameter $r_0 = 1.24 \text{ fm}$ for all systems. The qualitative agreement between the predictions of these two models could be an indication that there is a close connection between the macroscopic picture, based on a neck degree of freedom, and the microscopic neutron transfer and flow processes. Clearly more work along this line is needed to settle this point.

4. Conclusions

We have carried out a systematic study of the sub-barrier fusion-enhancement phenomenon. In this study we give theoretical interpretations for the asymptotic energy shifts $^8$) extracted from fusion data for a variety of heavy-ion systems.
We claim that the empirical asymptotic energy shift contains two terms. A background contribution, which depends on the liquid-drop properties of finite portions of nuclear matter, and fluctuations with respect to this background, arising from specific nuclear structure properties of the collision partners.

The background contribution can be approximately described by the neck-formation model of ref. 6), as discussed in ref. 8). The main deviations from this background were shown to arise from static deformation and from vibrational motion of the projectile and/or the target. A simple treatment of the static deformation effects was proposed on the basis of the sudden approximation. We found that deformation leads to an additional contribution to the asymptotic energy shift given by the analytical expression of eq. (16). This treatment was applied to the fusion of $^{40}$Ar with deformed Sm isotopes and the large deviations from the neck-formation model predictions were fully accounted for. The effects of vibrational motion were also considered in detail. Using the coupled-channel treatment of Dasso and Landowne 27) we have calculated the contribution of vibrational motion to the asymptotic energy shift. As in the case of static deformation, this contribution is given by a simple expression (eq. (24)). This treatment was used to explain the deviations found in the fusion of $^{74}$Ge with different targets. We were successful with the $^{58}$Ni+$^{74}$Ge and $^{64}$Ni+$^{74}$Ge cases but the vibrational contribution fell short from the deviation found in the $^{74}$Ge+$^{74}$Ge system. This discrepancy was attributed to its anomalous fusion barrier extracted from the data.

An attempt to relate neck-formation effects to the microscopic flow of nucleons between the collision partners has been made. For this purpose we have compared the barrier lowering due to neck formation to that arising from neutron flow. Although these quantities do not quantitatively agree, they were shown to be clearly correlated.

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