In this chapter we begin to investigate the broad area of physics concerned with electromagnetic radiation and waves. The next four chapters discuss various aspects of optics, the science of light. Here we first introduce electromagnetic waves and some of their properties including their structure, energy, and momentum. Laser (or optical) tweezers is an exciting new technique that allows manipulation of microscopic structures or of individual macromolecules even within living cells. We introduce the technique based on the momentum contained in an electromagnetic wave, and show that laser tweezers represents a novel rapidly growing experimental technique. A brief discussion of photons, the elementary quanta of electromagnetism, and the notion of wave–particle duality are given in order to understand the basis for a large array of spectroscopic techniques using the various portions of the electromagnetic spectrum.

1. ELECTROMAGNETIC WAVES

Electromagnetic (EM) radiation is created whenever charges accelerate. This occurs, for example, when time-varying currents run up and down the transmitter of a radio station or when atoms bounce around inside a fluorescent light bulb. The “news” that acceleration has occurred travels outward at the speed of light \( c \). Figure 19.1 is a picture of a sphere of radius \( r \) surrounding a negatively charged electron. At time \( t \), the radiation \( E \)-field is measured everywhere on the surface of the sphere. That field (a few of the vectors are shown in red) is due to the acceleration of the electron at a time earlier than \( t \). The relevant earlier acceleration is shown in the figure (in black). The time at which the fields shown were created is the present time \( t \) minus the time necessary for the radiation to travel a distance \( r \); that is, \( t - r/c \). At that earlier time the electron was at the center of our sphere.

The electric field radiated by the electron in Figure 19.1 has a magnitude given by

\[
E_{\text{rad}} = \frac{e}{4\pi\varepsilon_0 c^2 r} \sin (\theta) a(t - \frac{r}{c}),
\]

where \( e \) is magnitude of the electronic charge, \( \theta \) is the smallest angle between the acceleration direction and the line connecting the charge to the point of observation, and \( a(t - r/c) \) is the value of the charge’s acceleration at the time \( r/c \) before the present time \( t \). Because of the \( \theta \) dependence, the magnitude of the field is a maximum on the equator of the sphere (where \( \sin(90^\circ) = 1 \)) and zero at the poles (where \( \sin(0^\circ) = \sin(180^\circ) = 0 \)). In other words, if you look at a charge directly along its line of acceleration you don’t see any radiation; the maximum radiation is observed at right angles to the acceleration. The radiation \( E \)-field vectors are tangent to the sphere everywhere and point as shown. They are always perpendicular to the direction of propagation of the radiation.

The \( E_{\text{rad}} \) field of Equation (19.1) decreases with distance from the source as \( 1/r \) and not as the usual \( 1/r^2 \) dependence of the electrostatic field. We can demonstrate that this
must be true from the following geometric argument. The oscillating electron supplies energy at a certain constant rate so that the power \( P \) carried by the radiation is constant. Remember that power is proportional to the intensity, which is itself proportional to the square of the field. As this energy is carried away by the spherical radiation wave traveling at a constant speed \( c \), the total amount of energy crossing any spherical surface per second must be the same. As shown in Figure 19.2, because the surface area of a sphere increases with the square of the radius (remember that the surface area of a sphere of radius \( r \) is given by \( A = 4\pi r^2 \)), the total energy crossing a spherical surface per second at two different radii \( r_1 \) and \( r_2 \) can only be equal if the intensity decreases as \( 1/r^2 \). This follows because if the intensities \( I_1 \) and \( I_2 \) represent those at radii \( r_1 \) and \( r_2 \), then we must have \( P = I_1 4\pi r_1^2 = I_2 4\pi r_2^2 \), so that \( I \propto 1/r^2 \). From this, we can conclude that the \( E \)-field then must vary as \( \sqrt{1/r^2} = 1/r \) in agreement with the above expression for \( E_{\text{rad}} \).

Often electrons have heavy, positively charged, protons nearby (as in an atomic nucleus, e.g.). The electric field causing the electrons to accelerate will also cause the protons to accelerate, but because the protons are 2000 times more massive, their motion usually can be ignored. In that case, the total electric field near a neutral glob of matter equals the static Coulomb fields of the protons plus the static Coulomb fields of the electrons plus the radiation fields of the electrons. The static fields approximately cancel out, leaving just the radiation fields. That’s why we can’t measure the static electric field of a galaxy 10 billion light years from Earth, but can detect its radiation field just fine (as light, with our eyes; oh, okay, and maybe a telescope).

If the electron in Figure 19.1 oscillates up and down periodically, it emits a continuous series of concentric spheres of radiation, each with radius expanding at the speed of light. Some of these spheres are shown at one instant in Figure 19.3. A few \( E \)-vectors are also shown. \( E \) is large near the oscillating electron and smaller farther away. Suppose the period of oscillation of the electron is \( T \) seconds. The concentric spheres shown are generated every \( T/2 \) seconds and the distance between them is \( cT/2 \). If one sits at a fixed point in space, such as \( P \) in the figure, the sequence of \( E \)'s that pass through there will vary sinusoidally in time with a fixed amplitude. This passing wave of \( E \) is transverse. Furthermore, if we look at \( E \) at any instant over a small planar patch that is tangent to any one of the spheres of radiation, the value of \( E \) will be about the same everywhere in the patch. The size of such a patch of uniform \( E \) will be small in close to the radiating electron and

![FIGURE 19.1](image-url) A map of the \( E \)-field on a sphere of radius \( r \) due to an accelerating electron shown at an earlier time, \( t = r/c \), located at the center.

![FIGURE 19.2](image-url) A point source radiating at the center of two concentric spheres showing that the energy passing through \( A_1 \) and \( A_2 \) per second must be the same and thus that the intensity must decrease as \( 1/r^2 \).

![FIGURE 19.3](image-url) A series of spherical waves of radiation emanating from an oscillating electron at the center. Oscillating \( E \)-fields with decreasing amplitude with distance from the center are shown.
will be larger the farther out one goes. A set of \( E \)-vectors that are all the same at one instant over a plane is called an “electric field plane wave.”

Figure 19.4 depicts a series of time-lapse photographs of the electric field, produced by the radiating electron in Figure 19.3, over a small planar patch that is perpendicular to the direction of propagation of the radiation. Read the panels in cartoon fashion: left to right, top to bottom. Thus, at first \( E \) is large and pointing up, then small and up, then small and down, then large and down, then small and up, then large and up (one complete cycle after the first panel), and so on. The period of the oscillation equals “six panels” in this strip. The cartoon keeps running along in the same way, over and over. Because the electron in Figure 19.3 is always oscillating vertically, the \( E \)-fields in the panels of this strip are also always pointing vertically because we are at point \( P \) on the sphere’s equator (see Figure 19.1). Such a plane wave is said to be “linearly polarized.” More typically, the source of electromagnetic radiation is a large number of electrons that tumble about irregularly. Usually, such electrons will vibrate in concert with each other for only a short time. The cartoon for radiation from a collection of tumbling electrons will have panels where \( E \) is vertically oriented for a while, then oriented at some other angle, then oriented at another angle, and so on, with no connection between panels. Such a plane wave is said to be “unpolarized.”

Because time-varying \( E \) makes \( B \), there is a magnetic field that travels along with \( E \). When \( E \) is large, so is \( B \). When \( E \) is zero, so is \( B \). The magnetic field is perpendicular to \( E \) and both are perpendicular to the direction of propagation. The direction of \( B \) at any moment can be determined by the following usual right-hand rule equivalent to \((\mathbf{E}, \mathbf{B}, \mathbf{v})\) forming a right-handed coordinate system such as \((x, y, z)\): place the thumb of your right hand in the direction of propagation of the wave and your extended fingers in the direction of \( E \); curl your right-hand fingers \( 90^\circ \) to point in the direction of \( B \).

When far from the source, these fields can be described as plane waves shown schematically in Figure 19.5. Here an electromagnetic plane wave is shown to be composed of oscillating electric and magnetic fields traveling along the \( x \)-axis. Both \( \mathbf{E} \) and \( \mathbf{B} \) are found to lie in a transverse plane, perpendicular to the \( x \)-direction along which the wave travels. Furthermore, \( \mathbf{E} \) and \( \mathbf{B} \) are also perpendicular to each other and oscillate together, in phase with each other, as the wave travels along at speed \( c \). We can write each of the magnitudes of the fields in the form of traveling waves (as in Section 3 of Chapter 10)

\[
E(x, t) = E_{\text{max}} \sin \left( \frac{2\pi}{\lambda} x - \omega t \right), \\
B(x, t) = B_{\text{max}} \sin \left( \frac{2\pi}{\lambda} x - \omega t \right),
\]

where, as before,

\[
k = \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = \frac{2\pi}{T},
\]

with \( \lambda \) the wavelength and \( T \) the period of the wave (\( T = 1/f \) where \( f \) is the frequency of oscillation of the wave). The speed of the wave is given by \( v = c = \omega/k \), (equivalent to \( c = \lambda f \)).
see Equation (10.10)). As a consequence of Maxwell’s equations, the values of $E_{\text{max}}$ and $B_{\text{max}}$ are related to each other as

$$\frac{E_{\text{max}}}{B_{\text{max}}} = c. \quad (19.3)$$

We show shortly that this expression leads to the fact that both the $E$ and $B$ fields carry the same contribution to the total energy of the wave.

Remember that Figure 19.5 is like four snapshots of the fields (if they were somehow made visible) traveling along the $x$-axis. As time ticks on, the wave shown will move along the $x$-axis at speed $c$ with the $E$ and $B$ fields oscillating with period $T$ at any particular $x$ location, as in Figure 19.4 for the $E$-field. The wave also has some spatial extent in the transverse plane (not shown). A plane wave has a flat or plane wavefront (the locus of all points at which $E$ is in phase; e.g., all the crests of the $E$ wave). For this type of idealized wave, the amplitudes in Equation (19.2) are constants, not varying with the distance the wave has traveled (Figure 19.6).

Recall that at the end of the previous chapter we made an analogy between EM waves and waves on a string. There we loosely associated the velocity of the string with $E$ and the slope of the string with $B$. The correct analogies are that $E$ corresponds to transverse velocity and $B$ corresponds to stretch of the string. On a string, velocity is associated with kinetic energy and stretch is associated with potential energy. For a traveling wave on a string these energies are at a maximum together and they travel at the wave speed. The same is true of EM waves. The $E$ part of the energy and the $B$ part are in phase, having maxima together and zeroes together, and they both travel at the speed of light.

We have seen that there is an energy density associated with an electrostatic field given by Equation (15.22),

$$\frac{\text{PE}}{V} = \frac{1}{2} \varepsilon_0 E^2.$$

It can be shown that energy can also be stored in a magnetostatic field and that the energy density associated with $B$ is given by

$$\frac{\text{PE}}{V} = \frac{1}{2} \frac{B^2}{\mu_0}.$$

Given this, it should not be surprising that an electromagnetic wave made from oscillating $E$ and $B$ fields also has an associated energy density given by

$$\frac{\text{PE}}{V} = \frac{1}{2} \left( \varepsilon_0 E^2 + \frac{B^2}{\mu_0} \right), \quad (19.4)$$

where the $E$ and $B$ fields vary sinusoidally. We can rewrite this expression using Equation (19.3) to substitute for $B = E/c$ and the fact that $\mu_0 = 1/(\varepsilon_0 c^2)$ (from Equation (18.11)), where we have also used the fact that $E$ and $B$ are in phase so that Equation (19.3) holds not only for the maximum values but at all times. We then have that $B^2/\mu_0 = \varepsilon_0 E^2$, so the energy density can be rewritten as

$$\frac{\text{PE}}{V} = \frac{1}{2} \left( \varepsilon_0 E^2 + \varepsilon_0 E^2 \right) = \varepsilon_0 E^2. \quad (19.5)$$

Because each of the terms in the bracket in Equation (19.5) is equal, the electric and magnetic fields each contribute equally to the total energy density of the EM wave.

As an EM wave moves with speed $c$ in vacuum, it carries energy. If we imagine such a wave traveling along the $x$-axis (Figure 19.7), then in a time $\Delta t$ the wave will move a distance of $c \Delta t$. In that time a volume of the wave equal to $Ac \Delta t$ will sweep...
through a cross-sectional area \( A \) perpendicular to the wave velocity. The energy transported by the wave in time \( \Delta t \) through the area \( A \) is then

\[
\left( \frac{\text{PE}}{V} \right) A c \Delta t = \varepsilon_0 E^2 A c \Delta t,
\]

so that the EM wave energy per unit time per unit area, \( S \), is equal to

\[
S = c \frac{\text{PE}}{V} = c \varepsilon_0 E^2.
\]

\( \vec{S} \) is known as the Poynting vector. It points in the direction of travel of the wave, and is measured in units of J/s/m^2, or W/m^2. Because \( E \) is taken as a sinusoidal function, \( S \) varies with time as well.

The average value of \( S \) represents the intensity \( I \) of the wave, or the mean energy flow per unit time per unit cross-sectional area. Intensity is important because that’s what the eye detects when the EM radiation is in the visible range. Because \( E \) is given by Equation (19.2) and the average of the function \( \sin^2(x) \) over one period is equal to \( \frac{1}{2} \), we have that

\[
I = \frac{1}{2} \varepsilon_0 c E_{\text{max}}^2.
\]

The intensity of EM waves is a measurable quantity; detectors can measure the amount of energy per unit time and per unit area that reach them. On the other hand, the Poynting vector fluctuates with time, often much too fast to be detected directly. For example, visible light has a frequency of about \( 10^{15} \) Hz. In order to detect such rapid fluctuations a time resolution of about \( 1/10^{15} \) s, or 1 fs, is required and this is just at the edge of our current abilities.

**Example 19.1** An EM plane wave traveling along the \( x \)-axis has an effective cross-sectional area of 1.5 cm^2, a maximum electric field of 1500 N/C and a frequency of \( 4 \times 10^{15} \) Hz. Find each of the following quantities: its maximum \( B \) field, energy density, an expression for the Poynting vector, the intensity of the wave, and the energy striking a 0.5 cm^2 area with its normal along the \( x \)-axis in 10 s.

**Solution:** We first find the amplitude of the magnetic field from Equation (19.3) to be \( B = E/c = 5 \times 10^{-6} \) T. These values then allow us to calculate the energy density, from Equation (19.4) to be \( 2 \times 10^{-5} \) J/m^3. Alternatively we can use Equation (19.5) directly to find the same result. The Poynting vector then has an amplitude given by Equation (19.6) to be \( S_{\text{max}} = (\text{PE}/V)c = 6000 \) W/m^2 along the \( x \)-axis and varies at the very high frequency of \( 4 \times 10^{15} \) Hz. It can be written as \( S = 6000 \sin (2\pi \cdot 4 \cdot 10^{-15}t) \) and is directed along the \( x \)-axis. Its average value over time is the intensity given by Equation (19.7) as \( I = \frac{1}{2} S_{\text{max}} = 3000 \) W/m^2. Finally, if this wave strikes the given surface, the power reaching the surface is just \( P = IA = 0.15 \) W, so that in 10 s the energy absorbed will be 1.5 J.

**Example 19.2** If the maximum intensity of an EM wave is 1000 W/m^2 (about what it is in sunlight reaching the Earth), what is the maximum \( E \)?

**Solution:** Solving for \( E \) from Equation (19.7), we find \( E = \sqrt{2I/e_0 c} = 870 \) V/m, or about 9 V/cm. Such a field drives as much current through a cm of skin as a 9 V battery!
When an electric radiation field encounters a charge, it makes the charge jiggle with the same frequency as the radiation. A jiggling charge is accelerating, so it radiates as well, emitting “induced” radiation with the same frequency as the “incident” radiation. This induced radiation travels outward in all directions. The total $E$ observed at any point in space is the vector sum of all $E$’s, the incident $E$’s as well as all induced $E$’s. This is just the superposition principle for electric fields that we’ve discussed previously. We return to this in Chapter 22.

In addition to carrying energy, an electromagnetic wave also carries linear momentum, and hence can exert a force. Although the amount of momentum or force is usually small compared to ordinary forces we experience, the force generated by an intense light beam, for example, from a laser, is enough to provide an upward force on small particles to balance their weight and suspend them in air. The pressure exerted by EM waves is known as radiation pressure. Figure 19.8 shows a small drop of glycerol being suspended in water by the radiation pressure of a laser beam. The possibility of “trapping” micron-sized spheres was first demonstrated in the early 1970s. In the next section we discuss a new technique that uses radiation pressure to allow the direct manipulation of microscopic objects.

2. LASER TWEEZERS

First conceived and developed in the mid-1980s by Ashkin and colleagues at AT&T Bell Laboratories, laser, or optical, tweezers is a method of using radiation pressure to trap atoms, molecules, or larger particles. In applications with the simplest possible arrangement using a single laser beam, particles with sizes in the range of several hundred microns down to about 25 nm can be “trapped” and moved about using the radiation pressure of the EM radiation. How does radiation pressure trap such particles?

If a plane electromagnetic wave is incident on a particle, the radiation pressure on the particle would be such as to propel it along the direction of the beam. This is due to the fact that the reflected wave results in a net decrease in forward momentum of the wave. Conservation of momentum for the system composed of the EM wave and the particle then dictates that the particle must sustain a forward momentum. This process is responsible for the suspension of micron-sized spheres in gravity as in Figure 19.8 when the beam intensity is adjusted so as to just balance the sphere’s weight. A higher intensity beam would propel the sphere upwards, whereas a lower intensity beam would allow the sphere to fall but at a reduced acceleration compared to $g$. This analysis does not as yet explain how a laser or optical tweezers can trap a particle.

Consider a transparent sphere with dimensions large or comparable to the wavelength of the EM wave that impinges on it. We show in the next
chapter that at an interface between different materials, EM waves are bent, or refracted, as shown in Figure 19.9. This process is studied in detail later; for now, all we need to know is that the arrows drawn to represent the direction of travel of the wave will be bent at each surface. Refraction of an EM wave is the basis for laser tweezers. In Figure 19.9, the EM wave is shown coming to a focus above the sphere and two rays are drawn to typify the path of the wave. As shown in the insert, if the change in momentum of the wave is determined for this situation, the net change is in the forward direction, so that there is an equal and opposite change in the sphere’s momentum resulting in a net force on the sphere in the opposite direction, toward the focus point. Similarly if the focus point is below the sphere, there will be a restoring force due to the radiation pressure directed again toward the focus point. These longitudinal forces directed toward the focus point act to stabilize or “trap” the particle longitudinally.

Figure 19.10 shows the same arrangement with the sphere off-center but with the beam having a variation in intensity across its cross-sectional area. A similar momentum change analysis, shown in the insert, reveals that in addition to a force toward the focus point, in this case downward, there will also be a transverse force directed toward the more intense portion of the beam along its axis. In a real single beam laser tweezers arrangement, the beam will have its maximum intensity at the center and the sphere would be trapped transversely to lie along the center and at the focus point.

Typical forces capable of being exerted are in the pN ($10^{-12}$ N) range. In order to move the trapped particle about, either the laser beam itself or the sample, sitting on a microscope stage, is moved. An experimental station uses a good quality inverted microscope with an optical port for the laser as shown in Figure 19.11. Usually near-infrared

![Figure 19.10](image19.10.png)  
**FIGURE 19.10** Similar to the previous figure, but with the laser beam having a typical transverse intensity profile as shown and with the sphere off-center and above the focus point. The insert shows the momentum change of the extreme laser beam rays, now no longer symmetric. This analysis shows that, because the intensities are not symmetric, there will be a net change in the light momentum as shown and so the sphere feels an oppositely directed reaction force not only along the beam toward the focus point, but also transversely toward the beam axis.

![Figure 19.11](image19.11.png)  
**FIGURE 19.11** (left) Schematic of a basic laser tweezers experimental setup; (right) Laser tweezers experimental station with microscope at far end of optical table.
laser light with a wavelength of about 1 μm is used with biological samples. Although the beam is invisible to the human eye and therefore needs to be detected with an infrared-sensitive CCD camera for recording, its use usually avoids the problems of light absorption and subsequent heating of samples that can occur when using visible light. The sample can be directly viewed through the usual microscope eyepiece using a standard visible light source of the microscope. Care must be exercised to ensure that the laser beam is not directed on the sample when viewing by eye because the beam is invisible and, for a sufficiently intense laser beam, can cause damage to the eye. Laser beams have a cross-sectional intensity profile that is bell-shaped with a maximum in the center and therefore automatically act to trap particles in the transverse direction according to the above discussion.

Optical tweezers of biological samples takes place in an aqueous solvent, whether a solution in which the biomolecules of interest are placed or the cytoplasm of a cell. As the trapped object is moved, there will be an immediate viscous drag force acting that will balance the trapping force, so that the object will move at a constant velocity. This Stokes’ drag force (see Equation (9.6))

\[ F_j = 6\pi r \eta v \]

can be used to calibrate the trapping force achieved at a particular laser intensity and beam geometry. By measuring the maximum velocity with which a micron-sized plastic sphere of known radius can be dragged by the laser tweezers, the trapping force can be determined by its balance with the Stokes’ force because the viscosity and sphere radius are known. In this way, the maximum applied trapping force can be found as a function of the laser intensity.

If a sphere is attached to one end of a linear macromolecule such as DNA or a filamentous protein such as actin and the other end of the macromolecule is immobilized, laser tweezers can be used to stretch the macromolecule a given distance and measure the minimum applied trapping force at which the sphere just “pops out” of the trap (Figure 19.12). Under this condition, the applied trapping force is just equal to the force the macromolecule is exerting on the sphere, allowing a measurement of the elastic force exerted by the macromolecule.

There are already many applications for which laser tweezers have been used. Individual motile (swimming) organisms, such as E. coli bacteria, have been trapped by laser tweezers while they continue to live normally with flagella beating (Figure 19.13). Similarly laser tweezers can be used to manipulate subcellular organelles within a living cell. The IR laser passes through the cell membrane and can trap large organelles or structures, such as individual chromosomes, that can then be moved about. It can also be used to exert forces directly on a cell membrane (Figure 19.14). More quantitative measurements of forces have been made with laser tweezers as mentioned in the last paragraph. In this way the stiffness and breaking strength of such molecules can be measured. Time-resolved measurements on trapped micron-sized plastic spheres attached to the ends of an actin filament or a microtubule have been made to study the forces generated when single molecules of either myosin or kinesin move along the respective filaments (Figure 19.15). These motility assays have recently achieved measurements at subpiconewton force and nanometer displacement resolutions with a time resolution of about 1 ms, allowing the results of single molecule interactions to be studied in great detail.
3. POLARIZATION

We have seen that electromagnetic waves are transverse waves with their electric and magnetic fields both lying in the transverse plane (perpendicular to the wave velocity) and also perpendicular to each other. The direction of the electric field is known as the polarization direction of the wave. If, as the wave propagates, the electric field remains along the same direction in space, the wave is said to be linearly polarized. Sometimes, in this case, the wave may be referred to as vertically or horizontally polarized if the $E$ field points in either of those directions.

**FIGURE 19.14** Laser tweezers applied at the arrow exert mechanical forces on a membrane, stretching it from its undeformed contour (dashed curve).

**FIGURE 19.15** A time series of phase contrast images of a microtubule, deflected by laser tweezers (visible as a set of four bright spots), returning to its unbent position. The images are taken at $t = -0.04$ s (upper left; the laser is switched off at $t = 0$), $t = 0.12$ s (top right), $t = 0.52$ s (bottom left), and $t = 1.0$ s (bottom right). Laser tweezers can give information about the forces and displacements involved and can be used to micromanipulate individual macromolecules.
In more general terms, because \( \mathbf{E} \) is confined to the transverse plane, there are two independent orthogonal directions possible, let us say the \( x \)- and \( y \)-axes as shown in Figure 19.16. Depending on the source of the EM radiation, the electric field may or may not have a definite polarization direction. For example, light from the sun, from a flame, or from incandescent or fluorescent light bulbs has its origin in the independent motions of huge numbers of atoms or molecules and has no particular polarization direction. Such light is said to be unpolarized, meaning that the superposition of the \( E \) field directions from all of the individual sources of light (atoms/molecules) leads to a random orientation for \( E \) as a function of time.

Various methods can be used to change the polarization properties of EM radiation. We discuss some of these in more detail in our discussions of optics in the next chapters. Here we illustrate one particular method, the use of an absorption polarizer such as a sheet of Polaroid, for producing a linearly polarized light wave from unpolarized light. Polaroid sheets contain long chains of organic molecules that are preferentially oriented in one direction. When incident unpolarized light falls on such a sheet, the oscillating electric field component along the chain direction is preferentially absorbed because, simply put, the electrons are able to move along that direction and take up some of the energy corresponding to light polarized along the chains. In contrast, the electric field polarized perpendicular to the chains is not able to interact strongly with electrons because they have more limited mobility in the transverse direction and this electric field polarization passes directly through the otherwise transparent sheet. The net effect is that after passing through a Polaroid sheet, unpolarized light becomes linearly polarized along the Polaroid axis, which is perpendicular to the organic chain axis, as shown in Figure 19.17.

Other forms of EM radiation behave quite similarly, although the nature of the polarizer device will be different. For example, an unpolarized microwave beam (with a wavelength of several cm) can be polarized by passing it through a set of parallel wires, such as the metal baking rack used in a conventional oven. The polarization axis in this case is perpendicular to the wires for the same reason as in Polaroid film: electrons are better able to absorb the energy of the microwaves along the axis of the wire, leaving the transmitted microwaves preferentially polarized perpendicular to the wires.

Polaroid sunglasses function just as described above, having their polarization axis vertical. What is the advantage of Polaroid sunglasses over others that simply attenuate the total intensity regardless of the polarization direction? As was mentioned above, sunlight is unpolarized and so there would be no benefit in preferentially blocking one polarization direction over another in looking directly at sunlight. However, when unpolarized light reflects from a surface, more of the horizontally than vertically polarized light is reflected and this is commonly seen in the form of “glare”. As long as you hold your head upright, Polaroid sunglasses are quite effective in blocking this glare (Figure 19.18).

The polarization properties of a wave may be investigated using polarizers placed in the path of the wave. If we choose to orient a first polarizer with its axis vertical then regardless of the polarization of the incident wave, only vertically polarized waves will be transmitted through the polarizer, assuming an ideal polarizer. If the incident wave was unpolarized, then because on average horizontal and vertical polarizations are present in equal amounts, half the incident intensity will pass through the first polarizer. If the initial wave were already vertically polarized then all of its intensity would be transmitted.
whereas, on the other hand, if it were initially horizontally polarized no light would be transmitted.

Suppose that a second polarizer, often called an analyzer, is now placed in the path of the vertically polarized wave transmitted by the first polarizer. We can treat the vertically polarized wave as a superposition of two linearly polarized waves, one parallel and one perpendicular to the analyzer’s axis, making an angle \( \theta \) with that of the polarizer (see Figure 19.19). Only the parallel component will be transmitted through the analyzer. If the incident electric field on the analyzer is \( E_0 \), its transmitted component along the analyzer’s axis, is

\[
E_t = E_0 \cos \theta.
\]  

(19.8)

Clearly if the analyzer is rotated around, there will be two positions (\( \theta = \pm 90^\circ \)) at which the transmitted field vanishes and the polarizers are then said to be “crossed.” Because, according to Equation (19.7), the intensity is proportional to the square of the electric field, the transmitted intensity \( I_t \) is related to the incident intensity \( I_0 \) by

\[
I_t = I_0 \cos^2 \theta.
\]  

(19.9)

Equation (19.9) is sometimes known as the law of Malus.

\[ \text{Example 19.3} \]

A vertically polarized 0.15 W laser beam is incident on a polarizer with its transmission axis 45° to the vertical. The beam then passes through a second polarizer with its transmission axis horizontal. What is the intensity of the transmitted beam? What would be the intensity of the transmitted beam if the second polarizer were removed? What would it be with the second polarizer in place if the first polarizer were removed?

\[ \text{Solution:} \]

Using Malus’ law the intensity passing through the first polarizer will be \( I_1 = 0.15 \cos^2 45 = 0.075 \) W. Then the intensity passing through the second polarizer will be \( I_2 = I_1 \cos^2 45 = 0.038 \) W, because the second polarizer’s transmission axis makes a 45° angle with that of the first. If the second polarizer is removed, the transmitted intensity will simply be \( I_1 = 0.075 \) W, whereas if the first polarizer is removed there will be no transmitted intensity because the angle between the vertically polarized incident beam and the horizontal second polarizer is 90°.

In our discussions the transverse electric field vector was pictured in general as two orthogonal components along a pair of axes in the transverse plane. For unpolarized light each of these components varies randomly in time and, on average, has equal amplitude. For linearly polarized light along some arbitrary direction, the components of \( \vec{E} \) oscillate along two perpendicular axes.
in phase and add to give a resultant along a fixed linear direction as shown in Figure 19.20. Different relative magnitudes of the two components of \( \mathbf{E} \) result in linearly polarized light at different angles. For example, the larger the red component is compared to the blue, the closer the resultant \( \mathbf{E} \) points to the \( x \)-axis.

Another interesting type of polarization is known as circular (elliptical) polarization, where the \( \mathbf{E} \) field moves around the transverse plane in a circle (ellipse). We can think of this type of polarization as arising from \( x \)- and \( y \)-components of \( \mathbf{E} \) that are 90° out of phase as shown in Figure 19.21. Starting with the \( y \)-component at a maximum and the \( x \)-component equal to zero, as the \( y \)-component decreases the \( x \)-component increases; the \( x \)-component reaches a maximum when the \( y \)-component vanishes; and so on. With both \( x \) and \( y \)-components equal in magnitude, the \( \mathbf{E} \) field traces out a circular pattern in the transverse plane. In our example, the \( \mathbf{E} \) field traces out a counterclockwise directed circle in the transverse plane as viewed from the right. If the \( x \)- and \( y \)-components are unequal, elliptical polarization occurs. Note that the actual path of \( \mathbf{E} \) is a circular (or elliptical) helix in space as the wave moves along (as shown in Figure 19.21). These types of polarization are important in discussing the spectroscopic technique of circular dichroism (CD) in Chapter 23.

### 4. THE ELECTROMAGNETIC SPECTRUM

In our general discussion of electromagnetic waves in Section 1, we have seen that accelerating electric charges produce electromagnetic radiation in the form of transverse traveling waves of \( \mathbf{E} \) and \( \mathbf{B} \). There we introduced the notion of a frequency and wavelength, connected through their product with the speed of light \( c = f \lambda \), for the traveling wave expressions for \( \mathbf{E} \) and \( \mathbf{B} \). The range of possible wavelengths, or frequencies, is enormous. Figure 19.22 shows the electromagnetic spectrum with wavelengths, frequencies, and...
corresponding energies given using logarithmic scales. Electromagnetic radiation can arise through a large number of different types of processes, all having to do with accelerating electric charges. The broad categories of electric waves, radio waves, microwaves, infrared, visible, ultraviolet, x-ray, and gamma rays are used to distinguish the various parts of the EM spectrum that are produced in different ways. All electromagnetic waves are similar, in our classical wave picture, in having transverse electric and magnetic fields. They differ greatly not only in how they are produced but also in how they interact with matter.

We show in the next section that EM radiation, although for some considerations can be thought of as a classical wave, is actually composed of individual “wave-particles,” known as photons. These elemental quanta of energy have associated frequencies and wavelengths. The energy carried in each photon is proportional to its frequency. We saw this briefly in Equation (18.8) of the previous chapter in connection with RF photons in NMR. This proportionality is the origin of the energy scale labeled in Figure 19.22. Higher-frequency EM radiation corresponds to higher-energy photons.

The lowest-frequency oscillations of current along an antenna result in radio or TV signals. Associated frequencies range from about 10 kHz to about 1 GHz (10^9 Hz). Interestingly, radio signals can also arise from nuclear magnetic dipole transitions, as was discussed in connection with NMR (nuclear magnetic resonance) and MRI (magnetic resonance imaging) in the last chapter. The highest frequencies obtainable in oscillating electric circuits, up to about 10^{11} Hz, are associated with microwaves (including radar). We have seen that microwave radiation is also emitted in the phenomenon of ESR (electron spin resonance) in paramagnetic materials.

Still higher frequency radiation has its origin in various energy transitions in atoms, molecules, or nuclei. When an atom or molecule makes a transition from a higher energy state to a lower one, often the energy difference is emitted in the form of a photon. In macroscopic systems the number of emitted photons is enormous and constitutes electromagnetic radiation. The higher the frequency of radiation is, the larger the energy difference between the two transition energy levels, or states, involved. The lowest such energy transitions occur in different spin states within the nucleus (NMR transitions; radio frequency radiation) and in paramagnetic electron spin transitions (ESR; microwave radiation) as mentioned. Higher-frequency radiation, such as infrared, visible, ultraviolet, or x-rays, is produced by transitions between various electron energy levels, whether closely spaced rotational or vibrational, or further spaced electronic states (these are discussed in Section 6 below and also in Chapter 25). The highest-energy, highest-frequency radiation, gamma rays, consists of high-energy photons resulting from energy transitions of the protons and neutrons within the nucleus.

It is also very instructive to examine the wavelengths of the various forms of electromagnetic radiation as shown in Figure 19.22. Note that longer wavelengths correspond to lower energy radiation. This is due to the inverse relation between frequency and wavelength and is explained further in the next section in connection with photon energies. Visible light is but a very narrow window of the EM spectrum, although the only one visible to the human eye. Other animals have visual receptors that extend out into the infrared or ultraviolet regions. We discuss the functioning of the eye in Chapter 21. Nonvisible radiation interacts with our bodies in various ways. We feel warmth from infrared radiation, get sunburned from damage that ultraviolet radiation causes to our skin, and receive small amounts of molecular damage from x-rays at the doctor or dentist and from naturally occurring gamma radiation. In Section 6 we survey the interactions of various forms of radiation with matter.

5. THE QUANTUM THEORY OF RADIATION: CONCEPTS

Our discussion of electromagnetic radiation has thus far been in terms of electromagnetic waves produced by accelerating charges. Maxwell’s equations predict such waves and give a rather full account of their properties. In the early 1900s and
culminating in the 1920s and 1930s, a more complete theory of radiation was developed incorporating particliligike properties of radiation as well as wavelike properties. These additional properties cannot be accounted for by classical physics, but have their basis in quantum mechanics, our best theory of the microscopic world.

Our understanding of radiation is now based on a picture in which there is a fundamental quantum of radiation, known as the photon. Introduced by Einstein, the photon has both particliligike and wavelike properties. Photons carry discrete amounts of energy and momentum that can be localized in space, just like a particle. The energy of a photon is related to its frequency, a wavelike property, by

$$E = hf,$$

(19.10)

where $h$ is a new fundamental constant of nature known as Planck’s constant, and has the value $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$. Because $f = c/\lambda$, we can also write Equation (19.10) as $E = hc/\lambda$. The theory of special relativity (which we study briefly in Chapter 24) shows us that the energy and momentum of a photon are related as $E = pc$. From this we can conclude that the photon’s momentum depends only on its wavelength, another wavelike property, as

$$p = \frac{h}{\lambda}.$$

(19.11)

You should definitely be surprised that photons, with no mass, can have momentum, a property that up until now we have associated with a particle with mass. We show in Chapter 24 that these two fundamental equations for the energy and momentum of a photon can explain a number of phenomena that cannot be explained by a theory based solely on classical electromagnetic waves.

How are we to picture radiation having both wave and particle properties? We are accustomed to thinking of waves as extended disturbances and of particles as “pointlike” objects. Up until now these have been mutually exclusive concepts. Particles do not have wave properties and waves do not have particle properties. Quantum mechanics turns those ideas upside down as we show. For now, it is sufficient to qualitatively introduce the notion of a wave packet. Figure 19.23 shows a schematic representation of a wave packet, a wave that has a limited extent in space. Although not a pure frequency, a wave packet can change its spatial dimension in response to interactions with the external world. The greater the spatial extent, the closer the frequency content is to a pure single frequency (the limit being a perfect sine wave of infinite extent). In this way a single photon can behave more like an extended wave or like a particle depending on its spatial extent.

The intensity of a classical wave (being the energy per unit time per unit area and proportional to the square of the amplitude of the wave) does not correspond to a property of a single photon, but rather to the number of photons per second, each carrying a particular energy given by Equation (19.10). A more intense beam of light of a single color contains more photons per second traveling in the beam. For example, in a 1 W beam of laser light at a visible green wavelength of 514.5 nm there are many, many photons per second.

**Figure 19.23** A wave packet, localized in space but able to change its spatial extent on interaction with its environment.
Because there are so many photons at the usual ambient levels of light, we are not usually aware of the discrete nature of light or of the interactions of a single photon. Processes that rely on a single photon to have sufficient energy to cause an event to occur will be sensitive to the frequency of the radiation and not to the intensity of the beam. For example, in the photoelectric effect, a process involved in the photodetection of light and discussed in Chapter 24, a minimum energy threshold must be exceeded in order for the detection process (absorption of the photon with the ejection of an electron) to occur. A high-intensity beam of photons with subthreshold energies (huge numbers of photons, each with not enough energy) will not cause the emission of any electrons, whereas another beam of higher-frequency photons but with very low intensity may allow the detection of single photons (Figure 19.24).

We have stressed the common features of electromagnetic radiation up to this point. What most distinguishes the different forms of photons, aside from their method of production, is their interaction with matter. In the next section we discuss the large variety of such interactions and some of their consequences.

6. THE INTERACTION OF RADIATION WITH MATTER; A PRIMER ON SPECTROSCOPY

When electromagnetic radiation is incident on matter some fraction of the photons will usually be transmitted, passing through a sufficiently thin piece of matter without interacting, and the rest will be absorbed and will interact with the material. The particular type of interaction will depend on the energy of the photon and the structure of the matter. We have already discussed some of the interactions of radio and microwave radiation in connection with NMR and ESR. In this section we focus on photons of higher energies, especially the infrared, visible, and ultraviolet portions of the EM spectrum.

The energy level diagram is the crucial indicator of the type of possible interactions with radiation. Figure 19.25 shows a schematic diagram of typical energy levels with closely spaced rotational and vibrational energy levels, with energy differences of 0.01–0.1 eV. These correspond to overall rotational motions of molecules or to the relative positional vibrations of atoms in a molecule and are the lowest energy transitions other than nuclear or electron spin flip energy differences. Larger energy level spacings, with energy differences of about 1 eV, are due to the various electronic states (related to the valence electron’s mean distance from the nucleus), and the even larger binding energies of the inner electrons.

**Example 19.4** Calculate the number of photons per second in a 1 W beam from an argon ion laser with a wavelength of 514.5 nm.

**Solution:** Since each green photon carries an energy given by Equation (19.10) as 

\[ E_{\text{photon}} = hf = hc/\lambda = (6.6 \times 10^{-34})(3 \times 10^8)/(514.5 \times 10^{-9}) = 3.8 \times 10^{-19} \text{ J}, \]

the number of photons in the 1 W beam is given by

\[ N = \frac{1 \text{ J/s}}{3.8 \times 10^{-19} \text{ J/photon}} = 2.6 \times 10^{18} \text{ photons/s},\]

a very large number indeed.

![An intense photon beam with red photons, all with energy too low to be detected, produces no output from a photon detector, whereas individual more energetic blue photons each can be detected and produce an output signal.](image-url)
Transitions of electrons from the lower “ground state” to higher energy levels, with a change in energy equal to $\Delta E$, can occur upon the absorption of a photon with an energy, frequency, and wavelength such that

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \Delta E.$$  

Table 19.1 shows the general correspondence between the type of EM radiation and the atomic or molecular transitions produced. In general, the shorter the photon wavelength, the more energy imparted to the atom or molecule that absorbs the photon.

<table>
<thead>
<tr>
<th>EM Radiation (Increasing Energy)</th>
<th>Energy Level Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microwave</td>
<td>Rotational</td>
</tr>
<tr>
<td>Infrared</td>
<td>Molecular vibrational</td>
</tr>
<tr>
<td>Visible and Ultraviolet</td>
<td>Valence electronic</td>
</tr>
<tr>
<td>X-ray</td>
<td>Inner electronic</td>
</tr>
</tbody>
</table>

Two general types of interactions can be distinguished based on whether ionizing or nonionizing radiation is used. Ionizing radiation consists of x-rays or gamma rays and results in the ejection of one or more electrons; we study such processes in Chapter 26. Nonionizing radiation results in a large variety of possible interactions and there is a correspondingly large number of spectroscopic techniques used to probe the radiation–matter interactions in order to learn something about the structure of the sample.

Let’s imagine that we perform a spectroscopy experiment in which a beam of monochromatic (literally, single color, but this term is used to mean a uniform frequency or wavelength for nonvisible light where we don’t use the concept of color) photons is directed on a biomolecular sample to study the nature of the absorbed or re-emitted radiation. Figure 19.26 shows a typical experimental setup for spectroscopy. Although the wavelength of the incident light is varied, the appropriate detector, monitoring either the transmitted or scattered light, records a spectrum showing the variation in the measured parameter as a function of incident wavelength. Depending on the type of interaction, we discuss the information one can obtain from such measurements. Our presentation is by no means a complete summary of the various types of spectroscopy.
With infrared, visible, or ultraviolet light incident on a sample, some small fraction of the incident photons will be absorbed and can be detected using *absorption spectroscopy*. Most of the absorbed energy that has caused various transitions to occur ends up heating the solution via collisions of the molecules. If the incident intensity on the sample is $I_0$ then the intensity detected after traveling a distance $x$ through the sample with molar concentration $c$ of absorbing molecules will be

$$I = I_0 e^{-exc},$$  \hspace{3cm} (19.13)$$

where $\varepsilon$ is the molar extinction coefficient, dependent on the wavelength of the incident EM wave. When rewritten taking the logarithm using base 10 as $\log (I/I_0) = -exc \log(\varepsilon) = -exc/2.3$, we can introduce the absorbance $A$, from the Beer–Lambert law

$$A = exc/2.3 = \log (I_0/I).$$  \hspace{3cm} (19.14)$$

When the distance $x$ is taken to be 1 cm, the standard path length of special optical cells used in spectroscopy, then $A$ is commonly called the optical density. An optical density of 2 means that only $10^{-2} = 0.01$ or 1% of the light will be transmitted. Remember that $\varepsilon$, and thus $A$, depends on the wavelength. At the wavelength corresponding to a maximum absorbance, $A$ is known as the extinction coefficient, and, once known, can be routinely used to determine the concentration of a solution of absorbing molecules after a measurement of the absorbance.

**Example 19.5** A 1:10 dilution of a DNA solution is pipetted into a 1 cm path length quartz (non-uv absorbing) cuvette which is then put in an absorption spectrometer to determine its optical density at 260 nm. Measurements find that the absorbance is 0.24 OD. If the extinction coefficient is known to be 6600 M$^{-1}$ cm$^{-1}$, what is the molar concentration of the original DNA sample and what fraction of the incident light would be transmitted through a cuvette with the original DNA solution in it?

**Solution:** According to Equation (19.14) the molar concentration $c$ is given by $c = 2.3A/\varepsilon = (2.3)(0.24)/(6600)(1)$ M = 8.4 $\mu$M. Then because the measured sample was a 1:10 dilution, the original concentration of the sample was 84 $\mu$M. If the original DNA solution had been used directly the measured optical density would be, barring any nonlinear effects, 2.4 OD. Using Equation (19.14), we find the ratio $I_0/I = 10^{2.4} = 250$, so that only 0.4% of the intensity is transmitted.

In *infrared spectroscopy*, the infrared photon energies are just sufficient to excite vibrational excitations of various covalently attached atoms. In general, IR radiation will heat such samples because the internal energy is increased by the absorption of these photons. Imagining that the bonds correspond to springs (see Chapter 4), when the photon frequency corresponds to the spring resonant frequency there will be a large increase in the absorption of photons. Because the “effective spring constant” of a bond is specific to the type of atoms bound, IR spectra can be used to “fingerprint” the sample molecules. Methyl (C–H), carbonyl (C = O), and amide (N–H) bonds, for example, each have characteristic absorption energies that depend also on the local environment of the atoms. Because of the large number of similar absorption energies in large macromolecules such as DNA, IR spectra tend to be broad superpositions of many peaks. One major difficulty is that water, the universal solvent in biology, is a strong absorber of IR radiation and, because it is present in
relatively enormous quantity, masks the absorption peaks due to other molecules. The use of D$_2$O or other methods have overcome this problem. Much of the IR work with larger macromolecules compares two otherwise identical samples when one variable of the environment, for example, the pH, has been changed. The information obtained from such difference measurements can be used to learn about conformational changes that have occurred under these conditions.

Ultraviolet-visible (uv-vis) absorption spectroscopy probes valence electron excitations. It is precisely these interactions that determine the “chemistry” of the material in that these valence electrons make up the chemical bonds between atoms. Photon energies are sufficient to excite the stronger double and triple bonds and the strongest contributors to the absorption are ring-structures commonly found in biomolecules such as the amino acids tryptophan and tyrosine or the bases of nucleic acids (see Figure 19.27). This technique is routinely used in biological research to measure the concentration of molecules because the absorption is usually proportional to concentration. Measurements are very sensitive to the overall conformation of molecules and can be used to monitor such changes as the melting of DNA (Figure 19.28). Again, difference measurements are commonly used to study the specific effects of a particular perturbation on the sample.

Of the absorbed visible photons, some small fraction will be re-emitted (scattered) by the molecules with nearly unchanged photon energy (elastic scattering), but redirected spatially. If the scattering molecules of molecular weight $M$ are small compared to the wavelength of light, then the scattering will be uniform in all directions (isotropic). In this case, the intensity of the scattered light will depend on the wavelength of the incident light $\lambda$ in a characteristic and very strong way (inverse fourth power of $\lambda$)

$$I_{\text{scatter}} \propto \frac{I_{\text{incident}} M c}{\lambda^4},$$

where $c$ is the mass concentration (kg/m$^3$) of the scatterers. Light-scattering measurements, usually performed using monochromatic incident light, can be used to determine the molecular weight of the scatterers. For larger molecules, the scattering is no longer isotropic, and the spatial dependence of the scattered intensity can be used to give information on the size and shape of such molecules.

Lord Rayleigh, in the 1870s, first discovered the strong $\lambda^{-4}$ dependence of scattered light and was able to answer the age-old fundamental question: Why is the sky
blue? The answer lies in Equation (19.15) and the fact that when we look up at the sky, the light that we see is sunlight that has been scattered from small gas and dust parti-
cles in the atmosphere. Of all the visible colors that our eyes are able to see, shades of blue have the shortest wavelength and, according to Equation (19.15), are therefore scattered with greater intensities. Hence, the sky appears blue. The same argument also explains the brilliant colors of a sunrise or sunset. In those cases we are looking toward the sun and the blue light is predominantly scattered out of the sunlight headed toward our eyes, leaving reds and oranges to reach our eyes directly (Figure 19.29).

Some still smaller fraction of the absorbed visible light will be re-emitted as photons with a wavelength and energy different from the incident light. After a molecule absorbs a visible photon, thereby exciting its valence electron to a higher energy state, often the excited electron will lose some energy to heat via collisions with the solvent in a very short time ($10^{-12}$ s). Subsequent emission of a photon (typically within a time of $10^{-9}$ s) requires, by conservation of energy, the photon to have a lower energy than the incident photon, and therefore a longer wavelength (recall that $E_{\text{photon}} = \frac{hc}{\lambda}$). Thus, for example, if a solution of macromolecules has blue light shining on it, some small fraction of the light will emerge, let’s say, red. This process is known as fluorescence and although the fluorescent intensity is very small compared to the incident intensity it can be detected quite easily because of its different color. Fluorescence also occurs when ultraviolet light is absorbed and visible fluorescent photons are emitted. The detergent industry even adds fluorescent “brighteners” in order to enhance the appearance of clothing from extra visible light emitted from uv fluorescence.

**FIGURE 19.29** (top) The shorter wavelengths (blues) of sunlight are preferentially scattered by the upper atmosphere so that the sky will appear blue to an observer looking up. When the sun is on the horizon, and the blue light is scattered out, the remaining reds and oranges give the wonderful colors of sunsets and sunrises. (bottom) Sunrise over the coast in Maine and sunset over Tuscon, Arizona.
In some few types of molecular systems, the excited electron finds itself in a very long-lived state as far as molecular times go, roughly $10^{-3}$ s. After this long time a photon is emitted in the process of phosphorescence. This is the way in which fireflies generate their mysterious light using the chemical compound luciferase. Roughly half of all types of jellyfish also emit phosphorescence, also known as bioluminescence (Figure 19.30).

**Fluorescence spectroscopy** can use “intrinsic” fluorescent groups, known as chromophores, if they are present (in proteins tryptophan is a good chromophore) or “extrinsic” fluorescent molecules, or fluors, if attached to the molecule of interest. In either case, fluorescent light is usually detected at a 90° angle to the incident beam direction using some sort of color filter to only detect the fluorescent photons. The quantum yield, defined as the ratio of the number of fluorescent photons emitted to the total number of photons absorbed, is usually very small and quite sensitive to the local environment of the chromophore, including the pH, temperature, neighboring chemical groups, and concentration. Therefore, the fluorescence intensity can be a measure of the local conformation of the biomolecule and measurements can often be used to detect the binding of ligands, small molecules with specific attachment sites, or the polymerization of monomer proteins, such as actin, to form long threadlike filaments.

Aside from its use in spectroscopy, fluorescence is also quite useful in imaging molecules in microscopy. By labeling specific molecules with a fluorescent dye and modifying a microscope with a color filter device, striking images of the arrangement of molecules otherwise too small or dilute to see are possible using **fluorescence microscopy** (Figure 19.31). We return to this topic in the next chapter in a more detailed discussion of microscopy.

Often spectroscopic techniques can study not only the intensity of the scattered or absorbed radiation, but also its polarization and time-dependence. Polarization information can be quite useful in learning about the rotational motion of macromolecules. In fluorescence polarization measurements, if a brief pulse (~ns duration) of intense vertically polarized light is incident on a sample, then after absorbing a photon, chromophores will be able to rotate before emitting a fluorescent photon. In doing so, the polarization of the emerging fluorescence will have a horizontal component as well as a vertical component. By analyzing the time-dependence of these two independent intensity components in the subsequent fluorescent burst of emission, the rotational timescales of motion of the macromolecules can be determined. Several other important types of polarization measurement techniques are discussed in Chapter 23.
CHAPTER SUMMARY

Electromagnetic radiation is produced by accelerating electric charges and consists of coupled traveling $E$ and $B$ waves, perpendicular to each other and to the direction of propagation, and that decrease in amplitude as they travel outward as $1/r$. The waves have the form, in one dimension,

$$E(x,t) = E_{\text{max}} \sin (kx - \omega t),$$
$$B(x,t) = B_{\text{max}} \sin (kx - \omega t), \quad (19.2)$$

with

$$\frac{E_{\text{max}}}{B_{\text{max}}} = c. \quad (19.3)$$

The energy density of the EM radiation is given by

$$\frac{PE}{V} = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) = \epsilon_0 E^2, \quad (19.4, 5)$$

(Continued)
where the two terms have equal energy. The Poynting vector is the energy per unit time per unit area and is given by

\[ S = c \frac{PE}{V} = c \varepsilon_0 E^2, \]  
(19.6)

and points in the direction of propagation of the wave. The average value of \( S \) is given by the intensity,

\[ I = \frac{1}{2} \varepsilon_0 c E_{\text{max}}^2. \]  
(19.7)

Laser tweezers is a technique that traps small transparent objects at the focal point of a laser with a trapping force proportional to the laser beam intensity. Careful calibration can then measure the force acting on the individual object. A wide range of biological applications have been developed using laser tweezers, now capable of studying the forces generated by individual macromolecules.

In EM waves, the direction of the \( E \) field is known as the polarization direction. If the \( E \) field is confined to a linear direction, the wave is said to be linearly polarized. Random orientation of \( E \) in a wave is known as an unpolarized EM wave. The \( E \) field can also be made to sweep in a circle or an ellipse as the wave propagates in circularly or elliptically polarized beams. The law of Malus gives the intensity transmitted through an analyzing polarizer in terms of the incident intensity \( I_0 \) after passing through a first polarizer with an angle \( \theta \) between the two polarizers,

\[ I_t = I_0 \cos^2 \theta. \]  
(19.9)

Photons are the elementary constituents of electromagnetic radiation and each photon has an energy and a momentum given by

\[ E = hf, \]  
(19.10)
\[ p = \frac{h}{\lambda}. \]  
(19.11)

Photons with different frequencies (or wavelengths) constitute electromagnetic radiation from different parts of the electromagnetic spectrum (see Figure 19.22). The study of the interactions of electromagnetic radiation with matter is called spectroscopy. Non-ionizing radiation can be absorbed according to the Beer–Lambert law

\[ A = \log(I_0/I), \]  
(19.14)

where \( A \) is the absorbance and \( I \) and \( I_0 \) are the transmitted and incident intensity, respectively. Absorption can only occur if the incident photon energy matches a possible energy transition in the absorbing atom or molecule. Scattering may also occur, whereby the incident photon energy is absorbed and re-emitted, either at the same frequency (elastic scattering) or at a lower frequency (e.g., fluorescence). In elastic scattering, the scattered intensity varies as the inverse fourth power of the wavelength and this strong dependence on wavelength is responsible for the blue color of the sky and for the bright red/orange colors of sunrises/sunsets.

**QUESTIONS**

1. Why must the electric and magnetic fields of a spherical wave vary as \( 1/r \)? Fill in the details of the energy argument of Figure 19.2.
2. Compare electromagnetic waves with waves on a string. Give as full an accounting as you can of how they are similar and how they are different.
3. Even though the magnetic field of an electromagnetic wave is a factor of \( c \) weaker than the electric field, the energy contained in the magnetic field is the same as that of the electric field. Show how this follows from the definitions of the field energies.
4. What is the distinction between the Poynting vector and the intensity of an electromagnetic wave?
5. Explain in words how a focused laser beam can supply a longitudinal force in the direction of the focal point on a microscopic particle. See Figure 19.9.
6. If a laser beam has a transverse intensity profile decreasing symmetrically from a maximum in the center of the beam, explain in words how the focused beam can supply a transverse force toward the center of the beam on a microscopic particle. See Figure 19.10.
7. Assuming that micron-sized spheres can be attached to biological molecules and these can be then trapped in laser tweezers, suggest several experiments that can be done to study cellular and macromolecular properties.
8. Distinguish between unpolarized and polarized electromagnetic waves. Because all electromagnetic waves are transverse, aren’t they all polarized?
9. How does Polaroid film work to polarize light? If unpolarized light is incident on a Polaroid sheet, what fraction of the incident intensity is transmitted?

10. If vertically polarized light passes through a perfect polarizer and no light is transmitted, what can you conclude?

11. Unpolarized light passes through two consecutive polarizers with axes oriented at 45° from each other. What fraction of the incident intensity is transmitted through the second polarizer?

12. Explain why circularly polarized light is equivalent to two linearly polarized electric fields at right angles to each other and 90° out of phase. Describe the phase relations that produce a clockwise or counterclockwise circularly polarized plane wave as viewed from in front of the wave watching it approach. Using your two arms, kept at right angles to each other, devise a way to move your arms to simulate the electric fields of right or left circularly polarized light.

13. The visible spectrum of light ranges from around 400 nm violet to 750 nm red. Which color photon has more energy? More momentum? Does having more momentum mean that those photons travel faster?

14. A pure sine wave has infinite spatial extent and a single frequency. A square pulse has a finite spatial size and is composed of an infinite range of different frequencies. These two examples are limits for a wave packet. Discuss this idea.

15. What type of radiation would you expect to be emitted when a molecule makes a transition between neighboring rotational states with energy spacings of about 0.01 eV? Between neighboring vibrational levels with energy differences of about 0.1 eV? Between electronic states with energy differences of about 1 eV?

16. What fraction of the incident light is transmitted through a sample with an optical density of 1.0? Of 2.0? Of 1.5?

17. Can a fluorescent dye that absorbs strongly in the green appear to be blue? Red? Yellow?

18. Discuss the difference between inelastic and elastic scattering. Can there be absorption with elastic scattering? Can there be fluorescence with elastic scattering?

**MULTIPLE CHOICE QUESTIONS**

1. In electromagnetic radiation (a) the electric and magnetic fields are both parallel to the direction of propagation, (b) the electric field is perpendicular to and the magnetic field is parallel to the direction of propagation, (c) the electric field is parallel to and the magnetic field is perpendicular to the direction of propagation, (d) the electric and magnetic fields are both perpendicular to the direction of propagation.

2. Which of these is not an example of the “electromagnetic spectrum”? (a) X-rays used by your dentist. (b) γ-rays used to prevent food spoilage. (c) Microwaves used to boil water. (d) Ultrasonic waves used to image a fetus.

3. A TV wave has a wavelength of about 1 m. The frequency of such a wave is (a) 1 Hz, (b) 3 Hz, (c) 300 MHz, (d) $3 \times 10^{15}$ Hz.

4. The wavelength of violet light is about (a) 400 m, (b) 400 cm, (c) 400 μm, (d) 400 nm.

5. Electromagnetic waves emitted from the wiring in your house due to AC currents have a wavelength of about 5 times (a) $10^{-10}$ m, (b) $10^{-2}$ m, (c) $10^{3}$ m, (d) $10^{6}$ m.

6. The magnetic field inside a solenoid is proportional to the current flowing through the solenoid. Suppose the current through a solenoid is doubled, by how much is the energy associated with the magnetic field in the solenoid changed? (a) It remains the same. (b) It is doubled. (c) It is increased by a factor of four. (d) It is halved.

7. One light source is four times more intense than a second light source. The maximum electric field in the light from the first source is (a) four times greater than that from the second, (b) two times greater than that from the second, (c) one half as great as that from the second, (d) one quarter as great as that from the second.

8. The intensity of an extremely bright laser is $10^7$ W/m², about 10,000 times brighter than sunlight. The average electric field in sunlight is roughly $10^3$ V/m. The average electric field in the bright laser is about (a) $10^7$ V/m, (b) $10^5$ V/m, (c) $10^3$ V/m, (d) $10$ V/m.

9. The sun emits electromagnetic radiation uniformly in all directions. Given the distance from the sun to the Earth of about $150 \times 10^6$ km, and the mean radius of the Earth of about 6400 km, the fraction of the sun’s radiation that falls on the Earth is (a) $(6400/150 \times 10^6)^2$, (b) $(6400/150 \times 10^6)^2$, (c) $(6400/300 \times 10^6)^2$, (d) $(1/2)(6400/300 \times 10^6)^2$.

10. The central 1 cm diameter portion of a 10 W laser beam with a diameter of 2 cm is focused down to a 100 μm diameter spot size. The intensity at the focal point is (a) $1.3 \times 10^8$, (b) $6.4 \times 10^8$, (c) $3.2 \times 10^8$, (d) $8.0 \times 10^7$ W/m². (Assume the laser beam has a uniform intensity across its cross-sectional area.)

11. A small plastic sphere is initially located below and to the left of the center of a focused laser beam. The force on this sphere is directed (a) down and to the right, (b) up and to the left, (c) up and to the right, (d) down and to the left.

12. A typical force capable of being exerted by a focused laser beam in a laser tweezers experiment is (a) $10^{-2}$ N, (b) $10^{-7}$ N, (c) $10^{-12}$ N, (d) $10^{-17}$ N.

13. A plane wave of light propagating along the positive x-axis is linearly polarized along the z-axis. A polarizing sheet parallel to the y−z plane is at x = 1 m. The transmission (or polarizing) axis of the sheet is parallel to the y-axis. A second polarizing sheet is placed parallel to the y−z plane at x = 0.5 m. A light detector is placed at x = 1.1 m. The detector
will receive (a) maximum light when the transmission axis of the second sheet makes a 45° angle with respect to the y-axis, (b) maximum light when the transmission axis of the second sheet is parallel to the z-axis, (c) maximum light when the transmission axis of the second sheet is parallel to the y-axis, (d) no light no matter what is the orientation of the transmission axis of the second sheet.

14. If a laser beam with intensity $I_0$ is incident on a polarizer and the transmitted intensity is determined to be also approximately $I_0$, we can conclude that (a) the initial beam was unpolarized, (b) the initial beam was polarized perpendicular to the polarizer’s transmission axis, (c) the initial beam was circularly polarized, (d) the initial beam was polarized parallel to the polarizer’s transmission axis.

15. If a laser beam with intensity $I_0$ is incident on a polarizer and the transmitted intensity is determined to be approximately $I_0/2$, we can conclude that the initial beam (a) must have been unpolarized, (b) may have been unpolarized, circularly polarized, or linearly polarized at 45° to the transmission axis of the polarizer, (c) must have been circularly polarized, (d) must have been linearly polarized at 45° to the transmission axis of the polarizer.

16. The number of photons per second emitted by a 1000 W, 100 MHz FM radio station is about (a) $10^3$, (b) $10^5$, (c) $10^{11}$, (d) $10^{25}$.

17. A 440 nm photon has an energy of (a) $4.5 \times 10^{-19}$ J, (b) $1.5 \times 10^{-27}$ J, (c) $4.5 \times 10^{-17}$ J, (d) $3.8 \times 10^{-19}$ J.

18. A sample with an optical density of 2.5 transmits (a) 8, (b) 300, (c) 3, (d) 0.3 percent of the incident light.

19. The midday sky appears blue because (a) our eyes see blue better than other colors making up white light; (b) molecules in the sky reflect back light from the oceans which are blue; (c) molecules in the sky re-emit blue light more than other colors visible to our eyes; (d) hydrogen molecules in the sky have a strong absorption peak in the blue.

PROBLEMS

1. If a plane electromagnetic wave has a maximum magnetic field amplitude of $2 \times 10^{-7}$ T, find the peak value of the electric field of the wave.

2. A plane electromagnetic wave is traveling along the x-axis. If the electric field of the wave has a maximum value of $2 \times 10^{-4}$ N/C and lies along the y-axis, find the wave’s maximum magnetic field and its direction.

3. Given that the peak value of the magnetic field of an electromagnetic plane wave is $5 \times 10^{-7}$ T, find the intensity of the wave.

4. If the maximum trapping force from a laser tweezers on a 1 μm radius spherical particle in water at 20°C is $10^{-12}$ N, find the minimum flow velocity of the water, in mm/s, that will just free the particle from the trap.

5. If in the previous problem the calculated flow velocity of the water is doubled, by what factor must the intensity of the laser beam be increased to just maintain the trap?

6. A vertically polarized beam of light passes through a sheet of Polaroid with its transmission axis at a 30° angle to the vertical. What fraction of the incident intensity emerges from the Polaroid? If this beam is then incident on a second sheet of Polaroid with its transmission axis along the vertical, what fraction of the beam’s original intensity is transmitted?

7. Unpolarized light of intensity $I_0$ passes through a Polaroid with its transmission axis vertically oriented. What intensity emerges? If the transmitted light passes through a second Polaroid sheet with its transmission axis 60° to the vertical what fraction of the original incident light intensity $I_0$ emerges?

8. Three polarizers arranged in series are each oriented at 30° from the previous one. If an unpolarized light beam travels through the three polarizers and emerges with an intensity of 0.2 W/m², what was the intensity of the beam incident on the first polarizer?

9. A circularly polarized light beam is incident on a vertically oriented polarizer. If the incident beam has an intensity $I_0$, describe the transmitted beam intensity and polarization.

10. Two polarizers are in series with one another with an angle of 45° between their axes. If a circularly polarized beam with an intensity of 0.8 W/m² is incident on the first polarizer, oriented vertically, describe the intensity and polarization of the beam transmitted through the second polarizer. What happens if the two polarizers are interchanged keeping their axes’ orientation fixed? Does the transmitted intensity change? Does the polarization direction of the transmitted beam change?

11. Suppose a point source of light generates 60 W. Four meters away there is a light detector that is 75% efficient and has a detector area of 10 cm² oriented with its normal directed at the point source. What power will the detector record?

12. Suppose a 50 kW radio station, operating at a frequency of 106.5 MHz, emits EM waves uniformly in all directions.

(a) What is the wavelength of the radio waves?

(b) How much energy per second crosses a 1.0 m² area 100 m from the transmitting antenna?

(c) What is the maximum value of the electric field at this point, assuming the station is operating at full power?

(d) What voltage is induced in a 1.0 m long vertical car antenna at this distance?

13. Our nearest star (Proxima Centari) is 4.2 light-years away (1 light-year equals the distance light travels in a year). How far away is Proxima Centari in meters?
14. The Andromeda Galaxy (our nearest neighbor galaxy) is approximately 2 million light-years away (see the previous problem). How far away is Andromeda in astronomical units? 1 AU = $1.5 \times 10^8$ km.

15. The human eye is most sensitive to light having a wavelength of $5.50 \times 10^{-7}$ m, which is in the green–yellow region of the visible electromagnetic spectrum. What is the frequency of this light?

16. Suppose that a laser pointer is rated at 3 mW. If the pointer produces a 2 mm diameter spot on a screen, what is the radiation pressure exerted on the screen?

17. A laser pulse lasting $10^{-9}$ s from a Neodymium–YAG laser has an energy of 5 J. If the wavelength of the laser is 1.06 μm, how many photons are in the pulse? What is the power of the laser pulse? If the pulses are repeated at a rate of 10 Hz (10 pulses/s), what is the average power output of the laser?

18. Calculate the frequency range and photon energies (in eV) of visible light if the wavelength range is 400–750 nm.

19. If the wave packet of a photon contains frequencies in the range $6.8–7.1 \times 10^{14}$ Hz, what is the average wavelength and ± wavelength uncertainty spread of the photon?

20. Suppose an atom has two energy levels at $-3.40$ eV and $-1.51$ eV. If an electron makes a transition from the upper to the lower of these levels, what is the wavelength of the emitted photon?

21. A 1 ns laser pulse from a neodymium–YAG laser, at 532 nm, contains 5 J.
   (a) Find the energy and momentum of each photon.
   (b) How many photons are in the 1 ns pulse of laser light?
   (c) If all the photons are completely absorbed by an object, what is the force exerted on the object by the laser light?

22. The laser light pulses from the previous problem are passed through a device known as a frequency doubler that basically combines two pulses into a single one with twice the frequency.
   (a) Find the wavelength, energy, and momentum of the frequency doubled pulses.
   (b) Assuming 100% efficiency, how many of these photons are in the 1 ns pulse?
   (c) If all of these photons are absorbed by the same object as in the previous problem, compare the force exerted on the object by the frequency doubled pulse compared to the original pulse.

23. A solution of a purified protein is put in a 1 cm path length quartz optical cell and into a uv spectrophotometer. The optical density measured at a 260 nm wavelength is 0.05. If the molar extinction coefficient of this protein is 349 M/cm at 260 nm find the molar concentration of the protein and the % of uv light transmitted by the solution.

24. If the incident intensity on a 0.01 M protein solution in a 1 cm optical cell is reduced in the transmitted beam by a factor of 1500, what is the molar extinction coefficient of the protein at the measuring wavelength?

25. A detector measures the elastic scattering from a gas in a sealed glass cell. If the incident light intensity is halved and the wavelength of the light used is reduced by 20%, find the percent change in the scattered intensity. Does the amount of scattered light increase or decrease with the 50% reduction in incident intensity at the new wavelength?