In our discussions of geometric optics we completely ignored the fact that light can also behave as an electromagnetic wave. The wave packet picture of photons (introduced in Section 5 of Chapter 19) is compatible with treating light as a collection of particlelike photons that follow the rules of geometric optics only as long as the objects with which light interacts (mirrors, lenses, apertures, surfaces, etc.) are large compared to the wavelength of the light. This justifies our treatment of light thus far as traveling in straight lines except on refraction or reflection at the boundary between two media. Under other conditions, the wave packet will change its spatial extent on interacting with smaller objects and exhibit wavelike properties, some of which are discussed in this chapter.

We begin this chapter by re-examining, in the context of light waves, some concepts introduced earlier in Chapters 10 and 11 for traveling mechanical or sound waves. A major idea is the principle of superposition, as applied to waves overlapping in space and time, which leads to interference effects. Two waves of equal amplitude that are in phase will add constructively to produce a net wave with an amplitude twice as large whereas such waves that are 180° out of phase will add destructively to completely cancel each other. Another general property of a wave is diffraction, or bending, that occurs at an obstacle. These effects are studied for a variety of important geometries and their fundamental implications in limiting resolution in optics are discussed. The next chapter discusses a variety of applications in imaging that stem from these ideas.

1. DIFFRACTION AND INTEREERENCE OF LIGHT

1.1. PRELIMINARIES

We have seen that a monochromatic traveling plane light wave can be represented by a sine wave with a well-defined frequency and amplitude moving with a constant velocity \( v = \frac{c}{n} \). Having defined the frequency, the wavelength is dependent on the medium and is given by

\[
\lambda = \frac{v}{f} = \frac{c}{nf},
\]

and the intensity of the wave is proportional to the square of its amplitude. In a plane wave, all points on the wavefront are in phase and oscillate together (Figure 22.1). Such idealized plane waves can be fairly well represented by a laser beam, as we show in Chapter 25.

When light crosses a boundary between two different optical media, its frequency remains the same but its speed changes and therefore the wavelength of the

light will change. As a consequence of this, light traveling through different media will be shifted in relative phase leading to some interesting phenomena. As shown in Figure 22.2, when one beam of light passes through a slab of material with higher index of refraction its wavelength is shortened and after emerging from the slab with the original wavelength, its phase has been shifted with respect to a second reference beam. In traveling some distance $d$ in a particular medium, the number of wavelengths of light within that distance is given by $d \lambda = nd\lambda_o$, where $\lambda_o$ is the vacuum wavelength of the light. Thus for a given $\lambda_o$ of light, different beams traveling through different media will be shifted in phase by amounts related to the product $nd$ for each media. Each beam arriving at the viewer has traveled a different optical path, defined as the distance traveled multiplied by the corresponding index of refraction

$$Optical \ Path = \sum n_i d_i,$$

where the sum is over the different distances $d_i$ the beam travels in different media with indices $n_i$ to obtain the net optical path traveled. This optical path will determine the relative phase of EM waves traveling through different media. In the case shown in the figure, as long as the two waves originate in phase, their relative phase at the viewer will be determined solely by differences in optical paths. These differences will create optical consequences at the viewer that can be used to learn something about the media through which the beams have traveled as we show shortly.

But how can we ensure that the initial beams are in phase? One way is to generate a plane wave beam of light and then split it into two sources with a beamsplitter device or simply with two apertures. As shown in Figure 22.3, a real extended source of light can, with the use of a small aperture, be used to generate light that has a wavefront that oscillates in phase. Light emanating from the aperture can be imagined to spread in space according to an idea due to Huygens and known as Huygens’ construction. The method consists of imagining each point on the wavefront as a source of spherical wavelets that spread out in the forward direction at the speed of the wave; the new wavefront consists of the envelope, or tangent, to all the wavelets. This construction for the small aperture is shown to generate a spherical wave. At some large distance from the aperture, the wavefront is approximately plane and this method can be used to obtain a plane wave. Alternatively, a laser beam can be used because, as mentioned above, most laser beams behave as plane waves.
1.2. DIFFRACTION

When a wave meets an obstacle, or hole in an opaque material, and is partially obstructed, Huygens’ construction can be useful. If the opening is very large compared to the wavelength of light, the unobstructed portions of a plane wave continue on as a plane wave, with some bending of the light at the wall edges. This bending is known as diffraction and occurs with all types of waves. As the opening gets smaller and comparable to the wavelength, the diffraction increases and the wave is spread much more. This is shown schematically in Figure 22.4.

Diffraction of sound is obvious to us: we can hear around corners. Sound has a wavelength comparable to the dimensions of objects around us and thus is easily diffracted through large angles. Visible light, with its wavelength of about 500 nm, is only diffracted near sharp edges or at small openings with micron-sized dimensions. As long as diffraction effects are negligible, light can be understood in terms of geometrical optics. In Section 2 we show some of the effects of diffraction on light transmitted through different slits and in Section 3 we consider the fundamental limitations on resolution due to diffraction.

1.3. INTERFERENCE

As a first example of interference, consider the situation shown in Figure 22.5 in which a light beam is incident on a thin film coating a second medium with a different index of refraction. To be specific, suppose that the beam is incident from air onto an organic film (e.g., an oil or gasoline) with index of refraction $n$ and thickness $t$, coating the surface of water. At each surface there will be a division of the incident intensity into a reflected and refracted beam. If the incident light makes a small angle with the normal then there will be a series of multiple reflections, as shown in the diagram, resulting in the overlap of the reflected beams as seen by a person viewing from the air (we ignore the transmitted beams in what follows).

If we approximate the beams as normal to the surface then those labeled 1 and 2 have an optical path difference given by $O.P.D. = n(2t)$. What is the effect of this extra path difference on the relative phase of the two beams at the viewer? If the two beams that we are comparing were as in the situation in Figure 22.2, the optical path difference divided by the wavelength of light in air would be the number of wavelengths shifted between the two beams. If this number were equal to 1 then the two beams would again be in phase, but if it were equal to 1/2 then the two beams would be exactly out of phase.

In the case of reflection from a surface there is an additional effect that enters. If the light is reflecting from a medium with a larger index of refraction, as is the case...
from air to the organic film, then there is an additional phase change of 180°, or \( \pi \) radians. Note that this is the same effect we see when a wave on a string is reflected from a fixed end (see the discussion of Figure 10.16). In our thin film example, because the film has a larger index than the lower water layer and air, only ray 1 will have the additional \( \pi \) phase shift; the other reflected beams arise from reflection at the second surface between the film with greater index of refraction and the water or air, and therefore no additional phase shift occurs on reflection. When the optical path difference \( (2nt) \) is just equal to an integer number of wavelengths of the light, then with the additional half-wave shift, the multiply reflected beams will all be out of phase with ray 1 at the viewer, a condition known as destructive interference. Using our expression for the optical path difference, we can write this condition as

\[
2nt = m\lambda, \quad \text{(destructive interference),} \quad (22.3)
\]

where \( \lambda \) is the wavelength of light in the incident medium (air) and \( m = 0, 1, 2, \ldots \) is an integer known as the order number. When \( m = 0 \), even with an extremely thin layer of index \( n \) material, there is still destructive interference (see below). Although the waves will arrive out of phase, the cancellation will not be complete in general because the intensities of the multiply reflected beams are not equal. With two beams of equal intensity 180° out of phase, complete cancellation would occur leaving no light at all.

If, on the other hand, the optical path difference is equal to a half-integer multiple of the wavelength, then with the added half-wave shift on reflection, the two most intense waves, labeled 1 and 2, will undergo constructive interference

\[
2nt = \left( m + \frac{1}{2} \right)\lambda. \quad \text{(constructive interference).} \quad (22.4)
\]

In the case of an oil film on water or a soap film in air illuminated by white light, the reflected light reveals a set of brightly colored fringes as shown in Figure 22.6. The different colors arise from constructive interference at the corresponding wavelengths due to variations in film thickness.

**Example 22.1** White light illuminates a film of 325 nm optical thickness with index of refraction \( n = 1.5 \), corresponding to a real thickness of 217 nm. Which visible wavelengths will appear intense on reflection and which will be absent?

**Solution:** According to Equation (22.3), destructive interference will occur for \( \lambda = 2nt = 2(325) = 650 \text{ nm} \), corresponding to a shade of red light. This is the only visible color that will be totally absent; the next wavelength at which there is total destructive interference is at \( \lambda = nt = 325 \text{ nm} \) which is in the ultraviolet. According to Equation (22.4), constructive interference will occur for \( 2nt = 3\lambda/2 \), resulting in \( \lambda = 430 \text{ nm} \) and corresponding to violet light, as the only visible wavelength that will be intensified by constructive interference. Other values of \( m \) in Equation (22.4) do not lead to visible wavelengths. The resulting overall reflection will make the film appear bluish. Reflected colors can therefore be used as an indicator of film thickness.

Very thin films (with \( t < \lambda/10 \)) will appear to be black because there is a negligible optical path difference and the half-wave shift of the primary reflected
beam tends to cancel all the other reflected light from the second surface where there is no half-wave shift. Artificial membranes made from lipids, formed in a manner similar to soap films, appear black when their thickness corresponds to that of a bilayer and this color change is used as a signature of film thickness.

The ideas just presented are the basis for reflection-interference microscopy. This microscope technique can be used to visualize cell-to-surface contacts. A typical sample (Figure 22.7) is a suspension of cells covered with a glass cover slip above which lies immersion oil with an index of refraction matching that of glass. The indices of refraction of the three media involved, the cover glass, $n_g$, the solvent, $n_s$, and the cell, $n_c$, satisfy $n_g < n_s < n_c$, therefore interference on reflection can be used to discern cell–surface adhesion sites. Remembering that an extra $\pi$ phase shift occurs on reflection from a larger index medium, reflection at the solvent–cell interface produces a phase shift of $\pi$ radians. In addition to that phase shift, there will be an optical path difference ($2n_c t$) between the reflections at the two surfaces from the solvent layer of variable thickness $t$ (see Figure 22.7). At sites of cell attachment where this layer of solvent is vanishingly thin, there is no additional optical path difference and these sites will appear dark on reflection. In this method, total internal reflection is used to illuminate a very thin layer of the sample using the evanescent waves (see the discussion in Chapter 20 concerning Figure 20.14) and only the attached cells in a very thin surface layer will then be seen and magnified, often using fluorescent light from attached dyes (Figure 22.8).

Aside from the beautiful patterns of colorful fringes appearing on reflection from thin films (Figure 22.9), these films can be used in a number of ways to perform optical tasks. Thin coatings on lenses can be used to reduce troublesome reflections in optical systems such as compound lenses with many optical surfaces routinely used for camera and microscope lenses. Such nonreflective coatings can be detected by the characteristic faint bluish color of reflected light. Special multilayered coatings on a glass surface can be used as an interference filter to select a very limited wavelength (with a range of less than 1 nm) region to be transmitted. Interference filters are particularly useful in spectroscopy. They are also used in laser safety goggles to block only the narrow wavelength range of laser light.

---

**FIGURE 22.7** A cell, solvent, glass cover slip (from bottom up) sample in a reflection-interference microscope.

**FIGURE 22.8** (left) Total internal reflection fluorescence microscopy image of a fluorescently labeled cell surface showing focal contacts (darker regions are closer to glass slide with lightest imaged regions about 85 nm from glass). (right) Atomic force microscopy (see Chapter 8) image of the same cell, but the opposite surface seen in the left image. The cytoskeleton is visible here showing its contacts with the glass surface.
2. SINGLE-, DOUBLE-, AND MULTIPLE-SLITS AND INTERFEROMETERS

2.1. SIMPLE INTERFERENCE WITH A DOUBLE-SLIT

Consider the experiment sketched in Figure 22.10 in which a light wave is incident on two closely spaced slits, an arrangement known as Young’s double-slit experiment. Light passing through either slit will be diffracted, spreading out in waves that overlap on a screen some relatively large distance away. If the light waves at the two slits begin in phase, when the two waves arrive and overlap at some point on the screen their phase difference is simply related to the difference in the optical paths they traveled from either slit. In this case because the medium is simply air, the optical paths are the physical distances traveled and we can calculate the path difference using the geometry shown in the inset in the figure.

At positions on the screen where the optical path difference (shown in the figure to be \(d \sin \theta\)) is equal to an integral number of wavelengths of light, there will be constructive interference according to

\[ d \sin \theta = m \lambda, \quad \text{(constructive interference)}, \quad (22.5) \]

where \(d\) is the slit separation, \(\theta\) is the angle measured at the slits between the optic axis and the point on the screen, and \(m\) is the integer order number. A value of \(m = 0\) corresponds to the point on the optic axis at the screen where clearly, by symmetry, the path difference is zero and we expect the two arriving waves to be in phase; this is known as the central maximum. There is also a corresponding condition for destructive interference given by

\[ d \sin \theta = \left( m + \frac{1}{2} \right) \lambda. \quad \text{(destructive interference)}. \quad (22.6) \]

In this case, because the incident intensity at the two slits is equal, the destructive interference is complete and this equation gives the set of locations that are completely dark.

The light pattern on the screen will consist of a set of alternating bright and dark fringes (slit-shaped regions) spaced periodically and symmetrically about the optic axis. The first fringe to either side of the central maximum is known as the first-order maximum, the second bright fringe to either side is the second-order maximum, and so on. Because the distance to the screen, \(D\), is very large compared to the slit spacing, for small angles from the optic axis, the positions \(y\) at which bright fringes occur on the screen measured from the optic axis \(y = 0\) are given by \(\tan \theta \approx \sin \theta = y/D\) (see Figure 22.11). Substituting from Equation (22.5), the fringes are equally spaced along the screen at the positions \(y = m\lambda D/d\).

**Example 22.2** A double-slit pattern is observed on a screen 5 m away from the slits, which are separated by 0.05 mm. If the first bright fringe is observed to lie a distance of 4.6 cm from the optic axis on which the central interference maximum lies, find the wavelength of the light producing the pattern.

**Solution:** The angle that the observation point (bright fringe) makes with the optic axis is given by \(\sin \theta \approx \tan \theta = 0.046 \text{ m}/5 \text{ m} = 0.0092\). Substituting this expression into Equation (22.5) for interference maxima and choosing \(m = 1\) for the first-order, we find \(\lambda = d \sin \theta = 4.6 \times 10^{-7} \text{ m} = 460 \text{ nm}\), a blue color.
Let's consider the light pattern on the screen in a bit more detail. If \( E_1 = E_0 \cos(\omega t) \) is the oscillating electric field from slit 1 at a point on the screen, then at a point of constructive interference the electric field from the second slit will be \( E_2 = E_0 \cos(\omega t + m2\pi) = E_0 \cos(\omega t) \), because the phase difference must be a multiple of \( 2\pi \). Furthermore, because the intensity on the screen is proportional to \( E_{\text{net}}^2 = (E_1 + E_2)^2 \), we have

\[
I_{\text{constr}} = 4I_o,
\]

(22.7)

where \( I_o = E_0^2 \) is the intensity at the screen produced by either slit (this could be directly measured by covering one of the slits). Similarly, at an interference minimum where \( E_1 \) and \( E_2 \) are \( 180^\circ \) out of phase so that \( E_{\text{net}} = (E_1 - E_2) = 0 \), we have

\[
I_{\text{destr}} = 0.
\]

(22.8)

Figure 22.11 shows the predicted distribution of the intensity along the screen produced by interference, given, for small \( \theta \), by \( I = 4I_o \cos^2(\theta dy/DA) \). Note that although the maxima have four times the intensity of a single slit, the average value of the intensity (easily found as the value of \( I \) about which the curve is symmetric, the dashed line in the figure) is just equal to twice the intensity of a single slit or the total intensity of the incident light passing beyond both slits. This must be the case according to conservation of energy with the total energy separately passing through each slit adding up to the total detected at the screen. The effect of interference is to spatially redistribute the intensity of light in a characteristic interference pattern of light.

In order to observe this double-slit interference pattern the two incident light beams must have the same frequency and be in phase at the two slits (or at least have a definite time-independent phase relation); two such beams are said to be coherent. If two completely incoherent beams, those with no definite phase relation (see below), were used, one at each slit, no interference pattern would be observed and the intensity at the screen would simply be the sum of the individual intensities of each light beam according to

\[
I = I_1 + I_2. \quad \text{(incoherent light).}
\]

(22.9)

What determines whether light is coherent or incoherent? Actually there are various gradations in the coherency (or degree of phase integrity) of light in both time and space. Light can have a definite phase relation over various spatial distances across the transverse direction of a beam (leading to a well-defined wavefront and known as spatial coherence) as well as over various periods of time corresponding to definite phase relations over different distances along its direction of travel (leading to a well-defined wave shape and known as temporal coherence). For example, light from different portions of the heated filament of an incandescent light bulb has no definite phase relationship because the electrons generating the light at different locations do not interact with each other and emit their light in an uncorrelated manner. Incandescent light is thus incoherent both spatially and also temporally. In contrast, we show that laser light is an excellent source of (spatially and temporally) coherent light.

Nonlaser light can always be made spatially coherent by using an arrangement similar to Figure 22.3 to create a point source of such light generating a plane wave as discussed earlier. Thus, all across a plane wave light will be in phase because it has traveled the same path length from the point source. However, the longitudinal distance over which the light is coherent is determined by its temporal coherence, the time during which there is a definite phase relation. Light emitted by specific electron transitions from excited states to lower energy states in atoms or molecules (discussed further in Chapter 25) takes place over a finite “lifetime,” or coherence time \( \tau_{\text{coh}} \), of those electronic states.

The longer the coherence time, the greater the distance along the direction of travel over which there is a definite phase variation and so we identify this as the coherence...
length, \( \ell_{\text{coh}} \) given by \( \ell_{\text{coh}} = c\tau_{\text{coh}} \) (Figure 22.12). Greater coherence lengths correspond more closely to pure sine waves and a shorter coherence time corresponds to a larger range of frequencies \( \Delta f \), present with

\[
\tau_{\text{coh}} \propto \frac{1}{\Delta f}
\]

Incandescent light sources have very broad frequency (color) content and corresponding extremely short coherence lengths. If a colored filter is used to reduce the range of frequencies present then the coherence length of the light can be increased somewhat. The purer the color of the light beam is, the longer its coherence time and length will be. Some lasers generate extremely pure colors of light and have coherence lengths of many kilometers.

Now, to understand the effect of coherence on an interference experiment, first consider the simple case of two waves that have slightly different frequencies, but that start in phase. After traveling some distance, they will lose their common phase because of the frequency difference, as shown in Figure 22.13. In an interference experiment with light, the phase relations between different beams are of utmost importance. Light from any real source, such as the filament of an incandescent light bulb, can be made spatially coherent as we have seen, after focusing down to a point and using the distant plane wave produced, but it will still have a particular coherence length due to temporal coherence. If the coherence length is not longer than the distances involved in the experimental geometry, each beam will itself not have a definite phase over the entire path, even if both start out in phase together, and it will be impossible to observe any interference between different beams. Light from an incandescent bulb has a very short coherence time of about \( 10^{-10} \) s, corresponding to a coherence length of a few cm. Unless the optical path length differences are very small, interference experiments with incandescent bulbs are not generally possible.

### 2.2. SINGLE-SLIT DIFFRACTION

So far we have only very loosely defined diffraction to be the bending of light at a sharp edge or obstacle and have discussed in principle how Huygens’ construction can be used to determine the angular spread of the diffracted light. One of the early and most striking demonstrations of diffraction, and the one most responsible for the final acceptance of a wave picture for light in the early 1800s, is the bending of light around a small circular obstacle such as a coin. At that time, Newton’s theory of light, treating light as a particle, still dominated the scientific world. Fresnel’s early wave theory of light submitted to the French Academy of Sciences in 1818 quickly led Poisson, a nonbeliever of the wave theory and member of the Academy, to deduce a prediction of Fresnel’s theory that seemed absurd to him. Poisson claimed that the wave theory should lead to a central bright spot in the shadow of the object, a prediction that
invalidated the theory as far as he was concerned. Arago then performed the experiment and, to everyone’s amazement, discovered what is now termed the Poisson–Arago spot (see Figure 22.14). This dramatic event led to the rapid acceptance of the wave theory of light. Rather than analyze this experiment, we consider the mathematically simpler case of the diffraction of light at a narrow slit.

Figure 22.15 shows the experimental arrangement with a single-slit of width $a$ illuminated by a plane wave of monochromatic light and the pattern of light examined on a screen located a distance $D$ from the slit, with $D \gg a$. This is an example of Fraunhofer diffraction, in which the diffracted light is examined at a large distance from the slit, in the so-called far-field. If the screen were close to the slit, the diffraction pattern would be more complex as well as more mathematically difficult to analyze; this near-field diffraction is known as Fresnel diffraction.

The Fraunhofer diffraction pattern from a single-slit consists of a central bright maximum surrounded by a series of secondary maxima of decreasing intensity known as fringes (Figure 22.16). Notice in the figure that the central maximum is wider than the other secondary maxima; its width is inversely related to the slit width. The narrower the slit is, the greater the extent of diffraction and correspondingly the wider the central maximum. We can determine the locations of the maxima and minima, or fringe boundaries, in the diffraction pattern, by using a simple argument based on the...
phase relations of the wavelets emitted at the slit when they arrive at the screen. Those rays that emerge parallel to the optic axis remain in phase and produce a bright central maximum. To find the position of the first minimum on either side, consider those rays that are deviated by an angle $\theta$ from the optic axis, such that the path difference between a ray from one edge of the slit is one wavelength $\lambda$ more than that from the opposite edge as shown in Figure 22.17. Because all these rays are parallel, at this angular condition the rays will cancel in pairs as can be seen from the following argument. The ray from the center of the slit will have a path length to the screen of $\frac{\lambda}{2}$ more than the ray from the bottom of the slit and hence these two rays will destructively interfere. Using the same argument repeatedly, we can consider neighboring rays just above each of those in a stepwise fashion, rays that will cancel pairwise because the path difference will remain at $\frac{\lambda}{2}$ all the way up the slit, to conclude that there is no light at this point. At this angle we have, from the insert in the diagram,

$$\sin \theta = \frac{\lambda}{a}$$

(first diffraction minimum),

Equation (22.10)

defining the distance along the screen from the optic axis $y$ to the first minimum to be $y = D \tan \theta \approx D \sin \theta = D\lambda/a$.

The next larger angle at which we can have pairwise destructive interference and thus a diffraction minimum is shown in Figure 22.18. The slit is imagined to be divided into four equal portions with a total path difference of $2\lambda$ between the top and bottom, so that there will again be pairwise cancellation for rays within each separate half of the slit just as above. For this case we must have that

$$(a/2)\sin \theta = \lambda \quad \text{or} \quad a \sin \theta = 2\lambda.$$ 

Continuing this construction, the general condition for a diffraction minimum becomes

$$a \sin \theta = m\lambda,$$

(diffraction minima),

Equation (22.11)

where $m$ is a nonzero integer. Notice that this equation for the single-slit dark fringes is very similar to Equation (22.5) for the location of bright interference fringes in the double-slit experiment and the symbol meanings must be kept carefully in mind.

Equation (22.11) predicts that for a given wavelength of light, the width of the diffraction pattern on a screen is inversely related to the slit width; that is, for small angles $\sin \theta \sim \theta \propto \frac{1}{a}$. This means that the smaller the slit width is, the wider the observed fringe pattern on a distant screen. Conversely, slits that are very wide compared to the wavelength of light only show a faint fringe pattern near the geometric shadow of the slit edges, with no other diffraction effects occurring in this geometrical optics limit. This was the limit that we investigated in the previous two chapters.
The pattern of intensity variation in the diffraction fringes from a single-slit is more difficult to derive but is shown graphically in Figure 22.15. The central bright fringe is twice as wide as the other maxima and is much brighter. The secondary maxima are less than 5% as intense as the central maximum with the intensity rapidly decreasing for higher order maxima.

2.3. DOUBLE-SLIT INTERFERENCE RECONSIDERED

Returning to the double-slit experiment, the intensity profile of the interference fringes will be governed by diffraction from each slit. Instead of the profile shown earlier in Figure 22.11 where diffraction effects were ignored, the actual pattern observed, an example of which is shown in Figure 22.19, is a convolution (productlike combination) of the interference and diffraction effects and depends on the particular slit widths and separation. With the individual slit widths small compared to the slit separation, the bright central diffraction maximum will have many interference minima where no light falls on the screen in contrast to the diffraction pattern of a single-slit shown before in Figure 22.16. Light from each slit interferes with that from the other slit and produces a characteristic fringe pattern with a fringe spacing given in the small angle approximation by 

\[
\Delta y = \left( \frac{\lambda d}{D} \right)
\]

and the diffraction minima are spaced by

\[
\Delta y = \left( \frac{\lambda d}{D} \right)
\]
Because $d > a$ the fringe spacing is smaller than the widths of the diffraction peaks. Fringe intensity from Figure 22.11 is modulated by the single-slit diffraction patterns from each slit that essentially overlap on the screen.

**Example 22.3** Let’s return to Example 22.2 with two slits separated by 0.05 mm with a screen 5 m away and examine the problem in more detail. Suppose that the two slits are identical and each has a width of 0.01 mm. What will the pattern on the screen look like when illuminated with light at 460 nm?

**Solution:** In the previous example we were told that the interference fringes were spaced 4.6 cm apart on the screen. Having learned about diffraction from slits, we now know that the overall intensity on the screen will be modulated by the single-slit diffraction pattern. The central diffraction maximum lies within an angle given by $\sin \theta = \frac{\lambda}{a}$, where $a$ is the given slit width, so $\sin \theta = 460 \text{ nm} / 0.01 \text{ mm} = 460 \times 10^{-9} \text{ m} / 1 \times 10^{-5} \text{ m} = 0.046$, corresponding to a distance along the screen of $y \approx D \sin \theta = (5 \text{ m})(0.046) = 23 \text{ cm}$ from the optic axis to the first diffraction minimum. Because according to Example 22.2 each interference fringe is spaced 4.6 cm from the next, the fifth fringe from the central one, on either side, actually falls directly on the first diffraction minimum and therefore will have zero intensity and not be seen. Within the central diffraction maximum there then will be four fringes visible on either side of the central fringe on the optic axis, making a total of nine fringes within the central diffraction maximum. If we look further off axis, the next minimum in the diffraction pattern occurs when $\sin \theta = 2\lambda/a = 0.092$, so that it occurs at a distance of 46 cm from the optic axis. We find that the (46/4.6) = 10th interference fringe lies at this location and so is also not visible. Therefore within the second diffraction maximum on either side there are four interference fringes. The resulting pattern of fringes is somewhat similar to the photo shown in Figure 22.19, but with different numbers of fringes observed.

### 2.4. MULTIPLE SLITS AND DIFFRACTION GRATINGS

As the number of slits with the same width and spacing is increased beyond two, the patterns of light and dark on a distant screen at first become more complex, with each slit still creating the same Fraunhofer diffraction pattern as in our single-slit discussions but with the interference pattern within the diffraction peaks having more detail. The angular positions of the bright interference fringes are the same as for the double-slit, Equation (22.5) above, independent of the number of slits, namely

$$d \sin \theta = m\lambda,$$

where $d$ is the uniform slit spacing between any adjacent pair of slits and $m$ is the order number. In fact, the same exact reasoning holds in deriving this equation, because if light from two neighboring slits has a path difference of $m\lambda$, then light from any pair of slits, neighboring or not, will still have a path difference that is a multiple of the wavelength of light.

On the other hand, the nature of the minima and the width of the maxima both change with the number of slits. With an increasing number of slits, the secondary maxima dramatically decrease in intensity and the central maximum narrows in width. Figure 22.20 illustrates this sharpening of the central maximum with an increasing number of slits. The larger number of slits makes the condition for constructive interference from all the slits that much more stringent. With only two slits, points on the screen near
the central maximum peak have path differences that are only slightly different from an integer number of wavelengths and so there is only a gradual decrease in intensity away from the peak. With many slits, even if the rays from two neighboring slits have a path difference of only a small fraction of a wavelength, the path difference from the 100th slit away is increased by a factor of 100 and much more destructive interference occurs. This is the reason for the much sharper central maximum with many slits.

Diffraction gratings are devices that have a very large number of very narrow slits, separated by distances comparable to the wavelength of the light. The best gratings for visible light have more than 30,000 lines per inch (or spacings of less than 1 \( \mu m \) apart). There are two fundamental types of gratings for optical work: transmission gratings, of the type we have been discussing, and reflection gratings that have their fine rulings made on a mirrored surface. Diffraction gratings give very sharp interference peaks, so that with monochromatic light, such as that from a laser, there will be a series of small spots, one for each order of Equation (22.12).

The real utility of diffraction gratings is their ability to analyze polychromatic light as “spectrum analyzers,” dispersing all the colors present in a particular light source (Figures 22.21 and 22.22). Equation (22.12) indicates that for a given slit spacing, or its inverse, known as the grating constant which is the number of lines per unit distance,
different wavelengths of light will be diffracted at different angles. Gratings can thereby serve as prisms, dispersing light of different colors to produce a spectrum. According to Equation (22.12), light of longer wavelengths will be diffracted by larger angles in contrast to a prism which refracts light of shorter wavelengths more because of dispersion. For each order, the grating will produce an entire spectrum, except for the central maximum (zeroth order) at which all colors superimpose. In grating spectroscopy, a source of light is collimated, directed on a grating, and the diffracted light detected. We show the application of spectroscopy in the study of atomic physics in the next chapter.

2.5. INTERFEROMETERS

Optical devices known as interferometers split a light beam into two beams that travel different routes and are then brought together to interfere. One important example, known as a Michelson interferometer, is shown schematically in Figure 22.23. The beamsplitter $S$ divides the incident light into two portions, one reflected and one transmitted at its back surface. These are separately reflected by mirrors $M_1$ and $M_2$ and the reflected beams are recombined by the beamsplitter and observed by a detector $D$. The path differences in the two “arms” of the interferometer must be shorter than the coherence length of the light in order to observe interference effects at the detector. Typically one mirror is slowly and precisely moved along its axis and the fringes shift at a rate of 1 fringe per a path difference equal to the wavelength of light.

Interferometers can be used for a variety of purposes including, for example, measuring the wavelength of light or accurately measuring optical distances or changes in optical distances. This can be accomplished simply by counting fringes at the detector and knowing that each fringe corresponds to an optical path difference of one wavelength. Interferometers are often useful to check on the quality of optical components during and after manufacture. They are also useful to determine refractive indices of transparent materials by inserting a sample in one branch of the interferometer, thus increasing the optical path in that branch and measuring its optical distance compared to its physical distance.

3. RESOLUTION

In order to distinguish by eye two objects that are very close together, whether they be microscopic objects or stars in the sky, we can use lenses to magnify the objects. Aside from lens aberrations that can limit the quality of the images as discussed in the last chapter, diffraction imposes a fundamental
limit on our ability to discern two closely spaced objects. Most optical instruments use circular rather than slit apertures, therefore before discussing such limits on resolution we first briefly consider the diffraction of light by a circular aperture.

The far-field (Fraunhofer) diffraction pattern from a circular aperture consists of a central circular maximum, known as the Airy disk, surrounded by a set of circular fringes (Figure 22.24). A similar, but more complex, derivation to that for a slit shows that the angular spread of the Airy disk (first-order minimum location) is given by

\[ d \sin \theta = 1.22 \lambda, \]  

(22.13)

where \( d \) is the diameter of the aperture. Effectively, \( d/1.22 \) is the average width of the equivalent slit representing the circular aperture so that \( (d/1.22) \sin \theta = m \lambda \) resembles the single-slit diffraction equation. A photo of the intensity profile of the image of a circular aperture is somewhat deceiving because the secondary maxima are much dimmer (<5%) than the Airy disk.

Two closely spaced objects will each produce a diffraction pattern in the image of an optical system. When the two objects are so close that their Airy disks overlap in the image, it becomes very difficult to distinguish whether there are actually two objects present or just one. The Rayleigh criterion is the accepted condition for the resolution of two such objects: two objects are just resolved when the central maximum of one is superimposed on the first diffraction minimum of the other. From Equation (22.13), the Rayleigh criterion can be written as

\[ \theta_{\text{min}} = \frac{1.22 \lambda}{d}, \]  

(22.14)

where, because the angles are small, \( \theta_{\text{min}} \) represents the minimum angular separation (in radians) of two objects as shown in Figure 22.25 and \( d \) is the aperture size. Figure 22.26 shows the diffraction patterns observed when two distant point sources of light get progressively closer. As the angle subtended by the point sources at the aperture gets smaller the images of the two point sources coalesce and blur, so that eventually they cannot be resolved. Thus, in order to increase the resolution of an optical system, the shortest wavelength and the largest aperture possible are desired.
With a microscope it is more common to discuss resolution in terms of the minimum separation distance of two objects under optimal conditions, known as the resolving power, rather than resolving angle. A straightforward (but omitted) derivation shows that the resolving power is given by

\[ r_{\text{min}} = \frac{0.61\lambda}{n \sin \alpha} = \frac{0.61\lambda}{NA}, \tag{22.15} \]

**Example 22.4** For the human eye, with a pupil diameter of about 2 mm and using a wavelength of 500 nm, calculate the minimum angle separating two just resolvable points. Then find the actual minimum distance between such just resolvable points as well as the distance between their images on the retina.

**Solution:** Using Equation (22.14), we find that

\[ \theta_{\text{min}} = \frac{(1.22)(500 \times 10^{-9})}{2 \times 10^{-3}} = 3 \times 10^{-4} \text{ rad}, \]

corresponding to about 1 min of arc. The minimum spatial separation occurs when the objects are placed at the near point of the eye (taken as 25 cm). This distance between two just resolved points is then \((0.25 \text{ m})(3 \times 10^{-4}) = 75 \mu\text{m}\), somewhat less than 1/10 mm. So, thinking of the 1 mm divisions on a ruler, our eyes can resolve two objects that are apart by about 1/10 of the smallest mm division on the ruler. The corresponding separation distance between the central maxima of the two images on the surface of the retina (using a lens–retina distance of 2 cm) is 

\( (0.02 \text{ m})(3 \times 10^{-4}) = 6 \mu\text{m} \). Note that cones have an average spacing in the fovea of about 2 \( \mu\text{m} \), so that the detector size is three times smaller than the diffraction limiting image size. To have an effective resolution this high, it appears that at least one nonactivated cone must lie between two other activated cones. Only in the fovea do individual cones have 1:1 connections with nerve cells going to the visual cortex. Our eyes are exquisitely designed to provide the best possible resolution for the physical dimensions of our eyeball. There would be no improvement in the visual resolution of our eyes by having smaller cone cells, because diffraction is the fundamental limit and not the size of the cones.

**FIGURE 22.26** Diffraction patterns observed for the situation in Figure 22.25 with progressively closer objects.
where $\alpha$ is the acceptance angle of the light from the objects at the objective lens (Figure 22.27), $\lambda/n$ is the wavelength of light in the medium between the sample and lens, $0.61 = 1.22/2$, and the product $(n \sin \alpha)$ is known as the numerical aperture (NA) of the lens. For routine microscopy $n = 1$, whereas for higher magnifications, an oil-immersion objective is often used to increase the resolving power. In this case a drop of immersion oil (typical $n = 1.5$) is placed on the cover slip between the sample and the lens, increasing the resolving power (by decreasing $r_{\min}$) by about 50%. The larger the numerical aperture of a lens, the greater is its resolving power. Numerical apertures of 1.4 are commonly used at high resolutions with optical microscopes. Equation (22.15) then tells us that the very highest resolution obtainable with a light microscope is about $\lambda/4$. Because $\sin \alpha$ is limited to a maximum value of 1, the only way to further improve resolution in a microscope is to decrease the wavelength of the probing radiation. We show how this is done using an electron microscope in the next chapter.

**CHAPTER SUMMARY**

In discussing the different optical wave phenomena in this chapter, what is important is not the physical distance that light travels, but rather the optical path, defined by

\[ \text{Optical Path} = \sum n_i d_i, \quad (22.2) \]

where $n_i$ and $d_i$ are the index of refraction and distance traveled in the $i$th segment of the path.

Diffraction is the bending of waves around obstacles or the spreading of waves passing through an aperture. Interference is the superposition of waves in space leading to constructive and destructive interference. One example is interference in thin films of index $n$ and thickness $t$, where light of wavelength $\lambda$ reflected normally will experience destructive interference if

\[ 2nt = m\lambda, \quad \text{(destructive interference).} \quad (22.3) \]

This phenomenon is exploited in the reflection-interference microscope to study a thin surface layer and in the use of nonreflective glass coatings.

Young’s double-slit interference, in which coherent light passes through a pair of slits separated by distance $d$ and is viewed at a distance at an angle $\theta$, leads to constructive interference at angles such that

\[ d \sin \theta = m\lambda, \quad \text{(constructive interference).} \quad (22.5) \]

The order number $m$ is an integer. Each of the slits of width $a$, in turn, diffracts the light and produces a single-slit diffraction pattern governed by

\[ a \sin \theta = m\lambda, \quad \text{(diffraction minima),} \quad (22.11) \]

where $\theta$ is the angle to the first diffraction minimum. The overall pattern observed in a double-slit experiment is the convolution (productlike mix) of both patterns of intensity of light. A common device, the diffraction grating, has numerous closely spaced slits, of separation $d$, and gives an intensity diffraction pattern of brightness governed by the grating equation

\[ d \sin \theta = m\lambda. \quad (22.12) \]

Resolution is limited by diffraction. Rayleigh’s criterion for the threshold of resolution, the minimum angular separation $\theta_{\min}$ of two barely resolvable objects, viewed through an aperture of size $d$, is given by

\[ \theta_{\min} = \frac{1.22\lambda}{d}. \quad (22.14) \]

In a microscope, this can be shown to give a resolving power, or minimum separation of two just resolved objects, of

\[ r_{\min} = \frac{0.61\lambda}{n \sin \alpha} = \frac{0.61\lambda}{NA}, \quad (22.15) \]

where $\alpha$ is the light acceptance angle and $NA$ is the numerical aperture, defined in the equation.
QUESTIONS
1. Which of the following properties of light do not depend on the material medium in which the light travels: speed, frequency, wavelength, optical path, and index of refraction?
2. A garden hose has an adjustable nozzle which determines whether the water comes out as a tightly focused jet of water or as a wide-angled spray. As the nozzle is adjusted from a jet to a spray is the size of the exit aperture increasing or decreasing?
3. In a concert hall which sound waves bend more on leaving the hall at the open rear doors, high C or low C?
4. Why does a thin film of oil or gasoline on water appear multicolored as in Figure 22.6?
5. Why is the correct thickness of a nonreflective coating on a glass surface equal to λ/4 where λ is the wavelength of the light in the film? Assume the index of refraction of the coating is less than that of the glass.
6. Show that the two angles labeled θ in Figure 22.10 are in fact equal so that the path difference can be written in terms of the diffraction angle.
7. When listening to a weak radio station on your car radio often the reception will fade in and out while slowly driving along. What is this due to and can you use its source as a way to estimate the wavelength and frequency of the radio waves?
8. Interference is a phenomenon that takes an otherwise uniform energy density and re-distributes the energy into maxima and minima with the same total energy. Discuss this statement.
9. Why does the source of light for Young’s double-slit experiment need to be coherent? What pattern would be observed if an ordinary light bulb were used as a light source right behind the slits?
10. Sketch a picture of the central maximum and first peak to either side in the intensity pattern seen on a distant screen in a double slit experiment with slits of width 1/8 of the slit spacing. Check the angle at which the minimum in the diffraction pattern occurs relative to the interference peaks and be sure to draw the correct number of interference peaks within each diffraction peak. What does the pattern look like when one of the two slits is covered?
11. Why is no interference pattern seen when looking at a car’s headlights from a distance on a road at night? After all, the light comes from two “point sources” close together.
12. With regard to a diffraction grating, Equation (22.12) is called the grating equation. What is the meaning of d in this equation? What is d for a grating with 10,000 lines per cm?
13. Discuss the meaning of θ_{min} in Equation (22.14). In particular, where is the angle measured from, to where is it measured, what is the meaning of d and does a higher resolution mean a greater or smaller θ?
14. Suppose that the density of cones in the macula was a factor of 10 greater. Would the resolution of the human eye be increased with no other changes? Explain.
15. What is the purpose of immersion oil when used at the objective of a compound microscope? Explain how it works.

MULTIPLE CHOICE QUESTIONS
Questions 1 and 2 refer to two parallel light rays, initially in phase and having a 500 nm wavelength, that reach a detector after one of the rays travels through a 10 cm long block of glass with an index of refraction of 1.5 as in Figure 22.2.
1. The optical path difference between the two rays at the detector is (a) 10 cm, (b) 750 nm, (c) 15 cm, (d) 5 cm.
2. The total number of wavelengths shifted between the two rays when they reach the detector is (a) 100,000, (b) 200,000, (c) 300,000, (d) 500,000.
3. The equations for double-slit interference and single-slit diffraction, mλ = d sin θ and mλ = a sin θ, respectively, are very similar. Which of the following is true? (a) m can take the value zero in the first equation, but not in the second, (b) m can take the value zero in the second equation, but not in the first, (c) m can take exactly the same values in both equations because the two equations describe exactly the same interference pattern, (d) m is the mass of the electron in both equations.

For Questions 4 and 5, consider the four patterns shown to the right. A box corresponds to a bright spot.

4. The pattern most likely to have been produced by a single fairly wide slit is (a) a, (b) b, (c) c, (d) d.
5. The pattern most likely to have been produced by a diffraction grating is (a) a, (b) b, (c) c, (d) d.
6. The figure below shows an approximate plot of intensity as a function of position for a double-slit interference pattern. The vertical bars represent where light is observed. Between the vertical bars are dark spots. The most likely reason the maximum intensities are not all the same is (a) the beam isn’t centered properly, (b) the slit openings have a finite size, (c) that’s what is predicted for two slits that have infinitesimal openings, (d) there are actually more than two slits.
7. Which of the following best describes the angles at which dark spots will be observed in a double-slit diffraction pattern, if \( d \) is the slit separation, \( \lambda \) is the wavelength of the incident radiation, and \( m = 0, \pm 1, \pm 2, \pm 3, \ldots \)? \( \theta \) is measured relative to the incident direction and its sine equals (a) \( m\lambda d \), (b) \( (2m + 1)\lambda/(2d) \), (c) \( m\lambda d \), (d) \( 2m\lambda/(2m + 1)\lambda \).

8. Near-normal 500 nm light is reflected from a thin organic film (with \( n = 1.5 \)) on water. What minimum thickness results in destructive interference? (a) 83 nm, (b) 167 nm, (c) 250 nm, (d) 330 nm.

9. If the first-order double-slit diffraction minimum lies at the same place as the fourth-order interference maximum, how many fringes will be visible in the central diffraction maximum? (a) 3, (b) 5, (c) 6, (d) 7.

10. When two microscope slides are placed together and a laser beam is passed through the two at near normal incidence, if the slides are squeezed together, the pattern of reflected light moves. This is best explained by (a) the gap between the slides is changed by squeezing, (b) the thickness of the slides is changed by squeezing, (c) the index of refraction of the air in the gap between the slides is changed by squeezing, (d) the index of refraction of glass is changed by squeezing.

11. What is the approximate minimum coherence length of a 1 cm diameter 500 nm light beam needed to observe a fringe pattern in a Michelson interferometer with mirror-to-detector distances of 0.2 m? (a) 1 \( \mu m \), (b) 1 cm, (c) 0.5 m, (d) 1 km.

12. When a car is 500 m ahead of you, you see its tail lights as one long, red light. When the car is 100 m ahead of you, you see that the tail lights are actually several red lights placed close to each other. This is because (a) the pupil of your eye has a finite size, (b) the Doppler effect for light shifts the frequency of the tail lights, (c) light disperses in the lens of your eye, (d) light is made up of photons.

13. When a sheet of paper is 20 cm from your face you can see two small dots of ink. When the paper is a meter from your face the dots appear as a single dot. That is most likely because (a) the pupil of your eye has a finite size, (b) you are farsighted, (c) light disperses in the lens of your eye, (d) light from the dots is polarized.

14. To improve the resolving power of a microscope one can do all of the following except (a) increase the wavelength of light, (b) use immersion oil to index match the glass slide to the glass objective, (c) maximize the acceptance angle, (d) increase the power of the objective lens.

15. The maximum resolution of an optical microscope is about (a) 1 nm, (b) 1 mm, (c) 1 \( \mu m \), (d) 1 \( \AA \).

**PROBLEMS**

1. A thin film of oil with refractive index 1.5 on a water puddle is illuminated from directly overhead by white light. If there is an interference maximum at 600 nm and a minimum at 450 nm with no other minimum in between, what is the film thickness, assumed uniform?

2. A soap film with index of refraction 1.33 is surrounded by air and illuminated by white light. If the film is 265 nm thick, which wavelength in the range 400–750 nm will interfere constructively?

3. If a soap film of refractive index 1.33, surrounded by air, is illuminated by a red (633 nm) and a green (515 nm) light, find the minimum film thickness at which the reflected light will appear red. Repeat to find the thickness at which it will appear green.

4. If a 220 nm thick soap film (\( n = 1.33 \)) is placed on a glass slide (\( n = 1.5 \)) and illuminated with white light, find the wavelengths that interfere constructively in the reflected light. (Note: To do this problem you will need to reanalyze the derivation of Equations (22.3) and (22.4) when there is an additional \( \pi \) phase shift at the second interface.)

5. For a reflection-interference microscope, what is the minimum cell thickness that results in maximum contrast between adhesion sites and the cell region when illuminated with blue light (480 nm)? Take the index of refraction of the cell to be that of water.

6. In a double-slit experiment with red light (633 nm) using slits with 0.12 mm separation, what is the fringe spacing on a screen that is 3.5 m from the slits.

7. If the two slits in a double-slit experiment each have a width of 0.08 mm and a spacing of 0.24 mm, how many interference peaks will lie within the central diffraction maximum using 488 nm light?

8. In a double-slit experiment with 500 nm light, the slits each have a width of 0.1 mm. (a) If the interference fringes are 5 mm apart on a screen which is 4 m from the slits, determine the separation of the slits. (b) What is the distance from the center of the pattern to the first diffraction minimum on one side of the pattern? (c) How many interference fringes will be seen within the central maximum in the diffraction pattern? Draw a sketch of the pattern.
9. In a double-slit experiment with two slits of width 1.0 μm spaced 4 μm apart, suppose a beam of electrons is incident on the slits after being accelerated from rest through a potential difference of 100 V.
(a) We show in Chapter 24 that electrons (and all particles) have a wavelength given by \( \lambda = \frac{h}{p} \), where \( h \) is Planck’s constant and \( p \) is the particle’s momentum. What is the wavelength of the electrons in this problem?
(b) If the pattern of detected electrons is observed on a fluorescent screen 20 m from the slits, what is the width of the central diffraction maximum?
(c) How many interference fringes will be observed within the central diffraction maximum?

10. In a double-slit experiment, each slit has a width of 0.02 mm and they are spaced 0.14 mm apart. The pattern of light when a coherent 550 nm beam is incident on the slits is observed on a screen 4 m away.
(a) Find the spacing between interference fringes on the screen.
(b) Find the full width of the central diffraction maximum on the screen.
(c) How many fringes are visible within the central diffraction maximum?
(d) What happens to the pattern if the entire apparatus is immersed in water? Find new answers to each of the above parts.
(e) Describe what will happen to the pattern of light if a different polarizer is placed in front of each slit with their transmission axes at right angles to each other (assume the incident light is unpolarized). Will the intensities change? Will the pattern change? Give reasons for your answers.

11. Georges Seurat, a post-impressionist French painter, used a technique of painting with small dots of color placed close together on a canvas. From sufficiently far away, the dots cannot be distinguished and the painting looks normal in appearance.
(a) If the dots on the painting are separated by 1.5 mm and the painting is observed under light of 550 nm wavelength with eyes having a pupil diameter of 2 mm, find the minimum distance you must stand from the painting so that the individual dots cannot be resolved.
(b) Under the conditions of part (a), what would be the distance between the images of dots on the retina of the eye, 2.0 cm behind the lens of the eye.
(c) If a small scaled copy of a Seurat painting is made which is 100-fold smaller in size, the dots would be expected to be spaced only 0.015 mm, too small to be resolved with the naked eye. Using a compound microscope with a 10x eyepiece and 17 cm tube length, what maximum focal length of objective lens would be needed to test the scaled copy for authenticity (meaning that the painting is made from small dots, rather than continuous color).

12. A person’s eye has a pupil diameter of 0.2 cm and has a length of about 2.5 cm (from lens to retina). If light of 550 nm is used, find the following.
(a) The minimum distance between resolved images on the retina of the eye, ignoring any lens aberration.
(b) The distance apart that two point sources of light can have at the near point of the eye (\( N = 25 \) cm) and just be resolved.
(c) What microscope magnification would be needed for the eye to just clearly image these two point sources of light if they are only 500 nm apart?
(d) With a 10x eyepiece and a tube length of 17.0 cm, find the objective focal length needed to achieve the magnification in part c.

13. In a Michelson interferometer when using laser light of 488 nm, how many fringes are scanned at the detector if one of the mirrors is displaced by 2.4 μm.

14. A Michelson interferometer is used to determine the wavelength of a monochromatic light source. If 100 fringes are counted as one of the mirrors is scanned a distance of 31.7 μm, what is the wavelength of the light?

15. Given that the resolving power of the eye corresponds to about 1 minute of arc, how far away can a vehicle be at night where you can still resolve whether it is a car (with two headlights separated by 1.8 m) or a motorcycle?

16. The Hubble space telescope has a resolution of about 0.1 s of arc. If aimed at the moon (3.9 \( \times \) 10^5 km away) what is its resolving power? If aimed at Saturn (1.3 \( \times \) 10^9 km away)? If aimed at the nearest galaxy (about 2 \( \times \) 10^6 light years, where a light year is the distance light travels in a year)?

17. In a double-slit experiment with two slits of width 1.0 μm spaced 4 μm apart, suppose a beam of protons is incident on the slits after being accelerated from rest through a potential difference of 2500 V.
(a) What is the speed of the proton?
(b) What is the wavelength of the proton?
(c) If the pattern of detected protons is observed on a fluorescent screen 20 m from the slits, what is the center-to-center spacing between the constructive interference maxima?
(d) What is the full width of the central diffraction minimum?
(e) How many interference fringes will be observed within the central diffraction minimum?