THE APPARENT FRACTAL CONJECTURE*

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Abstract

This short communication advances the hypothesis that the observed fractal structure of large-scale distribution of galaxies is due to a geometrical effect, which arises when observational quantities relevant for the characterization of a cosmological fractal structure are calculated along the past light cone. If this hypothesis proves, even partially, correct, most, if not all, objections raised against fractals in cosmology may be solved. For instance, under this view the standard cosmology has zero average density, as predicted by an infinite fractal structure, with, at the same time, the cosmological principle remaining valid. The theoretical results which suggest this conjecture are reviewed, as well as possible ways of checking its validity.

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*The issue of whether or not the large-scale distribution of matter in the Universe actually follows a fractal pattern has divided cosmologists in the last decade or so, with the debates around this thorny question leading to a split of opinions between two main, and opposing, groups.

On one side, the orthodox view sustains that since a fractal structure is inhomogeneous, it cannot agree with what we know about the structure and evolution of the Universe, as this knowledge is based on the cosmological principle and the Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, with both predicting homogeneity for the universal distribution of matter. Moreover, inasmuch as the cosmic microwave background radiation (CMBR) is isotropic, a result predicted by the FLRW cosmology, this group is, understandably, not prepared to give up the...
standard FLRW universe model and the cosmological principle, as that would mean giving up most, if not all, of what we learned about the structure and evolution of the Universe since the dawn of cosmology.\(^1\)\(^-\)\(^4\)

On the other side, the heterodox view claims that the systematic and, under their methodology, unbiased interpretation of astronomical data continuously shows that the distribution of galaxies is not homogeneous, being in fact completely inhomogeneous up to present observational scales, without any sign of homogenization. Therefore, this other group not only disputes the traditional interpretation of astronomical data, but also the validity of the cosmological principle, going as far as to implicitly suggest that the CMBR may be a cosmological puzzle.\(^5\)\(^-\)\(^8\)

To reach those opposing conclusions, the validity of the methods used by both sides of this fractal debate are, naturally, hotly disputed, and so far there has not yet been achieved a consensus on this issue. However, even if one is prone to part of the orthodox argument, i.e. that we cannot simply throw away some basic tenets of modern cosmology, like the cosmological principle and the highly successful FLRW cosmological model, when one looks in a dispassionate way at the impressive data presented by the heterodox group, one cannot dispel a certain uneasy feeling that something might be wrong in the standard observational cosmology: the results are consistent and agree with one another.\(^9\)

Another interesting aspect to note about this fractal debate is that it is as old as modern cosmology itself. The first suggestion that the Universe could be constructed in a hierarchical, or fractal, manner dates back to the very beginning of cosmology.\(^10\)\(^-\)\(^12\) with contributions made even by Einstein\(^13\) (Ref. 14, and references therein). Consequently, what we are witnessing now is only the latest chapter of this old debate, which is now focused on the statistical methods used by cosmologists to study galaxy clustering. The previous chapter was between de Vaucouleurs,\(^15\)\(^16\) and Wertz\(^17\)\(^-\)\(^19\) on one side, and Sandage \textit{et al.}\(^20\) on the other side, and was mainly focused on measurements of galaxy velocity fields and deviations from uniform expansion, a topic which has also re-surfaced in the recent debate.\(^9\) Therefore, it is clear that despite being dismissed many times as “unrealistic,” the fractal or hierarchical concept has so far refused to die, being able to pass from one generation of cosmologists to another.\(^21\) Thus, considering its old history, and its incredible ability to survive, it is perhaps premature to say that we are about to see this issue being settled with the dismissal of the fractal concept.

From the brief summary above, it is clear that the two sides of the fractal debate are locked in antagonistic and self-excluding viewpoints. Nevertheless, it is the opinion of this author that this divide may not be as radical as presented by both sides, and that it is possible to build a bridge between both opinions, reconciling them by means of a change in perspective regarding how we deal with observations in cosmology. What I intend to show next is that there is already enough theoretical evidence to suggest that fractality can be accommodated within the standard cosmology, where it would appear as a real observational effect of geometrical nature, arising from the way we carry out observations of the large-scale structure of the Universe. At the same time the cosmological principle, uniform Hubble expansion, CMBR isotropy, and well-defined meanings for the cosmological parameters, such as \(\Omega\), can survive, together with the observational fractality obtained by the heterodox group mentioned above. This perspective has the advantage of preserving most of what we learned with the standard FLRW cosmology, and at the same time, making sense of Pietronero and collaborators’ data, which as seen above, can no longer be easily dismissed.

The key point in understanding how that can come about, is by re-discussing the meaning of observations in cosmology. In relativistic cosmology, astronomical observations occur along the past light cone, a fact which is often overlooked when one carries out astronomical data reduction in cosmological models. Observers often use cosmological formulae which do not take this key theoretical feature into consideration. They are often under the assumption that at the scales where observations are being made \((z < 1)\) one can safely ignore that, since this is the region where the Hubble law is very linear. However, Hubble law linearity has a range which does not coincide with a constant density (see below), and the observed average density is a key physical quantity for fractal characterization.\(^5\)\(^-\)\(^8\),\(^22\)

The first theoretical evidence that average density departs from local homogeneity at much lower values for the redshift \(z\) appeared in Ribeiro.\(^23\) There, observational relations were calculated along the past light cone for unperturbed Einstein-de Sitter cosmology, and it was clearly shown that deviations from local homogeneity start to occur at \(z \approx 0.04\), becoming very strong at \(z \approx 0.1\). A plot
of the average density $\langle \rho \rangle$ against the luminosity distance $d_L$ showed a continuous decrease in the average density, although not in a linear manner. Still, Ribeiro\textsuperscript{23} also showed that in the Einstein-de Sitter cosmology, the following limit holds:

$$\lim_{d_L \to \infty} \langle \rho \rangle = 0.$$  

Many years ago, Wertz stated that a pure hierarchical cosmology ought to obey the “zero global density postulate: for a pure hierarchy the global density exists and is zero everywhere”.\textsuperscript{(Ref. 17, p. 18)} Such a result was also speculated by Pietronero\textsuperscript{5} as a natural development of his fractal model. Therefore, what the above limit tells us is that the Einstein-de Sitter model does obey this key requirement of fractal cosmologies. In addition, the decay of the average density at increasing distances, another key aspect of a fractal model, is also obeyed by the Einstein-de Sitter cosmology. Notice that these two fractal features, present in all standard cosmological models,\textsuperscript{14,24} appear \textit{without any violation} of the cosmological principle, linearity of Hubble law, and CMBR isotropy. Moreover, cosmological parameters such as $q_0$, $\Omega_0$, and $H_0$ still have their usual definitions and interpretations.

What is clear from Ribeiro,\textsuperscript{23} and sequel papers,\textsuperscript{14,24} was that the homogeneity of the standard cosmological models is \textit{spatial}, i.e. it is a \textit{geometrical} feature which does not necessarily translate itself into an astronomically observable quantity. Although a number of authors are aware of this fact, what came as a surprise had been the calculated low redshift value where this observational inhomogeneity appears. Therefore, it was clear by then that relativistic effects start to play an important role in observational cosmology at much lower redshift values than previously assumed.

In another paper,\textsuperscript{25} the above results were further analyzed and it became clear why there seems to be no contradiction between strong \textit{observational} inhomogeneity and the linearity of the Hubble law for $0.1 \leq z < 1$ in Einstein-de Sitter cosmology. Due to the nonlinearity of the Einstein field equations, observational relations behave differently at different redshift depths. Consequently, while the linearity of the Hubble law is well-preserved in the Einstein-de Sitter model up to $z \approx 1$, a value implicitly assumed as the lower limit up to where relativistic effects could be safely ignored, the observational density is strongly affected by relativistic effects at much lower redshift values. A power series expansion of these two quantities showed that while the zeroth order term vanishes in the distance-redshift relation, it is nonzero for the average density as plotted against redshift. This zeroth order term is the main factor for the different behavior of these two observational quantities at small redshifts. Pietronero et al.,\textsuperscript{7} called this effect as the “Hubble-de Vaucouleurs paradox”.\textsuperscript{26} However, from the analysis presented by Ribeiro\textsuperscript{25} it is clear that this is not a paradox, but just very different relativistic effects on the observables at the moderate redshift range ($0.1 \leq z < 1$).

This effect explains why Sandage et al.\textsuperscript{20} failed to find deviations from uniform expansion in a hierarchical model: they were expecting that such a strong inhomogeneity would affect the velocity field, but it is clear now that if we take a relativistic perspective for these effects they are not necessarily correlated at the range expected by Sandage and collaborators. Notice that de Vaucouleurs and Wertz also expected that their inhomogeneous hierarchical models would necessarily affect the velocity field, and change the linearity of the Hubble law at $z < 1$, and once such a change was not observed by Sandage et al.\textsuperscript{20} it was thought that this implied an immediate dismissal of the hierarchical concept. Again, this is not necessarily the case if we take a relativistic view of those observational quantities.

The results discussed above show that some key fractals features can already be found in the simplest possible standard cosmological model, i.e. in the unperturbed Einstein-de Sitter universe. However, as the average density decay is not linear in this model, considering all these aspects we may naturally ask whether or not a perturbed model could turn the density decay at increasing redshift depths into a power law-type decay, as predicted by the fractal description of galaxy clustering. If this happens, then standard cosmology can be reconciled with a fractal galaxy distribution. Notice that there are some indications that this is a real possibility, as Amendola\textsuperscript{27} pointed out that locally the cold dark matter and fractal models predict the same behavior for the power spectrum, a conclusion apparently shared by Cappi et al.\textsuperscript{28} In addition, confirming Ribeiro’s\textsuperscript{23,25} conclusions, departures from the expected Euclidean results at small redshifts were also reported by Longair (Ref. 29, p. 398), and the starting point for his findings was the same as employed by Ribeiro\textsuperscript{23,25} — the use of source number count expression along the null cone.
Considering all results outlined above, I feel there is enough grounds to advance the following conjecture: the observed fractality of the large-scale distribution of galaxies should appear when observational relations necessary for fractal characterization are calculated along the past light cone in a perturbed metric of standard cosmology.

If this conjecture proves, even partially, correct, fractals in cosmology would no longer be necessarily seen as opposed to the cosmological principle. Notice that this can only happen in circumstances where fractality is characterized by an observed, smoothed out, and averaged fractal system, as opposed to building a fractal structure in the very spacetime geometrical structure, as initially thought necessary to do for having fractals in cosmology.\(^{13,30}\) Thus, the usual tools used in relativistic cosmology, like the fluid approximation, will remain valid. As a possible consequence of this conjecture, a detailed characterization of the fractal structure could provide direct clues for the kind of cosmological perturbation necessary in our cosmological models, and this could shed more light in issues like galaxy formation.

There is now underway an attempt to check the validity of this conjecture\(^{31}\) by means of a specific perturbative approach to standard cosmology.\(^{32}\)

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REFERENCES