COMMENT

No warp drive

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Abstract. The warp drive spacetime of Alcubierre is impossible to set up without first being able to distribute matter at tachyonic speed, put roughly, you need one to make one! However, over small distances, where the energy conditions possibly can be violated, one can envision opening the light-cones to increase the apparent speed of light.

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1. Introduction

There has long been interest in the possibility of overcoming the limitation of the finite value of the speed of light. The so-called warp drive is one recent attempt to realize this wish. At first sight following the arguments in [1] it seems a reasonable construction. This is worrying because it is known that 'time paradoxes' closely follow when the speed of light limit is overcome. But this alone cannot be used as an argument to dismiss the warp drive since many examples are known with closed-timelike curves, e.g. Gödel universe [2] and rotating (infinite) cylinders, see e.g. [3]. For various reasons they mostly tend to suffer from various forms of unphysicality. Although, there is active debate as to whether any physical examples can be found, the warp drive might seem to be one possible case. But far from the warp drive being possible to construct we will show that it also suffers from being unphysical due to a paradox that makes its construction impossible: one needs to transcend the speed of light in order to construct the warp drive in the first place.

To reach this conclusion we first consider how one goes about implementing a change in the metric in general relativity by manipulations of energy–momentum. This crucial aspect seems to have been ignored but will prove to be an insurmountable obstacle to this and related attempts to create hyperfast travel by alteration of the metric.

2. Matter ⇔ geometry

We are well aware that Einstein's equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$ can be run in either direction. Although normally we take a reasonable matter source and find the consequent geometric structure $G_{\mu\nu}$. One can consider more extreme matter sources that violate the so-called energy conditions, the most commonly used being the strong, weak and dominant energy conditions, see e.g. [2]. The energy conditions are formulated in terms of timelike vectors, thus being relevant to non-tachyonic matter.

Roughly speaking violating the strong energy condition is easier to justify on physical grounds. Such violations of the strong energy condition would occur during an inflationary

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expansion of the universe which can be driven by a classical scalar field source. Violations of the weak energy condition (and closely related dominant energy condition) are required for more exotic things such as Lorentzian wormholes, see e.g. [3]: although by 'stretching' the wormhole one can apparently manage with just violating the strong energy condition [4]. These violations of the weak or dominant energy condition can occur in quantum field theory, for example, the Casimir effect and similar examples reviewed in [3]. Matter sources that violate the weak energy condition were called *exotic* in [5]. However, there are limits to how large these violations can be: constrained by the so-called quantum inequalities [6].

Given the existence of these exotic matter sources one greatly extends the possibility of running Einstein's equations in the reverse direction: one first chooses an interesting geometric structure $G_{\mu\nu}$, or equivalently the metric $g_{\mu\nu}$, before finding the matter source to support this geometry. This is the idea behind the warp drive spacetime; but one does need to worry about whether the matter can be properly distributed within the background or starting geometry to produce the modified geometry.

3. Warp drive spacetimes

Alcubierre [1] has taken a metric with an unusual shift function (see e.g. [3]) which seemingly makes everything move at speeds greater than that of light to outside observers. Recall that the shift function gives a 'lateral displacement' to the foliations of space. This appears to be valid in that the lapse and shift functions can be specified arbitrarily. The arbitrariness in the lapse function led us earlier to the possibility of signature change where we wished to obviate the usual singularity theorems [7]. Here the freedom of choosing the shift function possibly gives rise to hyperfast travel.

Consider a spacetime that is to be altered by a distribution of matter $T_{\mu\nu}$. The specific details of this metric will not be necessary for our argument. In the warp drive spacetime the disturbance is to move at speeds v greater than the speed of light c. But this also means that the matter has to be distributed at this hyperfast speed before the geometry $G_{\mu\nu}$ is likewise affected. One cannot do this without first being able to distribute matter at hyperfast speeds: exactly what one is trying to accomplish in the first place. The warp drive is impossible, see figure 1. Why was this not realized initially in [1]? There Alcubierre considered the motion of the spaceship within the warp drive and found that it travelled on timelike curves which looked to outside observers to be hyperfast. But this is premised on the metric existing in the first place which in turn is caused by the (impossible) distribution of matter: the 'need one to make one' paradox was overlooked.

This problem was realized by Krasnikov [8] who showed, using the specific warp drive metric, that the leading edge of the matter distribution is spacelike separated from the interior. This may be true but more fundamentally any disturbance, caused by distributing matter, that is to move hyperfast is spacelike separated from the starting background geometry. In terms of the shift function of the modified warp drive metric, there is a limit on what values one can implement given the starting metric.

This problem also seems to have been obtained in a different guise: the presence of an event horizon in the warp drive for $v > c$ which causes divergences in the stress–energy tensor of any scalar fields present [9].

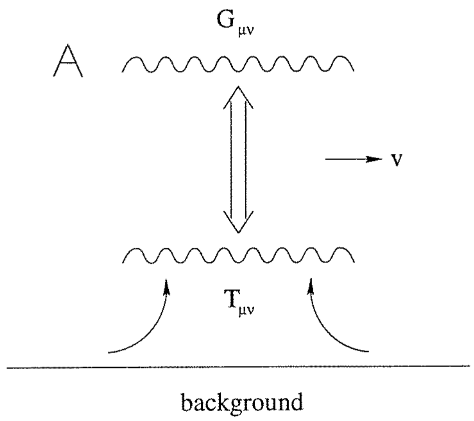

Figure 1. Impossibility of the warp drive spacetime. In order for the warp drive A to be set up and moved at speeds $v > c$ the matter distribution $T_{\mu\nu}$ producing it has to move tachyonically. Reference [1] only considered the behaviour of the spaceship within A and overlooked its spacelike separation from the starting geometry, but the matter causing the warp drive has to be distributed and moved in the background geometry.

4. Krasnikov's hyperfast tube

Krasnikov instead considered a modified metric which can be made by distributing the matter distribution $T_{\mu\nu}$ beforehand so not requiring a tachyonic matter source [8, 10]. This enables a hyperfast 'track' to be set up between two possibly distant points which could be used for interstellar travel, but not of course for the first trip that distributes the matter source. The resulting motion is always within the domain of dependence (see e.g. [2]) of the spacelike surface over which the matter source is distributed. Considerations of this metric show that it violates the weak energy condition drastically: it would require enormous amounts of negative energy [10, 11]. Similar manipulations of matter over spacelike surfaces could be used to create a new Robertson–Walker universe, although only within the domain of dependence of the spacelike surface, cf the discussion on Cauchy data for hyperbolic equations in [2, 12]. Of course, in practice such manipulations of matter are not credible as we pointed out for the somewhat related attempt to affect the final singularity of a closed universe to that of a 'smooth' so-called Omega point [13].

5. Alteration of the light-cone structure

It might still be possible to manipulate the light-cone structure of spacetime over small distances. We have in mind some future 'microchip' which wishes to surpass the lightspeed c limitation. Consider the metric

$$ds^2 = -(dt - k(t, x)\, dx)(dt + k(t, x)\, dx) + dy^2 + dz^2 \qquad (1)$$

which is related to Krasnikov's spacetime, cf [8], but not intended for space travel. Here we define k as

$$k(x, t) = 1 - (1 - \delta)\, \theta_\epsilon(t)[\theta_\epsilon(x) - \theta_\epsilon(x + \epsilon - D)] \qquad (2)$$

with θ the Heaviside step function and where the small constants δ, ϵ and the size of the disturbance D are defined as in [8]. The important point is that ordinary flat space

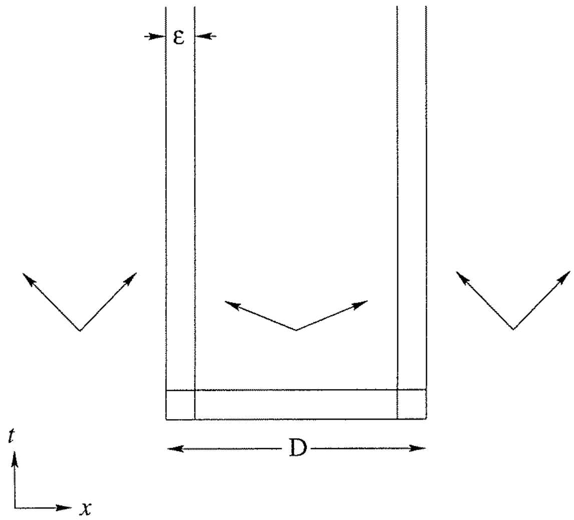

Figure 2. Geometry which causes an effective increase in the speed of light: the light-cones are opened up within the disturbance compared to flat space outside. This is related to an example of Krasnikov but is only intended to produce an effect over more realistic microscopic distances. For this example the dominant energy condition is violated.

corresponds to $k = 1$, while within the region D it is $0 < k \ll 1$. This small k corresponds to an opening of the light-cone, and thus having a larger effective speed of light in the x-direction, see figure 2. One can show, using GR TensorII software [14], that while the components T_{00} and T_{xx} are zero for this spacetime, the other transverse components are not. Without giving the cumbersome expressions for these components one can simply note that this is an obvious violation of the dominant energy condition. This might have been anticipated as there is a close relationship between the speed of light and the dominant energy condition (see p 94 in [2]). Although this metric is more benign than Krasnikov's (it is unlikely to give time paradoxes, cf [10]) and any exotic matter would only be required across small distances, it is still unlikely to satisfy the quantum inequalities. It is possible that a suitable geometry which also includes altering the light-cone in the other two spatial components can be found which further reduces the amount of exotic matter required and might just alter the energy conditions that are violated. Unfortunately, one expects that at least the weak energy condition will be violated as in Krasnikov's metric and that the holy grail, of merely violating the strong energy condition, would not suffice.

However, this example does suggest that the speed of light is somewhat malleable in general relativity, especially over small distances where the energy conditions are expected to be violated by quantum phenomena. Just as the speed of light is less than c in ordinary matter one can conspire to use exotic matter to increase the speed of light, at least to observers outside the disturbance.

Whether or not this could ever be realized practically it does once more highlight the interesting behaviour that can be obtained when one allows the full gamut of Einstein's equations: without invoking restrictions from other physical principles. But never overlook that we already start from a pre-existing background metric: in the warp drive example this alone precluded its possibility of formation, before one even worries whether *exotic* matter is actually available.

Since I first wrote this comment, this error seems to have also been made by [15]. Here the question of whether superluminal travel is possible is again only thought of in terms of violations of the relevant energy conditions. As I have tried to emphasize this alone is insufficient for the creation of warp drive or hyperfast (in the sense of distant travel) behaviour.

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