AdS/QCD and Confinement

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AdS/CFT Correspondence

A String Theory in 10d defined in $\mathbf{AdS_5} \times \mathbf{S^5}$ space is equivalent to an $\mathbf{SU(N)}$ supersymmetric gauge theory with $\mathbf{N} \to \infty$ in 4d Minkowski spacetime

Analogously, M-theory in 11d defined in $\mathbf{AdS_7} \times \mathbf{S^4}$ or $\mathbf{AdS_4} \times \mathbf{S^7}$ spaces is equivalent to $\mathbf{SU(N)}$ supersymmetric gauge theories with $\mathbf{N} \to \infty$ in 6d or 3d Minkowski spacetimes, respectively

In fact, these $\mathbf{SU(N)}$ gauge theories have extended supersymmetries (various fermions for each boson) and are conformal (**CFT**).

AdS/CFT (small) dictionary

String Theory $\Leftrightarrow$ SU(N) gauge theory ($\mathcal{N} = 4$)

$AdS_5 \times S^5$ $\Leftrightarrow$ 4d Minkowski

Dilaton (scalar field) $\Leftrightarrow$ $\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})$, (scalar Glueball)

Conformal Theories

Conformal theories do not have any scale.

Despite of that, they have applications in different areas of Physics, for instance:

- Theory of Phase transitions (condensed matter);
- Quantum Chromodynamics at high energies, that is, for $\mathbf{E \gg M_{Proton} c^2}$.

AdS space

The $\mathbf{AdS_5} \times \mathbf{S^5}$ metric is given by

$$\mathbf{ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}(dr^2 + r^2 d\Omega_5^2)}$$

$\mathbf{R}$ is a constant given by $\mathbf{R^4 = 4\pi g_S \ell_S^4}$, where $\mathbf{g_S}$ is the string coupling and ℓ_S the string length.

The coordinate r is usually called the Fifth coordinate, that is, it represents the Fifth dimension.

For any fixed $\mathbf{r}$ the term $\mathbf{-dt^2 + d\vec{x}^2}$ corresponds to a 4d Minkowski space.

The term $\mathbf{R^2 d\Omega_5^2}$ represents the 5d sphere $\mathbf{S^5}$ wiht radius $\mathbf{R}$.

Energy and the Fifth dimension

Let $\mathbf{p}^\mu = -\mathbf{i}\partial/\partial \mathbf{x}_\mu$ represents the momentum in 4d Minkowski space and $\tilde{\mathbf{p}}^\mu$ is the momentum seen by an observer in 10d. Then AdS metric implies that they are related by

$$\tilde{\mathbf{p}}^\mu = \frac{\mathbf{R}}{\mathbf{r}}\,\mathbf{p}^\mu$$

Further, defining a typical energy scale in 10d as $\sim \mathbf{R}^{-1}$, then the energy $\mathbf{E}$ seen in 4d is

$$\mathbf{E} \sim \frac{\mathbf{r}}{\mathbf{R}^2}$$

which means that the energy $\mathbf{E}$ of a 4d process is localised in the 5th dimension $\mathbf{r}$. So, the higher $\mathbf{E}$, the greater $\mathbf{r}$.

Energy and the Fifth dimension (2)

In terms of the Poincaré coordinate $\mathbf{z} = \mathbf{R}^2/\mathbf{r}$, the energy is

$$\mathbf{E} \sim \frac{\mathbf{1}}{\mathbf{z}}$$

so the higher $\mathbf{E}$ the smaller $\mathbf{z}$.

The other 5 extra dimensions of the hypersphere $\mathbf{S}^5$ codify the extended supersymmetry ($\mathcal{N} = \mathbf{4}$) in the 4d gauge theory.

Witten's Model for Holographic QCD

Witten proposed that QCD can be described by string theory in AdS space with a Black Hole in it.

In this case, the Horizon radius of the Black Hole defines a length scale that breaks conformal symmetry.

Supersymmetry is also broken imposing that fermions satisfy antiperiodic conditions on a compact dimension while bosons satisfy periodic conditions.

It is possible to calculate Glueball masses associated with fields modes in the AdS plus the Black Hole space satisfying a boundary condition on the Horizon.

Strings and the Scattering of Glueballs

In 2001, Polchinski and Strassler used the fact that a minimum energy scale ($\mathbf{E_{min}}$) in the SU(N) gauge theory corresponds to certain region in AdS space with the Fifth coordinate restricted by $\mathbf{r} > \mathbf{r_{min}}$ to calculate the glueball scattering amplitude.

$$\mathbf{E_{min}} \sim \mathbf{\Lambda_{QCD}}$$

In this work they reobtained the Veneziano amplitude corrected by the AdS warp factor thanks to its curvature, in such a way that they correctly describe hadronic scattering at fixed angles, overcomming a famous obstacle for the description of hadrons in terms of string theory.

The Hard-Wall Model

N. Braga and HBF considered string theory (and the corresponding supergravity fields) in an AdS Slice ($z \leq z_{max}$) satisfying boundary conditions (Neumann or Dirichlet, for isntance) on the "Wall" ($z = z_{max}$) and then calculating Glueball masses without the need of introducting a black hole.

Scalar Glueballs are described by the Dilaton (scalar) field in AdS satisfying the equation

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,\partial^\mu\phi\right) = 0$$

which implies

$$\left[z^3\partial_z\frac{1}{z^3}\partial_z + \eta^{\mu\nu}\partial_\mu\partial_\nu\right]\phi = 0$$

whose solutions are Bessel Functions $J_2(kz)$ where z is the Poincaré coordinate $z = R^2/r$ parametrising the Fifth dimension.

Scalar Glueballs masses in 3+1d

Scalar Glueball masses in the Hard-Wall model appear as a consequence of using boundary conditions on $z = z_{max}$ imposed on Bessel Functions.

Then, scalar Glueball masses are determined by the zeroes of Bessel Functions.

QCD_{3+1}	Lattice, N=3	AdS-BH	AdS Slice
0_{++}	1.61	1.61 (input)	1.61 (input)
0^{*}_{++}	2.8	2.38	2.64
0^{**}_{++}	-	3.11	3.64
0^{***}_{++}	-	3.82	4.64
0^{****}_{++}	-	4.52	5.63
0^{*****}_{++}	-	5.21	6.62

Lattice: Morningstar and Peardon; Teper 1997
AdS-BH: Csaki, Ooguri, Oz and Terning, JHEP 1999
AdS Slice: HBF and N Braga, JHEP 2003.

Scalar Glueballs masses in 2+1d

Analogously, in 2+1 d, scalar glueballs are described by Bessel Functions $\mathbf{J_{3/2}(kz)}$, which zeroes define their masses in the Hard-Wall model:

QCD_{2+1}	Lattice N=3	Lattice $N\rightarrow\infty$	AdS-BH	AdS Slice
$\mathbf{0}_{++}$	4.329	4.065	4.07 (input)	4.07 (input)
$\mathbf{0}^{*}_{++}$	6.52	6.18	7.02	7.00
$\mathbf{0}^{**}_{++}$	8.23	7.99	9.92	9.88
$\mathbf{0}^{***}_{++}$	-	-	12.80	12.74
$\mathbf{0}^{****}_{++}$	-	-	15.67	15.60
$\mathbf{0}^{*****}_{++}$	-	-	18.54	18.45

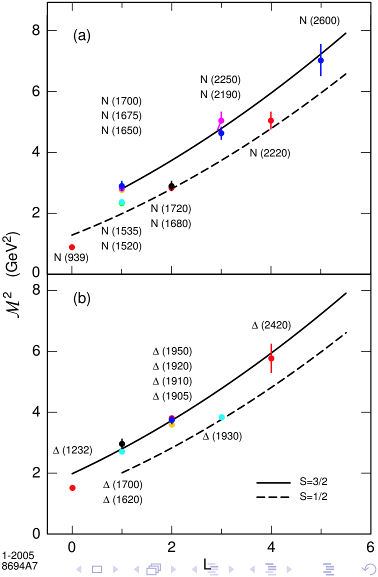
Lattice: Morningstar and Peardon; Teper 1997
AdS-BH: Csaki, Ooguri, Oz and Terning, JHEP 1999
AdS Slice: HBF and N Braga, JHEP 2003.

Light Baryons masses

The Hard-Wall model was extended to calculate light baryons masses of spin 1/2 and 3/2

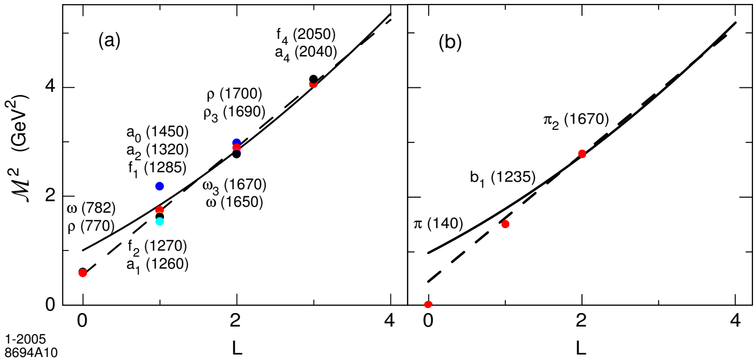
The masses are still given by the zeroes of Bessel Functions !

Teramond and Brodsky
PRL 2005, 2006;



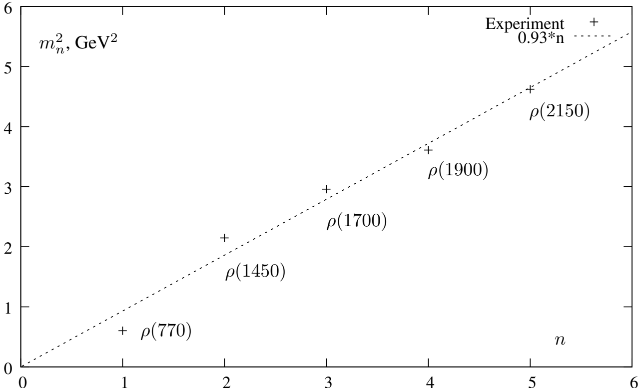
Light meson masses

Same for light mesons (Teramond and Brodsky PRL 2005, 2006)



More Light Mesons

The Hard-Wall model was also applied by Erlich, Katz, Son and Stephanov (PRL 2005) to calculate light vector meson masses



Glueballs of higher spins

The Hard-Wall model was also applied to calculate masses for Glueball with higher spins:

Dirichlet glueballs	lightest state	1^{st} excited state	2^{nd} excited state
$\mathbf{0}^{++}$	1.63	2.67	3.69
$\mathbf{2}^{++}$	2.41	3.51	4.56
$\mathbf{4}^{++}$	3.15	4.31	5.40
$\mathbf{6}^{++}$	3.88	5.85	6.21
$\mathbf{8}^{++}$	4.59	5.85	7.00
$\mathbf{10}^{++}$	5.30	6.60	7.77

More Glueballs with higher spins

Neumann glueballs	lightest state	1st excited state	2nd excited state
$\mathbf{0}^{++}$	1.63	2.98	4.33
$\mathbf{2}^{++}$	2.54	4.06	5.47
$\mathbf{4}^{++}$	3.45	5.09	6.56
$\mathbf{6}^{++}$	4.34	6.09	7.62
$\mathbf{8}^{++}$	5.23	7.08	8.66
$\mathbf{10}^{++}$	6.12	8.05	9.68

Masses of Glueballs $\mathbf{J}^{\mathbf{PC}}$ (with $\mathbf{J}$ even) expressed in GeV. The mass of $\mathbf{0}^{++}$ is an input from Lattice data [H.B.F., Nelson Braga and Hector Carrion PRD 2006]

Regge Trajectories for Glueballs and the Pomeron

Once the Glueball masses are determined and as we know their spins we can plot their $\mathbf{J} \times \mathbf{M}^2$ behavior.

The lines in these graphs correspond to their Regge Trajectories. For linear trajectories one has:

$$\mathbf{J} = \alpha_0 + \alpha' \mathbf{M}^2 .$$

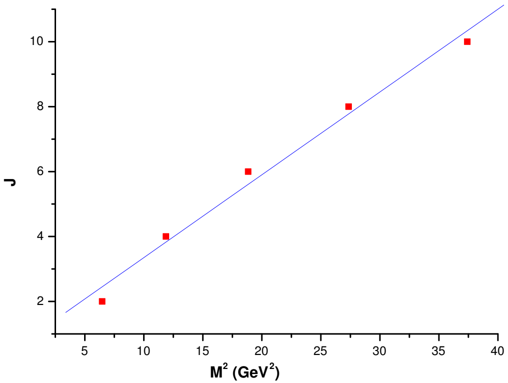
For Neumann b.c. and states $\mathbf{J}^{++}$ with $\mathbf{J} = 2, 4, \ldots, 10$ we found

$$\alpha' = (0.26 \pm 0.02)\mathbf{GeV}^{-2} \quad ; \quad \alpha_0 = 0.80 \pm 0.40$$

consistent with the Pomeron

$$\alpha'_{\mathbf{EXP}} = 0.25 \ \mathbf{GeV}^{-2} \quad ; \quad \alpha_{0\mathbf{EXP}} = 1.08$$

Regge Trajectories for Glueballs and the Pomeron (2)



Regge trajectory for Glueballs with Neumann b.c.

AdS/CFT and Confinement

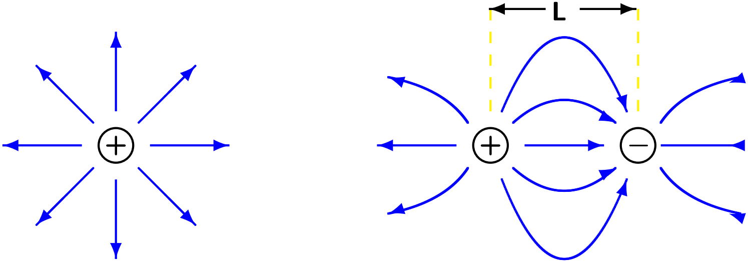
Maldacena (PRL 1998) showed how to use the AdS/CFT correspondence to calculate Wilson loops that describe the confining/deconfining behavior of gauge theories.

He calculated the Wilson loop for the $\mathcal{N} = 4$ supersymmetric $\mathbf{SU(N)}$ Yang-Mills Theory in $\mathbf{4d}$ from String theory in $\mathbf{AdS_5} \times \mathbf{S^5}$ space.

Non-Confining theory

The confining or non-confining behavior is related to the flux of gauge fields.

For instance, for a non-confining theory as QED we have for a monopole and a dipole the field lines

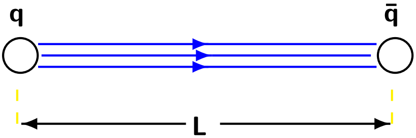


The biding energy for the dipole $\mathbf{q\bar{q}}$ is $\mathbf{E} \sim \mathbf{1/L}$

The Total (Relativistic) Energy becomes $\mathbf{2m}$ when $\mathbf{L} \to \infty$

Confining theory

For a confining theory such as QCD we expect that the field lines concentrate on a flux tube



The biding Energy for this configuration of the dipole $\mathbf{q\bar{q}}$ is $\mathbf{E} \sim \mathbf{L}$.

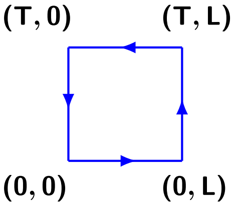
So that the Total Energy goes to ∞ when $\mathbf{L} \to \infty$.

Wilson Loops

For a non-Abelian gauge field $A_\mu \equiv \lambda^i A^i_\mu$, where $[\,\lambda^i\,,\lambda^j\,] = i\,f^{ijk}\lambda^k$ defines the non-Abelian group, the Wilson Loop corresponding to a closed contour **C** is given by:

$$W(C) = \langle\, 0\,|\, \mathrm{Tr}\,\{\mathcal{P}\,\exp(ig \oint_C \lambda^i A^i_\mu(y) dy^\mu)\}\,|\,0\,\rangle$$

In the particular case of a rectangular contour in 1+1 flat spacetime one has:



T= time interval
L = length

And the Wilson Loop behaves in this case as

$$W(C) \sim \exp\{-T\,[\,E(L)\,-\,2m\,]\,\}$$

Confinement Criteira

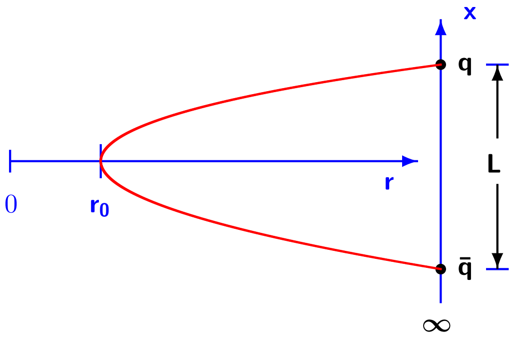
Given the Wilson Loop

$$\mathbf{W(C) \sim \exp\{-T\,[\,E(L)\ -\ 2m\,]\,\}}$$

- if $\mathbf{E(L)} \rightarrow \mathbf{2m}$ for $\mathbf{L} \rightarrow \infty$, **non**-confining
- if $\mathbf{E(L)} \rightarrow \infty$ for $\mathbf{L} \rightarrow \infty$, **confining**

Wilson Loops in AdS/CFT correspondence

Maldacena: Very heavy quark anti-quark pair (stationary configuration) in $\mathbf{r} = \mathbf{r_1}$ ($\rightarrow \infty$) on the axis $\mathbf{x^i} \equiv \mathbf{x}$ separated by a coordinate difference $\mathbf{L}$.



Wilson Loops in AdS/CFT correspondence (2)

Starting from the Nambu Goto Action

$$S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{\det G_{MN} \partial_a X^M \partial_b X^N}$$

where $a, b = \tau, \sigma$ and $M, N = 0, 1, 2, ..., 9$, choosing the background metric G_{MN} as the $AdS_5 \times S^5$

$$ds^2 = \frac{r^2}{R^2}(dt^2 + dx_i dx_i) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2$$

Using the parametrization $\tau = t\,;\ \sigma = x$ one has

$$S = \frac{1}{2\pi\alpha'} \int dx \sqrt{(\partial_x r)^2 + \left(\frac{r}{R}\right)^4}$$

The String Action is proportional to the world-sheet area. The principle of least action implies a geodesic solution connecting the two string ends (charges).

Wilson Loops in AdS/CFT correspondence (3)

First one needs to relate **L** and $\mathbf{r_0}$. Then

$$\mathbf{L} = \int \mathbf{dx} = \int \left(\frac{\mathbf{ds}}{\mathbf{dx}}\right)^{-1} \mathbf{ds} = \frac{\mathbf{2R^2}}{\mathbf{r_0}} \int_1^\infty \frac{\mathbf{d}\rho}{\rho^2 \sqrt{\rho^4 - 1}}$$

and the minimum value of the coordinate **r** in the geodesics in terms of **L** is given by

$$\mathbf{r_0} = \frac{\mathbf{2R^2}}{\mathbf{L}} \frac{\sqrt{2}\pi^{3/2}}{\Gamma(1/4)^2}$$

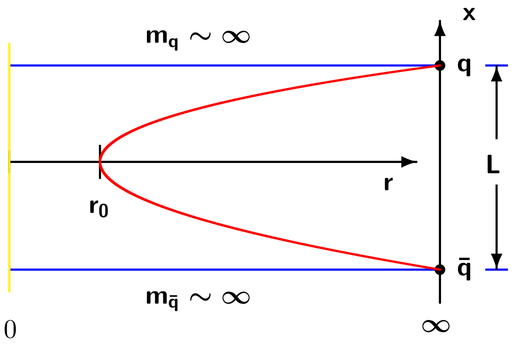
The energy of the configuration is $\mathbf{(1/T)} \times$ **Action**, so:

$$\mathbf{E} = \int \mathcal{L}\mathbf{dx} = \int \mathcal{L} \left(\frac{\mathbf{ds}}{\mathbf{dx}}\right)^{-1} \mathbf{ds} = \frac{\mathbf{r_0}}{\pi\alpha'} \int_1^\infty \frac{\rho^2\, \mathbf{d}\rho}{\sqrt{\rho^4 - 1}}$$

which is divergent.

Wilson Loops in AdS/CFT correspondence (4)

To regularize the energy expression let us subtrac a divergent quantity (the energy of “quarks”)



Wilson Loops in AdS/CFT correspondence (5)

So,

$$\mathbf{E'} = \int \mathcal{L} \left(\frac{\mathbf{ds}}{\mathbf{dx}} \right)^{-1} \mathbf{ds} - \int \mathbf{ds} = \frac{\mathbf{r_0}}{\pi\alpha'} \int_1^{\infty} \left[\frac{\rho^2}{\sqrt{\rho^4 - 1}} - 1 \right] \mathbf{d}\rho$$

which implies:

$$\mathbf{E'} = -\frac{4\pi \mathbf{R}^2}{\alpha' \Gamma(1/4)^4 \, \mathbf{L}}$$

This is a Coulomb potential (which is non-confining) for the charges at the ends of the string in AdS ($\mathbf{r} \to \infty$).

- S.J.Rey and J.T.Yee, Eur. Phys.J.C **22**, 379 (2001).
- J. M. Maldacena, Phys. Rev. Lett. **80**, 4859 (1998).

What kind of String Geometry has a Dual Confining Gauge Theory?

Generalisation of Maladacena's procedure:

- Brandhuber, Itzhaki, Sonnenschein and Yankielowicz JHEP 98
- Kinar, Schreiber and Sonnenschein NPB 2000

Interesting Geometries (10 dimensions)

$$ds^2 = -g_{00}(\xi)dt^2 + g_{ii}(\xi)dx^i dx^i + g_{\xi\xi}(\xi)dr^2 + d\tilde{s}^2$$

$i = 1, 2, 3$ and $d\tilde{s}^2$ represents 5 transverse dimensions.

Defining $\quad f(\xi) = \sqrt{g_{00}(\xi)\; g_{ii}(\xi)} \quad ; \quad g(\xi) = \sqrt{g_{00}(\xi)\; g_{\xi\xi}(\xi)}$

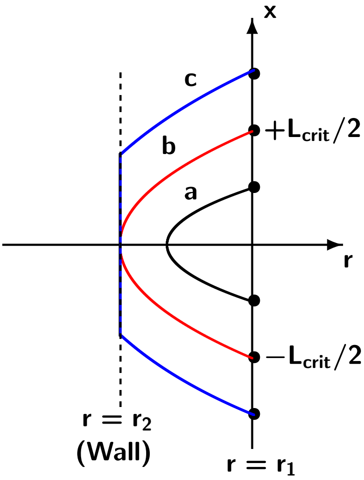
If the function $f(\xi)$ has a minimum (global for the metric) and its value is $\neq 0$:

$$f(\xi_{min}) \neq 0 \quad ; \quad f'(\xi)|_{\xi=\xi_{min}} = 0.$$

then the dual gauge theory is confining (for quarks at infinity, $\xi_1 \to \infty$).

Confinement in the Hard-Wall model

Considering a String in AdS space in the presence of a wall one finds three different situations:



$L < L_{crit}$ (case a)

$L = L_{crit}$ (case b)

$L > L_{crit}$ (case c)

Confinement in the Hard-Wall model (2)

Cases **a** e **b** coincide with the one analised by Maldacena and then are **non**-confining.

The energy of configuration **c** corresponds to the energy of configuration **b** plus the corresponding length along the wall at $\mathbf{r = r_2}$:

$$\mathbf{E'} = \frac{\mathbf{r_2}}{\pi\alpha'}\int_1^{\mathbf{r_1/r_2}}\left[\frac{\rho^2}{\sqrt{\rho^4 - 1}} - 1\right] \mathbf{d}\rho + \frac{\mathbf{r_2^2}}{2\pi\alpha'\mathbf{R}^2}(\mathbf{L} - \mathbf{L_{crit}})$$

Choosing $\mathbf{r_2 = R}$, one finds

$$\mathbf{E'} = \frac{\mathbf{R}}{\pi\alpha'}\int_1^{\mathbf{r_1/R}}\left[\frac{\rho^2}{\sqrt{\rho^4 - 1}} - \frac{1}{\rho^2\sqrt{\rho^4 - 1}} - 1\right] \mathbf{d}\rho + \frac{1}{2\pi\alpha'}\mathbf{L}$$

Obviously the last term ($\propto \mathbf{L}$) is confining.

Confinement in the Hard-Wall model (3)

Taking the limit $r_1 \to \infty$ the binding energy between the quark and antiquark at the ends of the String is approximately given by

$$E = \begin{cases} -\dfrac{4a}{3L} & L \leq L_{crit} \\ -4\sqrt{\dfrac{a\sigma}{3}} + \sigma L & L \geq L_{crit} \end{cases} \qquad (1)$$

where $a = 3C_1R^2/2\pi\alpha'$, $\sigma = 1/2\pi\alpha'$ and $C_1 = \frac{\sqrt{2}\pi^{3/2}}{[\Gamma(1/4)]^2}$.

This potential energy is very close to the Cornell potential

$$V(L) = -\frac{4a}{3L} + \sigma L + \mathrm{const.}$$

which describes the spectra of heavy mesons with $a = 0.39$ and $\sigma = 0.182 GeV^2$ for the Charmonium.

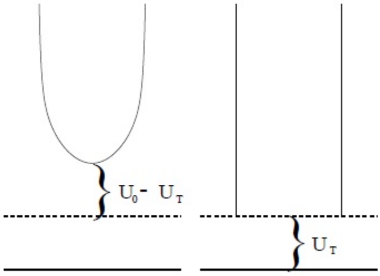
Wilson Loops at Finite Temperature

The Maldacena approach to calculate Wilson Loops can be extended to the Finite Temperature case.

In this case one considers Witten's model where there is a Black Hole in the AdS space.

The temperature of the field theory is identified with the Black Hole Hawking temperature.

There are two typical configurations for strings in this space:
Low temperatures × High temperatures



This metric is **non**-confining since the horizon function vanishes at $\mathbf{U = U_T}$

Quark-Gluon Plasma

At high temperatures the strong interactions become deconfined forming the quark-gluon plasma.

The quark-gluon plasma is strongly interacting and behaves like a perfect liquid, that means a liquid with very low viscosity.

The first theoretical estimate for the QGP viscosity was given by Policastro, Son and Starinets (PRL 2001) using Witten's model (black hole in AdS space) at finite temperature.

Other applications of AdS/CFT correspondence

Other problems in Particle Physics as the Deep inelastic scattering (DIS) can also be described using AdS/QCD models, for instance:

- Polchinski and Strassler, JHEP 2002;
- Ballon Bayona, HBF and Braga, JHEP 2008 a, b, c;
- Miranda, Ballon Bayona, HBF and Braga, JHEP 2009;
- Ballon Bayona, HBF, Braga and Torres, JHEP 2010.
- Ballon Bayona, HBF, Ihl and Torres, JHEP 2010.