Hadrons in AdS/QCD models
IFT, UNESP, São Paulo, March 20th, 2013

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* Supported also by CNPq and CAPES
Summary of the talk:

- **Brief Review**: hard wall AdS/QCD model, hadronic spectrum,…
- **Deep inelastic scattering** in AdS/QCD.
- **Hadronic form factors** from D4-D8 brane model.
- **Drell Yan process**: Calculation of the parameters \(\lambda, \mu, \nu\), of angular distribution of dileptons produced in proton proton collisions.
- **Other Results**
AdS/CFT correspondence, J. Maldacena, 1997

(simplified version of a particular useful case)

**Exact equivalence between String Theory in a 10-dimensional space and a gauge theory on the 4-dimensional boundary.**

Remarks:
String theory space = AdS\(_5\) \times S^5  
AdS = anti-de Sitter; \( S = \) sphere  
Gauge theory: SU(N) with very large N  
(supersymmetric and **conformal**).

At low energies string theory is represented by an effective supergravity theory \( \rightarrow \) gauge / gravity duality
Anti-de Sitter space (Poincaré patch)

\[ ds^2 = \frac{R^2}{(z)^2} \left( dz^2 + (d\vec{x})^2 - dt^2 \right) \]

The 4-dim boundary is at \( z = 0 \)

**Holographic** bulk ↔ boundary mapping


States of the boundary gauge theory with energy \( E \) ↔ Region of AdS space

\[ Z \sim \frac{1}{E} \]
Holographic relation suggests:

Cut off in AdS space: $0 < Z < Z_{max}$ \[\leftrightarrow\] infrared cut off in gauge theory.

This idea was introduced by Polchinski and Strassler (PRL 2002) to find the scaling of high energy scattering amplitudes at fixed angles.


- Glueballs $\leftrightarrow$ Normalizable modes of a scalar field in an AdS slice with size $Z_{max} = 1/\Lambda_{QCD}$

- Masses of scalar glueballs $J^{PC} = 0^{++}, 0^{++*}, 0^{++**}, ...$ with good agreement with lattice results.

This kind of model was then called the Hard wall AdS/QCD model and extended to other hadrons.
Deep inelastic scattering in AdS/QCD.

Inclusive cross section characterized by the hadronic tensor

\[ W^{\mu\nu} = i \int d^4 y \, e^{iq \cdot y} \langle P, Q | [J^\mu (y), J^\nu (0)] | P, Q \rangle \]

Dynamical variables: \( q^2 \), \( x = - \frac{q^2}{2P \cdot q} \)  
Bjorken variable
The Structure functions $F_{1,2}(x, q^2)$ contain informations about the distribution of constituents inside the hadron.

Polschinski and Strassler (2003) found prescriptions for calculating the structure functions, using the hard wall model (depending on the kinematical regime, here we just review the case when supergravity approximation holds).
The matrix element of the hadronic current is given by a 10-dimensional supergravity interaction action. For scalars:

\[ \eta_\mu \langle P_X | \tilde{J}^\mu(q) | P \rangle = i \int d^{10}x \sqrt{-g} A^m(\Phi_i \partial_m \Phi^*_X - \Phi^*_X \partial_m \Phi_i) \]

\[ \leftrightarrow \]

Gauge theory \leftrightarrow Supergravity (\sim low energy String theory)

(Just one hadron in the final state)

Result for fermions (\(\tau\) is the twist = \(d - s\)):

\[ F_2(x, q^2) = 2F_1(x, q^2) = C Q^2 \left( \frac{\Lambda^2}{q^2} \right)^{\tau-1} x^{\tau+1} (1 - x)^{\tau-2} \]

(For the hard wall)
Hadronic structure functions at small $x$.

Center of mass energy $\sqrt{s}$ is larger at small $x$: $s \sim q^2/x$

So, we expect more hadrons in the final state.

$\Delta = \text{scaling (conformal) dimension of hadronic operator.}$

↔ minimum number of constituents of the state.

Bulk/Boundary relation: $\Delta$ is “regulated” by the 5-dimensional mass of the dual bulk field

$$m_5^2 = \frac{\Delta(\Delta - 4)}{R^2}$$
So, we summed over final hadronic states with all allowed values of $\Delta$ (in Polchinski Strassler article $\Delta_{\text{initial}} = \Delta_{\text{final}}$), finding a behaviour similar to **GEOMETRIC SCALING**:

$$F_2(x, q^2) \sim \left( \frac{q^2}{\Lambda^2} \right)^{1/2} x^{-1/2}$$
Our AdS/QCD result imply a similar scaling with $\lambda = 1$

(without summing all values of $\Delta$ the result is a negative $\lambda$)
D4-D8 brane model for hadrons
Sakai and Sugimoto (2005)

• D8 (probe) branes embedded in D4 brane space.
• Holographic model for \( \text{(large } N_c, \text{ strongly coupled)} \) QCD.

D4 brane background:

\[
ds^2 = \left( \frac{U}{R} \right)^{3/2} \left[ \eta_{\mu\nu} dx^\mu dx^\nu + f(U)d\tau^2 \right] + \left( \frac{R}{U} \right)^{3/2} \left[ \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right]
\]

With: \( f(U) = 1 - (U_{\kappa\kappa} / U)^3 \), \( \tau \) is a compact dimension.
Period of \( \tau \) is related to minimum value of \( U \) \( \rightarrow \) mass scale.

In this model \textit{mesons} correspond to fluctuations of the D8 brane solutions in the D4 background.
Vector and axial vector mesons are described by $U(N_F)$ gauge field fluctuations.

4-dim effective action (after field redefinitions, ...)

$$\mathcal{L}_{eff}^{4d} = \frac{1}{2} \text{tr} \left( \partial_\mu \tilde{v}_\nu^n - \partial_\nu \tilde{v}_\mu^n \right)^2 + \frac{1}{2} \text{tr} \left( \partial_\mu \tilde{a}_\nu^n - \partial_\nu \tilde{a}_\mu^n \right)^2 + \text{tr} \left( i \partial_\mu \Pi + f_{\pi} A_\mu \right)^2 + M_{v_n}^2 \text{tr} \left( \tilde{v}_\mu^n - \frac{g_{v_n}}{M_{v_n}^2} v_\mu \right)^2 + M_{a_n}^2 \text{tr} \left( \tilde{a}_\mu^n - \frac{g_{a_n}}{M_{a_n}^2} A_\mu \right)^2 + \sum_{j \geq 3} \mathcal{L}_j$$

**Vector and axial-vector mesons:** $\tilde{v}_\mu^n$, $\tilde{a}_\mu^n$

→ The effective actions show up with a set of prescriptions for calculating masses and couplings. (everything is solved numerically)

• Important: D4-D8 model realizes vector meson dominance.
Form factors for vector and axial-vector mesons in the D4-D8 model: C.A.Ballon Bayona, H.B.-F., N.Braga, M.A.C.Torres, JHEP 2010

VMD (Vector meson dominance)
Interaction with a photon mediated by the exchange of vector mesons
Generalized form factors for vector mesons:

\[ \langle v^{m} a(p), \epsilon | J^{\mu c}(0) | v^{\ell b}(p'), \epsilon' \rangle \]

\[ = \epsilon^{\nu} \epsilon'^{\rho} f^{abc} \left[ \eta_{\nu \rho}(2p + q)_{\sigma} + 2(\eta_{\sigma \nu} q_{\rho} - \eta_{\rho \sigma} q_{\nu}) \right] \left( \eta^{\mu \sigma} \frac{q^{\mu} q^{\sigma}}{q^{2}} \right) F_{v^{m} v^{\ell}}(q^{2}) \]

Where, in the model:

\[ F_{v^{m} v^{\ell}}(q^{2}) = \sum_{n=1}^{\infty} \frac{g_{v^{n}} g_{v^{n} v^{m} v^{\ell}}}{q^{2} + M_{v^{n}}^{2}} \]

\( g_{v^{n}} \) is the coupling between the photon and the vector meson,

\( g_{v^{n} v^{\ell} v^{m}} \) is the 3 vertex on vector mesons, ..... .

\[ g_{v^{n}} = \kappa M_{v^{n}}^{2} \int d\bar{z} K(\bar{z})^{-1/3} \psi_{2n-1}(\bar{z}), \]

\[ K(\bar{z}) \equiv 1 + \bar{z}^{2} \]

\[ g_{v^{n} v^{\ell} v^{m}} = \kappa \int d\bar{z} K(\bar{z})^{-1/3} \psi_{2n-1}(\bar{z}) \psi_{2\ell-1}(\bar{z}) \psi_{2m-1}(\bar{z}), \]

... and similar expressions for the axial-vector mesons.
Results: appropriate decrease with $q^{-4}$ for large $q$.

$q^4$ times the form factors. Left panel: $F_{v^i v^i}$, right panel: $F_{a^1 a^i}$, for $i = 1$ (solid line), 2 (dashed line), 3 (dot-dashed line), 4 (dotted line).
Interesting quantities in the elastic case:

- Form factors for vector mesons with transversal and longitudinal polarizations

\[ F_{TT}(q^2) = \frac{\langle p, \epsilon_T | J_0(0) | p', \epsilon'_T \rangle}{2E} , \quad F_{LT}(q^2) = \frac{\langle p, \epsilon_T | J_x(0) | p', \epsilon'_L \rangle}{2E} \]
\[ F_{LL}(q^2) = \frac{\langle p, \epsilon_L | J_0(0) | p', \epsilon'_L \rangle}{2E} . \]

In the D4-D8 model we found:

\[ F_{TT}^{(v^m)} = F_{TT} \quad , \quad F_{LT}^{(v^m)} = \frac{q}{M_{\nu m}} F_{TT} \quad , \quad F_{LL}^{(v^m)} = \left( 1 - \frac{q^2}{2M_{\nu m}^2} \right) F_{TT} . \]

That imply the large \( q^2 \) behaviour expected from QCD:

\[ F_{TT}^{(v^m)} \sim q^{-4} , \quad F_{LT}^{(v^m)} \sim q^{-3} , \quad F_{LL}^{(v^m)} \sim q^{-2} \]
AdS/QCD model for dilepton production in proton proton collisions (Drell Yan process)


$P_1, P_2 \rightarrow \text{incident protons;} \quad k_1, k_2 \rightarrow \text{dilepton (lepton pair)}$

$X, Y \rightarrow \text{Final hadronic states}$

Fixed variables: $P_1, P_2$ and $q$ (momentum of the virtual photon)

\[
d\sigma = \frac{e^4}{(q^2)^2} W^{\mu\nu} L_{\mu\nu} \frac{d^3 \vec{k}_1}{(2\pi)^3 2|\vec{k}_1|} \frac{d^3 \vec{k}_2}{(2\pi)^3 2|\vec{k}_2|}
\]

Leptonic tensor (just QED)

\[
L_{\mu\nu} = \sum_{S_{L1}} \sum_{S_{L2}} \langle k_1, S_{L1} | J^{L}_{\mu}(0) | -k_2, S_{L2} \rangle \langle -k_2, S_{L2} | J^{L}_{\nu}(0) | k_1, S_{L1} \rangle
\]
\[
= 4 \left[ k_1 \cdot k_2 \eta_{\mu\nu} - k_\mu k_\nu - k_\nu k_\mu \right]
\]

Hadronic tensor:

\[
W^{\mu\nu} = \frac{1}{4} \sum_{S_{H1}} \sum_{S_{H2}} \int d^4 x e^{-i q \cdot x} \langle P_1, S_{H1}, P_2, S_{H2} | [ J^\mu_H(x), J^\nu_H(x) ] | P_1, S_{H1}, P_2, S_{H2} \rangle
\]

\[S_{Hi} = \text{spins}\]
Hadronic tensor can be decomposed into 4 invariant structure functions $W_i$ expressed in terms of 4 independent scalars (like $q^2$, $s$, $P_1 \cdot q$, $P_2 \cdot q$, or other convenient choice).

Useful quantities: Helicity Structure functions:

$$W_{\sigma, \sigma'} = \eta^{\mu}_{(\sigma)} \eta^{\nu*}_{(\sigma')} W_{\mu \nu} \quad ; \quad \sigma, \sigma' = (-1, 0, 1)$$

With polarizations:

$$\eta^{\mu}_{(0)} = Z^\mu \quad ; \quad \eta^{\mu}_{(\pm 1)} = \frac{1}{\sqrt{2}}(\mp X - iY)^\mu.$$  

More convenient form of helicity structure functions:

$$W_T = W_{1,1} \quad ; \quad W_L = W_{0,0} \quad ; \quad W_\Delta = \frac{1}{\sqrt{2}}(W_{0,1} + W_{1,0}) \quad ; \quad W_{\Delta\Delta} = W_{1,-1}$$
Cross section (in virtual photon rest frame):

\[ d\sigma \sim \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right) \]

Where \( \theta \) and \( \phi \) are the lepton angles (virtual photon rest frame):

\[ \vec{k}_1 = |\vec{k}_1|(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) = -\vec{k}_2 \]

With:

\[
\begin{align*}
\lambda &= \frac{W_T - W_L}{W_T + W_L}, & \mu &= \frac{W_\Delta}{W_T + W_L}, & \nu &= \frac{2W_\Delta\Delta}{W_T + W_L}
\end{align*}
\]

Helicity structure functions depend on the choice of coordinates (relation between the spatial axis \( X, Y, Z \) used for helicities and lepton directions and the momenta \( P_1, P_2, q \)). We used the Collins-Soper frame (Collins and Soper, Phys. Rev. D 77) (same used by experimentalists)
Perturbative QCD predictions in Collins-Soper frame:

\[ \lambda_{pert} = \frac{q^2 + \frac{1}{2}q_T^2}{q^2 - \frac{3}{2}q_T^2}, \quad \nu_{pert} = \frac{q_T^2}{-q^2 + \frac{3}{2}q_T^2}. \]

Where:

\[ q_T = (q_P^2 - q_p^2 - m_\gamma^2)^{1/2}, \quad q_p = \frac{q \cdot (P_1 - P_2)}{\sqrt{s}}, \quad q_P = -\frac{q \cdot (P_1 + P_2)}{\sqrt{s}} \]

These perturb. results are NOT valid for \( q_T^2 << |q|^2 \)

where there is an effective strong coupling (logarithm terms)

We used our model to investigate this region of small \( q_T \)
The model: dilepton production through the exchange of vector mesons

With vector mesons and baryons described by the hard wall model. Final states: excited baryons of spin $\frac{1}{2}$.
(We consider only lepton pairs produced by virtual photons)
Vector mesons in the hard wall model:

\[
S = -\frac{\kappa}{4} \int d^4x \int_0^{1/\Lambda} dz \sqrt{-g} \text{Tr} \ F_{MN} F^{MN}
\]

Kaluza Klein expansion, (gauge \( A_z = 0 \))

\[
A_\mu(z, x) = f^0(z) a_\mu(x) + \sum_{n=1}^{\infty} f^n(z) v^\mu_n(x)
\]

With the conditions:

\[
f^0(z) = 1, \quad f^n(z) = c_n z J_1(m_n z),
\]

\[
\kappa \int_0^{z_0} dz \sqrt{-g} h^2 f^n(z) f^m(z) = \delta^{nm}
\]

\[
\frac{1}{\sqrt{-g} h^2} \partial_z \left[ \sqrt{-g} h^2 \partial_z f^n(z) \right] = -m_n^2 f^n(z)
\]
After some field redefinitions we find an effective 4-d action

\[ S = \int d^4x \sum_{n=1}^{\infty} \text{Tr} \left[ -\frac{1}{4} \eta^{\mu\sigma} \eta^{\nu\rho} (\partial_{\mu} \tilde{v}_n^{\nu} - \partial_{\nu} \tilde{v}_n^{\mu}) (\partial_{\sigma} \tilde{v}_n^{\rho} - \partial_{\rho} \tilde{v}_n^{\sigma}) \right. \]

\[ -\frac{1}{2} m_n^2 \eta^{\mu\nu} \tilde{v}_n^{\mu} \tilde{v}_n^{\nu} + g_{vn} \tilde{v}_n^{\mu} a_{\nu} + \cdots \]

\[ - \sum_{n,m,\ell} g_{vm_\nu m_\nu \ell} \int d^4x \eta^{\mu\alpha} \eta^{\nu\beta} \text{Tr} \left\{ [v_n^{\mu}, v_m^{\nu}] \partial_{\alpha} v_{\beta}^{\ell} \right\} \]

With masses determined from boundary conditions at \( z = 1/\Lambda \) and couplings determined from the integration in the fifth dimension.

\[ g_{vn} = m_n^2 \kappa \int_0^{1/\Lambda} dz \sqrt{-g} \frac{z^4}{R^4} f^n(z) \]

\[ g_{vm_\nu m_\nu \ell} = \kappa \int_0^{1/\Lambda} dz \sqrt{-g} \frac{z^4}{R^4} f^n(z) f^m(z) f^\ell(z) \]
Baryons: similar Kaluza Klein expansion, boundary conditions, ...., effective 4-d free action

\[ S_F = \sum_{n=1}^{\infty} \int d^4 x \tilde{u}^n(x) \left[ \eta^{\mu\nu} \gamma_\mu \partial_\nu - M_n \right] u^n(x), \]

Interaction of fermions and vector mesons (start at 5-d, KK expansions, ....)

\[ S_{F\Phi V} = \sum_{n,m,\ell} \int d^4 x \eta^{\mu\nu} \tilde{u}^n \gamma_\mu \left[ g_{n\ell}^+ \phi^m \mathcal{P}_+ + g_{n\ell}^- \phi^m \mathcal{P}_- \right] \nu^m_{\nu} u^\ell, \]

\[ g_{n\ell}^+ = \kappa_F \int_0^{z_0} dz \sqrt{-g} \frac{z}{R} \phi^n(z) f^m(z) \phi^\ell(z), \quad \mathcal{P}_\pm = (1/2)(1 \pm \gamma^5). \]

\[ \psi = \sum_{n=1}^{\infty} \left[ \phi^n(z) \mathcal{P}_+ + \tilde{\phi}^n(z) \mathcal{P}_- \right] u^n(x), \quad \phi^n(z) = N_n z^{5/2} J_{\tilde{M}R-1/2}(M_n z), \]
Optical theorem: imaginary part of the forward proton-proton-photon scattering amplitude gives the hadronic tensor $W^{\mu\nu}$

So, it is just applying the Feynman rules for the effective model
We calculated $W^{\mu\nu}$ in the frame where:
\[ \vec{P}_1 + \vec{P}_2 - \vec{q} = 0 \]

Equivalent to final hadrons frame:
\[ \vec{P}_X + \vec{P}_Y = 0 \]

This simplifies the integrations. We first find the invariants: $W_1, W_2, W_3, W_4$, then the helicity structure functions in Collins Soper.

We considered a kinematical regime compatible with the region analysed, for di-muons by FNAL E866/NuSeA Collaboration, L.Y. Zhu at all, Phys. Rev. Lett 2009.

They found, from Fermilab data:

\[ \langle \lambda \rangle = 0.85 \ ; \ \langle \mu \rangle = -0.026 \ ; \ \langle \nu \rangle = 0.04 \]
Numerical set up.

- We fixed the size of the AdS slice by the mass of the ρ meson 0.776 GeV. This gives $\Lambda = 0.323$ GeV.

and chose for the proton and photon momenta the form:

$$
P_1 = \left( \sqrt{p^2 + M_1^2} , 0 , 0 , p \right) ; \quad q = \left( \sqrt{m_\gamma^2 + q_2^2 + q_3^2} , 0 , q_2 , q_3 \right)
$$

$$
P_2 = \left( \sqrt{q_2^2 + (q_3 - p)^2 + M_1^2} , 0 , q_2 , q_3 - p \right)
$$

Important issues for the Numerical computations:

- When do we stop the infinite series in baryons and vector mesons?

For the baryons (on shell states): $M_{n_X} + M_{n_Y} \leq E_F$.

For the vector mesons: we investigated numerically the convergence of the series and found that we should use 15 states for the vector mesons.
Results:

<table>
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<th>Kinematical regimes</th>
<th>Our results</th>
<th>Perturb. results</th>
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</thead>
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<td>$p$</td>
<td>$\sqrt{s}$</td>
<td>$q_T$</td>
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<td>38.8</td>
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<tr>
<td>22.7</td>
<td>38.9</td>
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</tr>
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</table>

TABLE I: Angular parameters calculated for $m_{\gamma}^2 = 104 \text{ GeV}^2$. $p, \sqrt{s}, q_T$ are in GeV.

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TABLE II: Angular parameters calculated for $m_{\gamma}^2 = 56.3 \text{ GeV}^2$. $p, \sqrt{s}, q_T$ in GeV.
We found a decrease in the parameter $\lambda$ for $q_T \to 0$, while perturbative expression $\to 1$.

$m_{\gamma}^2 = 104$ GeV$^2$

We are looking forward for more experimental results at small $q_T$. 

$m_{\gamma}^2 = 56.3$ GeV$^2$
Experimental results

FIG. 1: Parameters $\lambda, \mu, \nu$ and $2\nu - (1 - \lambda)$ vs. $p_T$ in the Collins-Soper frame. Solid squares (open circles) are for E866 $p + p$ ($p + d$) at 800 GeV/c. The vertical error bars include the statistical uncertainties only.
Other Results:

Production of negative parity baryons in the holographic Sakai-Sugimoto model

A. Ballón-Bayona, H.B.-F., N. Braga, M. Ihl, M. Torres
Other Results:

Pion and vector meson form factors in the Kuperstein-Sonnenschein holographic model


The KS model is based on the D3-brane background with a conical singularity in type IIB superstring theory first studied by Klebanov and Witten.
Other Results:

Scalar and vector mesons of flavor chiral symmetry breaking in the Klebanov-Strassler background

Matthias Ihl, a Marcus A.C. Torres, a Henrique Boschi-Filho a and C.A. Ballon Bayona b,c JHEP (2011)

Dymarsky, Kuperstein and Sonnenschein (DKS) proposed and embedding of flavor D7- and anti-D7-branes in the Klebanov-Strassler (deformed conifold) geometry, breaking the supersymmetry, but still being stable.
Thank you!!