Student understanding of the first law of thermodynamics: Relating work to the adiabatic compression of an ideal gas

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We report on an investigation of student understanding of the first law of thermodynamics. The students involved were drawn from first-year university physics courses and a second-year thermal physics course. The emphasis was on the ability of the students to relate the first law to the adiabatic compression of an ideal gas. Although they had studied the first law, few students recognized its relevance. Fewer still were able to apply the concept of work to account for a change in temperature in an adiabatic process. Instead most of the students based their predictions and explanations on a misinterpretation of the ideal gas law. Even when ideas of energy and work were suggested, many students were unable to give a correct analysis. They frequently failed to differentiate the concepts of heat, temperature, work, and internal energy. Some of the difficulties that students had in applying the concept of work in a thermal process seemed to be related to difficulties with mechanics. Our findings also suggest that a misinterpretation of simple microscopic models may interfere with student ability to understand macroscopic phenomena. Implications for instruction in thermal physics and in mechanics are discussed. © 2002 American Association of Physics Teachers.

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I. INTRODUCTION

Articles on the teaching of thermal physics have addressed a number of important issues, such as the proper presentation of relevant concepts and principles, the order in which topics should be treated, and the merits of a macroscopic or microscopic approach.1 However, for the most part, the discussions have not been informed by a detailed understanding of how students learn this body of material. Research has helped identify a number of specific difficulties.2,3 In particular, the tendency to confuse the concepts of heat, temperature, and internal energy has been well documented.4 However, these insights have had little impact on how thermal physics is typically presented in a standard introductory course.

In mechanics, the situation is different. The teaching of kinematics and dynamics in recent years has been strongly influenced by findings from research.5 Reports from assessments of the effectiveness of instruction have been published. Our experience suggests that a similar approach to curriculum development on other topics is necessary.6 We have found that the level of detail needed for developing effective instructional materials requires in-depth probing of student understanding with a variety of questions involving different contexts and administered in different formats. In this paper, we report on the results from such a study in thermal physics and discuss some implications for instruction.7 The emphasis is on the first law of thermodynamics, and, in particular, on the change in the internal energy of an ideal gas through thermodynamic work.

II. OVERVIEW OF INVESTIGATION

We were interested in whether university students who have studied thermal physics are able to apply the first law of thermodynamics to a simple physical phenomenon. Specifically, we wanted to examine the extent to which students, given a familiar situation, could (1) recognize that the first law is relevant to predicting or explaining an observation, (2) decide whether there is heat transfer or work done and, if so, determine the sign(s), and (3) relate these quantities to a change in the internal energy of the system.

A. Previous research

Much of the previous research has involved processes in which heat is the only means of energy transfer. Several of these studies indicate that students tend to treat heat as a substance residing in a body.8 Although most of the studies have been at the precollege level, experienced instructors have noted that many university students have similar difficulties.9 Studies in physics and chemistry courses have revealed the failure of many university students to distinguish between state and process quantities (for example, temperature and heat transfer), as well as to apply the ideal gas law correctly.10,11 There has been little or no previous research on student ability to apply the concept of thermodynamic work.

B. Instructional context of the study

Most of the investigation was carried out at the University of Washington (UW) among students in the introductory algebra-based course and a second-year course in thermal physics. Both courses have a standard lecture format. The algebra-based course devotes about two weeks to thermal physics. The number of students in each lecture section of this course ranges from fewer than 100 to more than 200 students. About half of these enroll in an accompanying laboratory course. Enrollment in the second-year course typically varies between 25 and 50 students. Physics majors comprise about 30% of the students in this course. Almost all of the students in the second-year course have taken introductory calculus-based physics (which does not include thermal physics at UW). Students from a junior-level statistical physics course also participated in the study. Additional data were obtained from introductory calculus-based physics courses at other large research institutions in which thermal physics is part of the first-year curriculum.
The first law of thermodynamics was expressed differently in the two courses at UW in order to be consistent with the respective textbooks. In the second-year course, it was expressed as \( \Delta U = Q + W \), where \( U \) is the internal energy of a system, \( Q \) is the heat transferred to the system, and \( W \) is the work done on the system. In the algebra-based course, the first law was expressed in terms of the work done by the system, that is, as \( \Delta U = Q - W \). In both courses, the application of the first law was limited to ideal gas systems and mostly to quasi-static processes.

All of the students had been taught a general definition for the work done on a system (\( W_{on} \)) by an agent that exerts a force on that system (\( F_{on} \)). Those in second-year thermal physics had previous experience with the definition \( W_{on} = \int F_{on} \cdot ds \) in a course on mechanics. This definition was used in the thermal physics course to derive an expression for the work done on (or by) a gas in terms of its pressure and a change in its volume (\( W_{on} = -\int P \cdot dV \)). In the algebra-based course, in which the scalar product was not used, work was defined as the product of a displacement and the component of the force parallel to that displacement. A nonintegral expression for the work done by a gas (\( W_{by} = P \Delta V \)) was derived for isobaric processes only.

C. Methods

As in most of our research, we used two primary methods. We have found that the focus on real objects and events in an individual demonstration interview is effective for probing the ability of students to relate the concepts of physics to the physical world. Written questions that are administered to large groups of students allow us to estimate the prevalence of specific student difficulties. In interviews, students are asked for details or clarification. On written questions, they are asked for explanations. In the discussion of the results for both written and interview questions, we quote specific students. For every case, however, the quotes are representative and not idiosyncratic.

We began our investigation with individual demonstration interviews with students from the two courses at the University of Washington. The interviews were designed to elicit ideas that students have about thermal phenomena and, more specifically, to probe their ability to apply the first law to the adiabatic compression of air, considered as an ideal gas. We deliberately chose an adiabatic process because we wanted to focus on the concept of work. During the interviews, the students were shown a plastic bicycle pump and were told to imagine that the open end would be sealed while the handle of the pump was rapidly pushed inward. They were asked to predict what would happen to the temperature of the air inside the pump and to explain their reasoning.

A correct explanation includes the following elements. (1) Because the compression of the air in the pump is carried out quickly, the process is approximately adiabatic and, therefore, \( Q \approx 0 \). (2) Because the force that the piston exerts on the gas and the displacement of its point of application are parallel, the work done on the gas is positive. (Alternately, the expression \( W_{on} = -\int P \cdot dV \) could be used, which also yields a positive result.) Therefore, according to the first law, \( \Delta U \) is positive and the temperature of the (ideal) gas will increase. In this paper, we include students' explanations in which heat transfer is not explicitly mentioned among those considered "correct," provided a correct link between the change in temperature and the work done was made.

We intentionally chose a task that could not be solved without the first law. The ideal gas law (\( PV = nRT \)), which relates the temperature \( T \) of a sample of \( n \) moles of dilute gas to the product of the pressure \( P \) and the volume \( V \) for equilibrium states, is insufficient. The force applied to the piston increases the pressure on the gas and decreases the volume. However, the first law is needed (directly or indirectly) to determine if the product of these two variables (and hence the temperature) increases, decreases, or remains the same.

Written problems were administered either on course examinations or as ungraded quizzes. Unless otherwise noted, all were given after the relevant instruction was completed.

The bicycle pump problem is a written version of the interview task described above. In version A, students are asked to predict what will happen to the temperature of the gas when the piston is pushed inward rapidly and to justify their prediction [see Fig. 1(a)]. In version B, the text states that the temperature of the gas increases, and students are asked to account for this change [see Fig. 1(b)]. A slightly different context is used in the written insulated cylinder problem, in which the compression is accomplished by adding masses to a piston that seals an insulated cylinder (see Fig. 2).
Student comments on the concept of work during the interviews prompted the design of additional written questions about the work done on or by an ideal gas during a specific process. Two of these are illustrated in Figs. 3 and 4. Because some of the difficulties we found seemed to reflect difficulties with mechanics, we posed analogous problems in that context for the purpose of comparison. An example is shown in Fig. 5.

III. INABILITY TO RECOGNIZE THE RELEVANCE OF THE FIRST LAW OF THERMODYNAMICS

As mentioned, one objective of our investigation was to determine whether students could recognize the relevance of the first law to the analysis of a physical situation, specifically the adiabatic compression of a gas. We worked toward this objective by conducting individual demonstration interviews based on the bicycle pump task.

Two rounds of interviews were conducted. They differed with respect to the follow-up questions that were posed after the initial bicycle pump task. The first round involved 22 students: 7 from the algebra-based course and 15 from the second-year course. In the second round, 14 students participated: 9 from the algebra-based course and 5 from the second-year course. The students, who were all volunteers, were drawn from several sections of each course that were taught by different instructors. Almost all of the students had final grades at or above the mean in their respective classes. All instruction in thermal physics had been completed at the time of the interviews.

In total, about 75% of the students who were interviewed stated correctly that the temperature of the air in the bicycle pump would increase. Assessing student ability to predict the outcome of this experiment was not the only goal. We were interested in what the students’ explanations revealed about their understanding of the role of work in changing the temperature of a gas. Their ability to recognize and to weigh the relevance of the various concepts and principles that they had studied was of particular interest. The pattern that emerged had two major, interdependent features: (1) failure to consider the concept of work and (2) misapplication of ideal gas concepts. In the following discussion, the quotes are from dialogues between the interviewer (I) and individual students (S). The data and quotes from the interviews are supplemented with information from responses to the written questions described above.

A. Failure to consider the work done

The degree to which students deliberated during the interviews before stating a prediction or giving an explanation varied widely. At one extreme were students (about 30% of the total) who responded almost immediately that the temperature would increase and had to be prompted for an explanation by the interviewer. This category includes some students who recalled the result from a similar lecture demonstration and others who acknowledged that their answers were based on instinct or experience and not on formal reasoning. For instance, one student said “I made that prediction without thinking because it just feels right.” At the other extreme were students (about 25% of the total) who discussed the first law and/or the ideal gas law sometimes for several minutes before giving an answer. In some cases, they could not arrive at an answer in spite of prolonged deliberation.

A characteristic feature of the first set of interviews was that few students spontaneously invoked the concept of work. We concluded that if we wanted them to discuss the concept, we needed to be more direct. This observation prompted a change for the second round of interviews. These interviews started the same way as those in the first round. However, students who did not spontaneously mention work were given an increasing amount of guidance as the interview progressed.

To encourage students who had not mentioned work to do so, we first asked whether the term energy could be related to the bicycle pump process. Several stated that they could not
Table I. Student responses to the written versions of the bicycle pump problem (see Fig. 1). In version B of the problem, the text states that the temperature of the gas increases. Therefore, only student explanations are reported. In the two cases in which results were obtained from multiple sections of the same course (some before and some after instruction) the results from individual sections all were within 5% of the mean value for the course, and therefore have been combined. For clarity, results reported are expressed as percentages, rounded to the nearest 1%.

<table>
<thead>
<tr>
<th></th>
<th>Algebra-based course</th>
<th>Algebra-based course</th>
<th>Calculus-based course</th>
<th>Thermal physics course</th>
<th>Statistical physics course</th>
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<td></td>
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<td>UW</td>
<td>UMCP</td>
<td>UW</td>
<td>UW</td>
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<tr>
<td>N=114</td>
<td>One section</td>
<td>One section</td>
<td>Three sections</td>
<td>Four sections</td>
<td>One section</td>
</tr>
<tr>
<td>Correct response (T increases)</td>
<td>67%</td>
<td>n/a</td>
<td>58%</td>
<td>86%</td>
<td>100%</td>
</tr>
<tr>
<td>Correct explanation based on:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>work</td>
<td>10%</td>
<td>9%</td>
<td>3%</td>
<td>25%</td>
<td>14%</td>
</tr>
<tr>
<td>microscopic work</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>13%</td>
<td>43%</td>
</tr>
<tr>
<td>adiabatic equation ($PV = const$)</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Incorrect explanation</td>
<td>57%</td>
<td>91%</td>
<td>55%</td>
<td>35%</td>
<td>43%</td>
</tr>
<tr>
<td>Incorrect responses</td>
<td>33%</td>
<td>n/a</td>
<td>36%</td>
<td>14%</td>
<td>0%</td>
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<tr>
<td>Blank</td>
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<td>n/a</td>
<td>6%</td>
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See how it could be helpful. Others hesitated, and when pressed by the interviewer, turned to potential energy. For example, one student said:

S: I guess...could it be that the potential energy is increasing? I mean that the air can create a force when it’s let go, so the potential energy increases.

In all cases, students who were asked about energy were unable to proceed further, and were subsequently asked whether they could apply the concept of work to the process. All of the students indicated that they recalled that work was relevant. However, some acknowledged a lack of understanding of the connection between work and the temperature of the gas, as illustrated by the following exchange:

I: If I do work on a gas, why is that important?
S: If I do work on a pencil, I don’t increase its temperature. But with a gas...it seems, well, it is related somehow.

Finally, if questions about work did not prompt students to apply the concept, the algebraic statement of the first law (that is, $ΔU = Q + W$), was provided. The students were explicitly asked whether this equation could help in analyzing the problem. Only 4 of the 16 students from the algebra-based course and 6 of the 20 from the second-year course gave correct explanations based on the concept of work, with or without prompting from the interviewer. The dialogues demonstrated that the failure of students to mention work spontaneously reflected serious difficulty in understanding both when and how to apply the concept, not merely a failure to remember it. The specific difficulties that arose when students tried to apply the concept of work will be discussed later.

The written versions of the same task were administered at the University of Washington in the algebra-based and second-year courses and in the junior-level course in statistical physics and in introductory calculus-based courses at the University of Maryland, College Park (UMCP) and the University of Illinois, Urbana/Champaign (UIUC). Altogether, more than 500 students were involved. In most cases, the question was administered after all relevant instruction had been completed. In a few cases, however, it was administered before instruction began. The amount of instruction did not seem to affect the results. In all classes, a majority of the students predicted that the temperature of the gas would increase. Correct explanations were given by fewer than 10% of the students in the introductory courses and by about 50% in the more advanced courses. The results are summarized in Table I.

The insulated cylinder problem was administered in one section of the UW algebra-based course ($N=179$) after all instruction was completed. In contrast to the bicycle pump problem, only about 10% of the students correctly predicted that the temperature of the gas would increase. The differences between these questions that could account for this apparent discrepancy are discussed later.

B. Misinterpretation of the ideal gas law

The failure of students to analyze the problems in terms of work and the first law of thermodynamics was prevalent and persistent. As with many other topics, we found that student difficulties with general principles are often intermingled with difficulties related to the specific system to which the principles are applied. Thus student understanding of the first law of thermodynamics cannot be discussed without some discussion of student understanding of the ideal gas law. In the interviews and written problems, we found that misinterpretation of the ideal gas law was common. Our observations are consistent with those of Rozier and Viennot, who described student difficulties with the ideal gas law at both macroscopic and microscopic levels.

1. Incorrect reasoning at the macroscopic level

In both the interviews and on the written problems, most students cited the ideal gas law and argued that changes in pressure and/or volume resulting from the compression would lead to an increase in the gas temperature. (As noted earlier, there is not enough information to reach such a conclusion.) We observed several distinct types of incorrect arguments that are similar to those reported by Rozier and Viennot, who attributed some of these difficulties to faulty
reasoning with multi-variable relationships. However, as in other research by our group, we found that general reasoning difficulties could not be completely separated from difficulties with specific concepts.25

Some students focused on only two macroscopic gas variables at a time and assumed (implicitly) an invalid relationship between them. Several stated that the volume decrease of the gas in the cylinder would cause a pressure increase (implicitly holding the temperature constant) and that the pressure increase would then cause a temperature increase (implicitly holding the volume constant). One student wrote: “Decreasing the volume increases the pressure, which increases the temperature. \(P V = nRT\).”

Other students appeared to ignore one variable entirely. For example, on the written problems, about 10% referred only to the fact that the pressure increases when the piston is pressed inward. One such student wrote: “There was a quick change in pressure. The pressure went up and so did the temperature.” The direct relationship between \(P\) and \(T\) that is implied may be an inappropriate generalization of the case of constant volume processes.

About 10% of the students made a related but more surprising error by treating temperature as inversely proportional to volume. For example, one student wrote: “In the ideal gas law, if we change the volume to a smaller amount, the temperature will rise.” Even under the assumption of constant pressure (or if pressure is neglected altogether), it does not follow from the ideal gas law that \(T\) varies inversely with \(V\). The evidence presented below suggests that arguments of this nature may be related to incorrect microscopic ideas.

2. Incorrect reasoning at the microscopic level

During the interviews, students were often quick to introduce ideas related to particles or molecules. Often they made an inappropriate connection between volume and temperature, apparently assuming a relationship between the number density of gas particles and their average speed or energy, and thus the temperature. Some students made vague statements to this effect, such as: “The smaller volume forces the molecules of gas to increase in speed, therefore increasing the temperature,” or “The molecules of gas are closer together, making the energy higher.”26

Some students were more explicit about the connection between number density and gas temperature. They proposed a causal mechanism based on an increased frequency of collisions between gas particles. For example, one student said: “The molecules are getting compressed, and they have less space to move around, so they are bumping into each other a lot more, and the temperature increases.” Often students suggested that the collisions would produce or release heat: “…more collisions per unit time equals more heat generated.”

In their lectures and textbooks, the students had been given a simple microscopic model in which gas molecules interact only through elastic collisions. The molecules themselves are assumed to have no internal energy. Moreover, there is no potential energy associated with the distances between molecules. In this model, the temperature of the gas is related to the average molecular kinetic energy, which can be changed through heat transfer or collisions with a moving piston.27 Therefore it is necessary to examine the system’s interactions with its surroundings to make judgments about heat transfer, work done, and internal energy. The failure of students to do so was widespread.

In particular, in the collision mechanism proposed by the students, a change in internal energy is achieved through interactions internal to the system, rather than interactions between the system and its environment. Students who referred to intermolecular collisions were unaware (or unconcerned) that the process they proposed could not increase the temperature without violating the principle of energy conservation. Moreover, they failed to consider the implications for gases in equilibrium states.

It is interesting to note that microscopic arguments were used by about 30% of the students in interviews, whereas as few as 10% expressed such ideas on the written problems even though the questions asked were essentially the same. The difference in format may help account for this apparent discrepancy. During the interviews, some students who initially referred to macroscopic quantities very quickly introduced microscopic ideas to support their assertions when asked for further details. Consequently, we believe that some students’ arguments that involve macroscopic quantities are in fact tightly linked to microscopic ideas.28 An underlying microscopic picture could account for the otherwise surprising tendency of students to assume an inverse relationship between \(V\) and \(T\) on written problems. Of course, answers to a single question administered in a single format do not capture the complexity of student thinking.29

IV. INABILITY TO APPLY THE CONCEPT OF WORK IN THERMAL PHYSICS

During the interviews, it quickly became apparent that many students did not treat the first law of thermodynamics as a cause–effect relationship in which work can bring about a change in the internal energy of a physical system. In an earlier study of student understanding of the work-energy and impulse-momentum theorems, we had encountered a similar inclination to view an equation in physics solely as a mathematical relation.30 There was a strong tendency to treat the theorems as formulas, not as mathematical models of important physical principles.

We were able to identify a number of specific difficulties related to the first law. These can be grouped into two broad, overlapping categories: (a) difficulties in discriminating among related concepts (heat, work, temperature, and internal energy) and (b) difficulties in applying the definition of work to an ideal gas undergoing a specific process. The following examples are taken from both the interviews and written problems.

A. Difficulty in discriminating among related concepts
(heat, work, temperature, and internal energy)

Many students seemed to confuse quantities associated with processes (such as heat transfer and work) with those associated with states (such as temperature and internal energy). For some, this confusion was reflected in a tendency to refer to the “change in heat” or the “change in work.” In writing, many used the symbol for a given quantity (for example, \(P\)) and the symbol denoting a change in that quantity (for example, \(\Delta P\)) interchangeably. There is evidence presented below that the inability to distinguish between two
seemingly similar concepts was often at a sufficiently deep level that it precluded students from correctly applying the first law.

1. Confusion among heat, temperature, and internal energy

We found that many students had difficulties with the concept of heat similar to those reported among precollege students, notably a tendency to confuse heat, temperature, and internal energy. Some students admitted confusion on this point, even wondering why both $\Delta U$ and $Q$ are present in the algebraic form of the first law, because both seem to describe the “heat in an object.” Some of the responses reported here might be seen as primarily reflecting careless use, or misinterpretation, of admittedly confusing terminology. Several articles on the teaching of thermal physics note that even instructors and textbook authors, who presumably have mastered the concepts, often use technical terms imprecisely. Therefore, an analysis of the language used by students is not sufficient for making judgments about understanding. To help distinguish conceptual and linguistic issues, it is useful to examine the ability of students to make correct predictions concerning physical phenomena.

An excerpt from an interview serves as an example. In this case, the student spontaneously used the first law in attempting to analyze the compression of the air in the bicycle pump:

S: The internal energy is equal to the energy of the system minus the work. So [writes] $U = Q - W$ [sic].

The interviewer questioned the student on the distinction, if any, between “internal energy” and “energy of the system.” The student responded by acknowledging confusion.

S: I’m getting confused with how to apply the thing.
I: How are you getting confused?
S: First of all, if the heat of the system is the same as internal energy, or the temperature, or if it’s related to $Q$.

The student continued to struggle but was unable to arrive at a prediction. Her inability to distinguish the quantities related by the first law prevented her from applying it even in this relatively simple case.

Results from the written insulated cylinder problem, which was administered in the algebra-based course, provide a further illustration. In contrast to the written bicycle pump problem, the majority of students answered incorrectly. Some incorrect responses reflected a failure to recognize either that work had been done or that work can change the temperature of the gas. Both of these difficulties are discussed in greater detail below. The most common incorrect answer, however, was that the temperature of the gas would remain the same when the volume decreases because the cylinder is insulated. For example, one student stated that “... an insulating jacket will hold all the heat in.” About half of the students explicitly stated that the insulation would either prevent a change in temperature or in internal energy. It seems that many students used the concepts of heat and internal energy, and not just the words, interchangeably.

2. Confusion among work, heat, and internal energy

Although some students treated heat transfer as a process that can produce a change in internal energy, many others treated heat as essentially the same thing as internal energy. In both cases, heat may be associated so closely with changes in temperature that the concept of work may seem superfluous. Therefore, it is not surprising that a number of students admitted that they did not see how work is connected to the temperature of a gas. Many others, in attempting to connect the concept of work with temperature change, seemed to associate work with heat. During the interviews, several students used the word “heat” rather than “work” to refer to the mechanical energy transfer in the process in which the piston is pressed inward. In their responses to written problems, students expressed many similar ideas. For example, one wrote: “The heat caused by pressing the piston inward is gained by the gas, so the temperature increased.”

Our discussions with students during the interviews suggested that such statements do not simply represent careless or confused use of language. In some cases, there was a genuine confusion between the mechanical energy transfer that physicists refer to as “work” and the nonmechanical energy transfer that physicists refer to as “heat.” This confusion prevented many students from correctly applying the first law. In particular, students who failed to recognize heat and work as independent means of energy transfer often indicated that any process that involves work must also involve a heat transfer.

When this issue arose during the interviews, we asked students explicitly whether it was possible for a process to involve work but no heat transfer. One student, who had used the concept of work to make her initial prediction concerning the bicycle pump, responded as follows:

I: Coming back to work and heat, is it possible to have work being done on the environment but no heat transfer?
S: Oh, well, what he [the instructor] was saying is that work equals $PdV$. So change in volume [pauses] no, there has to be heat transfer.

This statement suggests that work and heat are not independent. The student continued by invoking the ideal gas law:

S: Since work is related to $P$ and $V$, that’s going to relate to temperature.

The student went on to state that no work is done in an isothermal process, essentially arguing that because the temperature does not change, no heat is transferred, and if no heat is transferred, then no work is done. The tendency to make an inappropriate connection between work and heat transfer has also been reported in studies of chemistry students.

B. Difficulties in applying the definition of work to an ideal gas undergoing a specific process

During the interviews, some students were asked directly about the sign of the work done on the gas during the bicycle pump compression. Although most answered correctly that the work is positive, few applied the definition that they had been taught. The responses revealed a failure to recognize that the sign of the work has a physical significance in terms
of the change in internal energy of a system. References to the sign of work being a matter of “convention” were common. The following exchange is illustrative.

I: How can you find the sign of work?
S: Isn’t it a convention? It is. I think it’s negative, but I can’t remember why that was. If the volume is decreasing that means that there is work being done on that, so the $\Delta V$ would be negative. But the work will be positive.

I: What is the sign of the work done by the piston on the gas [in this process]?
S: That work is...I don’t know. I’m guessing it would be positive. I don’t know the correlation between the positive and the negative part.

The incomplete or incorrect responses of the students indicated the presence of underlying difficulties related to the definition of work. In a number of cases, we broadened the scope of the interviews to pursue specific issues that had arisen. We also developed written questions that involved nonadiabatic processes, as described below.

1. Failure to recognize that the sign of work is independent of the coordinate system

Some students seemed to apply a simplified definition of work, in which they considered either the direction of a force or the direction of the displacement of the point of application of the force. For example, one student said:

S: [the sign of work] is not completely arbitrary. The distance is negative, so that means...the work done on the piston is negative. You use the distance the piston is traveling.

Some students spoke of “the direction of work,” as though it were a vector quantity. Others explicitly related the sign of the work done to the choice of coordinate system. For example, when one student referred to negative work, the interviewer asked:

I: What’s negative work?
S: I don’t know. In the opposite direction.
I: Opposite to what?
S: Against your coordinate axes.

2. Failure to recognize that work done on and by a system in a given process have the same absolute value

Several students made comments during the interviews that suggested incorrect ideas about the absolute value of the work done on or by a gas in a given process. In some cases, dialogues led the interviewer to ask about the work done on the gas if it expands while the interviewer’s hand is still pressed against the handle of the bicycle pump. Several students responded that there is no such work.

The students’ comments were consistent with the belief that no work is done by a gas in a compression or on a gas in an expansion. Their statements were also reminiscent of those in most textbooks, in which thermodynamic work is described as being done on a system in a compression and by a system in an expansion. This practice necessitates the adoption of a sign convention for work. The convention depends on whether the first law is written in terms of $Q + W$ or $Q - W$. For example, the textbook used in the UW introductory algebra-based course, which adopts the $\Delta U = Q - W$ form for the first law, states that “If work is done on the system, then $W$ will be negative.”

The textbook used in the second-year course, which uses $\Delta U = Q + W$, states that “work done on a system is positive.” (Variations of these statements appear in other textbooks.)

In contrast, in most treatments of mechanics, work that is done on a system by an agent exerting a force on that system can be either positive or negative. The sign depends only on the relative directions of the force and the displacement of the point of application of the force (or the displacement of the system itself if it is a point particle). The change in kinetic energy of a system is usually analyzed through the work-kinetic energy theorem exclusively in terms of the work done on the system. The choice of whether to consider the work done on or by the system does not depend on the specific circumstances.

In order to apply the first law correctly, the students must be able to apply consistently a correct criterion for deciding whether work is done on or by the system in a given process. Therefore we were interested in how students would make this choice in specific situations. We gained some insight from some students’ claims, mentioned above, that no work is done on a gas in an expansion. (The situation they had been asked about was not a free expansion.) The students typically argued that the gas is causing the motion, as illustrated by the following excerpt:

S: If the gas were to expand, the work done by the gas would be positive.
I: Would there be work done on the gas in that case?
S: No.
I: Why not?
S: Because the work is being done on the piston in that case.

Another student, referring to the compression of the air in the bicycle pump, said:

S: I’m causing the movement, not the system, so the work is being done by me, not by this [the air in the pump].

Other students claimed that work is done on and by an object in the same process, but that the absolute values will be different. For example, referring to the compression of the gas in the bicycle pump, one student said:

S: There’s some kind of force in the opposite [outward] direction. There is work done by the gas, but the work I do is greater. The net force is in this [inward] direction.

Another seemed to associate the motion of the piston with an imbalance between the forces exerted on and by it:

S: In this case, you did work on the system, it kind of lost the war...Your hand was obviously a whole lot stronger than the gas inside, you did positive work on the system. That’s how I got the sign convention of it.

The arguments made are consistent with the belief that a passive object (the gas) does no work, or does less work than an active object (the hand). Either the piston or the gas is thought of as active (and the other as passive), depending on the direction of motion.

Several interpretations of the dialogues with students are possible. They may reflect a misunderstanding of discussions of sign conventions. The students’ comments may also reflect
underlying difficulties with concepts and principles from mechanics and their relation to thermodynamics. These interpretations are not mutually exclusive; both may reflect the different ways in which the concept of work is usually treated in mechanics and thermodynamics. This inconsistency may make it more difficult for students to transfer a concept initially learned in one context to the other.

3. Failure to recognize that work is path dependent in the general case

During interviews and informal conversations, a few students made comments that suggested they were relating the work done on a system to the net change in volume. (This procedure is valid if the process is adiabatic, but not otherwise.) To probe student understanding of the path dependence of work, we designed several problems in which two different paths between an initial state \( X \) and a final state \( Z \) are depicted on a \( PV \) diagram. Thus at least one of the paths is nonadiabatic.

The problem in Fig. 3 was given on an examination in a section of the UW second-year course \( (N=42) \) and in the UIUC calculus-based course \( (N=392) \). The students were asked (1) to determine the sign of the work done on the gas in processes \( X-1-Z \) and \( 2 \) to compare the absolute values of the work done on the gas in processes \( X-1-Z \) and \( X-2-Z \). The work done on the gas during process \( X-1-Z \) is negative. A comparison of the areas under the paths shows that it is smaller in absolute value than that done in process \( X-2-Z \).

About 45% of the students in the second-year course and 40% of the students in the UIUC calculus-based course answered correctly that the work done on the gas in process \( X-1-Z \) is negative. The tendency to focus on the initial and final states, rather than on the paths taken, was common. In both classes, the most common incorrect answer was that the work done on the gas is zero. In comparing the work done on the gas in the two processes, about 25% of the students in the second-year course and about 45% of the students in the calculus-based course answered that the absolute values would be the same. One student explained that “…the initial and final states are the same for both processes.” Several students explicitly stated that “The work is independent of the path taken.” It is possible that these students were overgeneralizing their experience with conservative forces in their mechanics courses.

Students were especially likely to neglect the path dependence of work in cyclic processes, such as the one shown in the \( PV \) diagram in Fig. 4. This problem was administered in the UW algebra-based course \( (N=112) \). The students were asked whether the net work done on the gas in the cycle is positive, negative, or zero. About half of the students said that the net work done on the gas was zero, typically pointing out that there was no net change in volume. For example, one student wrote:

\[
\text{No net work was done because the cycle started and ended at the same place. Since there is no displacement, then no work was done.}
\]

For some students, the belief that the work done would be zero took precedence over other considerations. For example, one student wrote “\( W= \) area under curve” and shaded the appropriate area, but finally answered: “Since there was no change in volume overall, there was no work done on the system.”

Another category of incorrect responses to questions based on \( PV \) diagrams seems to stem from a failure to recognize that the absolute value of the work done on \( X \) and by \( Z \) the gas must be the same. For example, in reference to the process depicted in Fig. 3, one student said: “From 1-Z work is being done on the environment, so the work done on the gas from 1-Z is zero.” As mentioned earlier, similar ideas were expressed by students during the bicycle pump interviews. It seems that difficulties of this nature can interfere with student ability to use \( PV \) diagrams to determine the work done in a specific process.

V. INABILITY TO APPLY THE CONCEPT OF WORK IN MECHANICS

In both the interviews and on written questions, many students’ comments about work were reminiscent of incorrect ideas about forces that had been documented. Previous research has demonstrated the tendency of students to ignore forces exerted by “passive” objects—both in static and dynamic situations. In the former case, many students neglect the force by the passive object altogether (such as the force exerted on a book by a table), arguing that the passive object is simply “in the way.” In the latter case, many students assume that a force exerted in the direction of motion is greater than the corresponding reaction force. It seemed that some of the difficulties encountered by students in applying the concept of work to ideal gas processes were rooted in difficulties with mechanics. We decided to probe student understanding of work in situations free from the complications of thermal physics.

A. Extension of the investigation in thermal physics to mechanics

We designed some written questions about the work done on objects that can be treated as point particles. The questions were administered on ungraded quizzes in the UW introductory algebra-based and second-year courses and in the UMCP introductory calculus-based course. The students had previously studied mechanics. Instruction on the first law of thermodynamics had also taken place.

1. Mechanical work problem

The mechanical work problem involves two or more situations in which a hand pushes on a block that is moving on an inclined plane. The students are asked to determine the sign of the work done: (1) on the block by the hand, (2) on the block by the earth, and (3) on the hand by the block (or to state explicitly if there is no such work).

In cases 1 and 2, the force exerted by the hand is directed upward along the incline but the motion of the block is different (see Fig. 5). In case 1, the block is moving up the incline with increasing speed. In case 2, the block is moving down the incline with decreasing speed. Some versions of the quiz included a third case, in which the hand pushes down the incline as the block moves down with decreasing speed (the presence of friction is noted in this case). In all cases, the sign of the work done on the block by the hand can be determined by comparing the direction of the force exerted by the hand to that of the block’s displacement over a short time interval. Newton’s third law can be used to find
that the corresponding work done on the hand by the block is of opposite sign. The sign of the work done on the block by the earth can be found by comparing the directions of the displacement of the block and the gravitational force exerted on the block.

2. Results from the mechanical work problem

Almost all of the students gave the correct answer for the sign of the work done on the block by the hand in case 1. However, correct explanations were given by only about 10% in the UW algebra-based course, about 20% in the UMCP calculus-based course, and about 45% in the UW second-year thermal physics course. An additional 5% to 15% in each class referred to work as the product of force and displacement, but failed to take into account the vector nature of these quantities. One such student wrote that the work done by the block "...would be negative work since the distance used to calculate work depends on where a person chooses to place the coordinate axis." Other students were less explicit but nonetheless described a specific coordinate system. For example, one wrote, "The work done on the block by the hand is positive, because I defined my coordinate system to be positive in the up direction." Students who drew and/or referred to coordinate systems based their answers on a variety of quantities (for example, the directions of the applied force, displacement, velocity, and acceleration). Shown in Fig. 6 is the response of one of these students.

Explanations for the sign of the work done on the block by the hand in which the hand is considered the "cause" of the motion were made by between about 5% and 15% of the students in each course. In the UW algebra-based course, about 10% of the students stated that the corresponding work done on the hand by the block would be zero. An additional 20% answered that there would be no such work. Many incorrect responses were consistent with the belief that the block does no work because it is not the agent causing the motion: "The work done by the block on any object is zero because it doesn’t make anything move." Others indicated that the work done by the block on the hand was nonzero, but smaller in absolute value than that done by the hand, "...the work done on block by hand is more than vice versa."

It is important to note that statements such as these were made even by students who demonstrated some understanding of Newton’s third law. For example, one student from the UW algebra-based class, referring to the work done by the block on the hand in case 1, wrote, "[There is] no such work, the block may have an equal and opposite force on the hand, but the block is not actually doing any work." Another student wrote that the work done by the block "...would be zero...because it exerts a force but its force is not responsible for traveling a certain distance."

3. Identification of specific difficulties

Some specific difficulties with mechanical work closely mirror those found in thermodynamic contexts: (a) association of the sign of work with the choice of coordinate system and (b) difficulties in relating the work done on and by a system. Both reflect a failure to apply correctly a general definition of work. Some of the issues that had arisen in thermal contexts, such as references to sign conventions, did not appear in responses to the mechanical work problem.

References to a particular coordinate system were made by about 10% of the students in the UW algebra-based course and about 25% of the students in the UMCP calculus-based course. (None of the students in the UW second-year course did so.) One student wrote, "There really is no such thing as negative work since the distance used to calculate work depends on where a person chooses to place the coordinate axis." Other students were less explicit but nonetheless described a specific coordinate system. For example, one wrote, "The work done on the block by the hand in case I is positive, because I defined my coordinate system to be positive in the up direction." Students who drew and/or referred to coordinate systems based their answers on a variety of quantities (for example, the directions of the applied force, displacement, velocity, and acceleration). Shown in Fig. 6 is the response of one of these students.

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B. Comparison of student performance on problems in mechanics and thermal physics

The mechanical work problem and a version of the bicycle pump problem were administered as part of the same ungraded quiz in the three courses mentioned above. (The results for the individual questions are included in Tables I and II.) Both the overall success rates and the distribution of types of explanations on the bicycle pump problem were essentially the same as in those cases in which there had been no questions on mechanical work. Apparently, the presence of the mechanical work problem did not suggest to students that they apply the concept of work to the thermal physics problem.46

We compared the responses of individual students on the two problems. For the purpose of these comparisons, a response to the bicycle pump problem had to include correct reasoning to be considered correct; all parts of the mechanical work problem had to be answered correctly. In the UW algebra-based and UMCP calculus-based courses, we found no noticeable difference in the success rates on the bicycle pump problem of the students who had answered the mechanical work problem correctly and those who had not. In the UW second-year course, students who answered the mechanical work problem correctly had a slightly higher success rate on the bicycle pump problem than those who had not done so. However, in all of the classes, the numbers of students who were successful in either of these tasks were small. Overall academic strength may also be a factor. Therefore, no definitive conclusion can be drawn.

There is another significant aspect to the comparison of performance on the mechanics and thermal physics problems. We interpret correct answers on all parts of the mechanical work problem as evidence of the ability of a student to apply the concept of work in a situation involving a point particle in which the directions of both the relevant force and displacement are either explicitly stated or can be easily inferred. This ability would seem to be necessary for applying the concept of work in more complex situations involving deformation and internal energy. However, in the mechanical work problem students do not need to recognize the relevance of the concept of work in a given situation. To demonstrate functional understanding of work, students must be able to apply the concept, but first they must recognize when they need to do so.

VI. CONCLUSION

Results from a number of studies in the context of mechanics have demonstrated that students often ignore general principles in their attempts to solve problems.47 In earlier research, our group identified several difficulties that students have in applying the work-energy and impulse-momentum theorems in mechanics.48 The failure to apply general principles to thermal physics problems was evident throughout the present study. Students frequently did not select, or even consider, the first law of thermodynamics as the relevant principle.

When presented with the questions described in this paper, most students, like many physicists, first attempted to use the ideal gas law. Physicists, however, quickly recognized the lack of information and turned to the first law.49 In contrast, we found that students typically did not recognize that their use of the ideal gas law was not valid. Their confidence in this law seemed to be a significant barrier to consideration of the first law of thermodynamics. Even references to the concept of work, whether by the interviewer or by statements in the written problems, did not seem to trigger application of the first law. When directly asked to use the first law, students revealed many difficulties that precluded correct application. The results were similar among first-year and second-year students. The courses in which they were enrolled differed in the amount of time spent on thermal physics, the range of topics covered, and the depth and mathematical complexity with which topics were treated. The courses also differed greatly in class size and in the background and interests of the students. However, student difficulties were the same in character and surprisingly similar in prevalence.

Although considerable emphasis is placed upon the first law in typical treatments of thermal physics, the importance of this general principle is lost on many students. It was evident throughout this investigation that many students had not developed an understanding of the relative importance of the concepts and principles they had studied. For many students, the ideal gas law was the most significant equation in the course.

Throughout this study, many students confirmed their incorrect macroscopic arguments with reference to an incorrect microscopic model. On the basis of experience in this and other investigations, we question the desirability of introducing the study of a macroscopic quantity or process (for example, gases or electric current) by first presenting students with an already developed visual microscopic model (for example, molecules or electrons).50 Although a microscopic model may provide a causal explanation that may be appealing to students, we have found that conceptual and reasoning difficulties with such a model may undo the benefit to students of a visual explanatory mechanism. We are also concerned that the early introduction of a microscopic model, when not necessary to account for the phenomena or when not strongly suggested by the evidence, may give students a false impression about the nature of science. In particular, the early introduction of the kinetic theory of gases may reinforce the notion that such a microscopic model proves the ideal gas law. Students may consequently fail to recognize that the model is designed to agree with experimental observations of macroscopic phenomena. In much of the curriculum developed by our group, including materials on thermal physics, students are provided with opportunities to become familiar with macroscopic phenomena and to use their observations as the basis for developing models with predictive power. Inferences about the existence of processes that are not directly observable are drawn only when necessary to account for experimental evidence. An example is provided by a series of tutorials by our group on optics in which phenomena that defy interpretation in terms of a ray model, such as interference fringes, provide the motivation for the development of a simple wave model for light.51

Difficulties with the concept of work precluded correct application of the first law for many students. In many introductory physics courses, students encounter thermal physics shortly after studying the concept of work in mechanics. In other courses, especially in chemistry, the study of thermodynamic work is not typically preceded by consideration of mechanical work. Our results suggest that we should ensure that students can apply the concept of work in simple mechanical contexts before introducing the complexities of thermal physics.

Linguistic complications have often been blamed for con-
ceptual confusion experienced by students. It has long been noted that in everyday usage, “heat” is strongly associated with temperature. The term “heat” is even used ambiguously in some textbooks. Appeals to instructors and authors to use terminology carefully appear frequently in the literature on teaching thermal physics.\(^2\) Certainly, precise use of technical terms is to be strongly encouraged. However, for many students the problem goes beyond incorrect or inexact use of terminology and reflects genuine inability to distinguish among closely related concepts. In such cases, a linguistic remedy is unlikely to be sufficient.

Although the student difficulties described in this article are serious and numerous, we believe they can be addressed. A concerted effort must be made to help students integrate the concepts and principles they have studied into a coherent conceptual framework that they can use to account for simple thermal phenomena. We have begun to develop tutorials for this purpose.\(^3\) As described elsewhere, the development of instructional materials by our group takes place in an iterative cycle of research, curriculum development, and instruction, with assessment playing a critical role.\(^4\) Preliminary versions of a tutorial on the first law have been tested and appear to be successful in our classes. Our experience thus far suggests that the effectiveness of the tutorial is increased if it is preceded by others that explicitly address difficulties with mechanical work and the ideal gas law. However, before we can feel confident about the replicability of the results, more cycles in the development process are necessary. In this as in other cases, ongoing research is necessary to ensure that instruction is well matched to the needs of students.

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\(^{11}\)In addition to the specific references below, see the summary in the chapter by G. Erickson and A. Tiberghien in Children’s Ideas in Science, edited by R. Driver (Open University Press, Philadelphia, 1985).


\(^{14}\)The article mentioned in Ref. 3 contains an extensive list of articles on research on student understanding in introductory mechanics and on the development and assessment of curriculum based on research.

\(^{15}\)For examples of the development and assessment of curriculum by the Physics Education Group in topics other than mechanics (for example, electricity and magnetism, geometrical and physical optics, and modern physics) see Ref. 25 and articles in Ref. 3 by members of the Physics Education Group.

\(^{16}\)The research reported in this paper is described in greater detail in M. E. Loverude, “Investigation of student understanding of hydrostatics and thermal physics and of the underlying concepts from mechanics,” and in C. H. Kautz, “Investigation of student understanding of the ideal gas law,” Ph.D. dissertations, Department of Physics, University of Washington, 1999 (unpublished).


\(^{18}\)See, for example, A. B. Arons, A Guide to Introductory Physics Teaching (Wiley, New York, 1997), Part III, pp. 118–121.


\(^{23}\)In the courses included in this study, students typically apply the first law of thermodynamics to closed systems, so there is no term for the chemical potential.

\(^{24}\)For a discussion of important conceptual issues that need to be addressed in teaching this material, see A. B. Arons, Teaching Introductory Physics (Wiley, New York, 1997), Part III, pp. 80–82, 124–129.

\(^{25}\)A description of the use of the individual demonstration interview by the Physics Education Group can be found in R. A. Lawson and L. C. McDermott, “Student understanding of the work-energy and impulse-momentum theorems,” Am. J. Phys. 55, 811–817 (1987) and in other articles in Ref. 3 that report on research by the group. Since 1994, most of the interviews have been both videotaped and audiotaped.

\(^{26}\)None of the students raised questions about the insulating capabilities of the pump or the speed with which the handle was pushed inward. If any had done so, they would have been told to assume the pump is perfectly insulating.

\(^{27}\)In both interviews and on written problems, a small number of students gave other explanations that were acceptable. For example, some students in the more advanced courses gave a microscopic work argument. Typically, these students argued that the speed of the gas particles would increase as a result of collisions with the moving piston and that the temperature would therefore rise. A handful of students in these courses also correctly applied the equation \(PV = nRT\) for adiabatic processes to predict that the temperature would increase. During interviews, such students were asked to try to account for this relationship between \(P\) and \(V\).

\(^{28}\)All the students assumed (explicitly or implicitly) that they could treat the air in the pump as an ideal gas. If this issue had been raised, the students would have been told to do so.

\(^{29}\)The version of the first law that was used in that particular student’s course was provided.

\(^{30}\)Results from three sections of the calculus-based course at the University of Maryland serve as an example of the similarity of student performance before and after instruction. In the two classes in which the question was administered after instruction, a correct prediction concerning the temperature was made by 61% (\(N = 66\)) and 54% (\(N = 83\)), respectively. In the class in which the question was given before any instruction, 59%
At UIUC, the question was posed in multiple-choice format with no explanations required. Therefore, we have not included this data in Table I. About 65% of the students answered correctly \((N = 119)\), a figure consistent with that obtained in other introductory courses.

23See Ref. 11.

22Additional evidence that student performance on certain types of questions is essentially the same before and after instruction can be found in several of the articles in Ref. 3 that report on research by the Physics Education Group. Other evidence is reported in R. R. Hake, "Interactive engagement versus traditional methods," Am. J. Phys. 66, 64–74 (1998).

21See Refs. 2, 3, 4, and 11.


19See Ref. 22.


17In one version, the students were asked explicitly about the relationship between the two pages.

16See Ref. 44.

15For articles in which the failure to use energy principles in mechanics is documented and discussed, see the section in Ref. 3 on problem-solving performance.

14See the article mentioned in Ref. 16 and the first article in Ref. 29.

13We have posed the bicycle pump task in informal situations to a number of friends and colleagues who are physicists.

12There have been several articles that support the introduction of microscopic models during the study of topics in classical physics. For examples in the context of thermal physics, see the first two articles in Ref. 1. For an example in the context of electric circuits, see B. A. Thacker, U. Daniel, and D. Boys, "Macroscopic phenomena and microscopic processes: Student understanding of transients in direct current electric circuits," Am. J. Phys. 67, S25–S31 (1999). In the article, data are presented that indicate that students who had been given a microscopic model outperformed others who had not. However, there is additional evidence that students who had developed a sound macroscopic model performed as well on the questions posed in this study as those who had been given a microscopic model.

11For other tutorials developed by the Physics Education Group, see L. C. McDermott, P. S. Shaffer, and the Physics Education Group at the University of Washington, Tutorials in Introductory Physics (Prentice–Hall, Upper Saddle River, NJ, 2002).

10For illustrations of the iterative process used by the Physics Education Group in the development of curriculum, see the second article in Ref. 25, the first article mentioned in Ref. 29, and articles in Ref. 3 by the Physics Education Group.