Foundations of Physics, Vol. 29, No. 5, 1999

The Electric Field Outside a Stationary Resistive Wire Carrying a Constant Current

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Received May 29, 1998; revised February 22, 1999

We present the opinion of some authors who believe there is no force between a stationary charge and a stationary resistive wire carrying a constant current. We show that this force is different from zero and present its main components: the force due to the charges induced in the wire by the test charge and a force proportional to the current in the resistive wire. We also discuss briefly a component of the force proportional to the square of the current which should exist according to some models and another component due to the acceleration of the conduction electrons in a curved wire carrying a dc current (centripetal acceleration). Finally, we analyze experiments showing the existence of the electric field proportional to the current in resistive wires.

1. INTRODUCTION

Consider a circuit like that in Fig. 1, where a stationary resistive wire connected to a battery carries a constant current I. Will it exert a force on a stationary charge q located nearby?

One force which will be there regardless of the value of the current is that due to the induced charges in the wire. That is, the point particle q induces a distribution of charges in the conducting wire and the net result will be an attraction between the wire and q. Most authors know about

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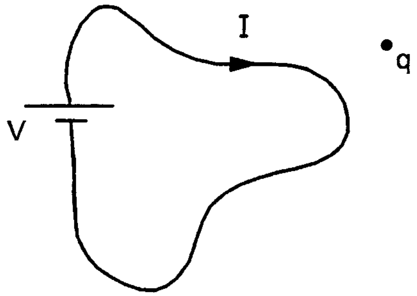

Fig. 1. A resistive stationary wire connected to a battery and carrying a *dc* current *I*, with a stationary point charge *q* nearby.

this fact, although forgetting to mention it. Moreover, they do not consider it in detail nor give its order of magnitude.

Is there any other force between the wire and the stationary charge? Many physicists believe that the answer to this question is no, and this opinion has been held for a long time. There are three main ideas leading to this belief. We analyze each of them here.

(A) The first idea is related to the supposition that a stationary resistive wire carrying a constant current is essentially neutral in its interior and along its surface. And this leads to the conclusion that a resistive current carrying wire generates only a magnetic field outside it. For more than a century scientists have been used to believing this statement. Clausius, for instance, based all his electrodynamics on this belief. In 1877 he wrote, "We accept as criterion the experimental result that a closed constant current in a stationary conductor exerts no force on stationary electricity" (quoted in Ref. 1, p. 589). Although he affirmed that this is an experimental result, he did not cite any experiments which tried to find this force. His electrodynamics led to this prediction: "The law formulated by me leads to the result that a constant stationary closed circuit exercises no force on a stationary charge" (Clausius statement in 1880, quoted in Ref. 1, p. 589). As we will see, he based his electrodynamics on an incorrect principle, as there is a force between a stationary charge and a stationary wire carrying a constant current. This force has been shown by Jefimenko's experiment (Refs. 2 and 3, pp. 299–319, and 509–511). We confirm the existence of this force by the calculations of this work.

Even in electromagnetic textbooks we can find statements like this. As we will see, the electric field inside and outside a resistive wire carrying a constant current is due to surface charges distributed along the wire. On the other hand, Reitz *et al.*, for instance, seem to say that no steady surface charges can exist in resistive wires (Ref. 4, pp. 168–169):

> Consider a conducting specimen obeying Ohm's law, in the shape of a straight wire of uniform cross section with a constant potential difference, $\Delta\varphi$, maintained between its ends. The wire is assumed to be homogeneous and characterized by the constant conductivity *g*. Under these conditions an electric field will exist in the wire, the field being related to $\Delta\varphi$ by the relation $\Delta\varphi = \int \vec{E} \cdot d\vec{l}$. It is evident that there can be no steady-state component of electric field at right angles to the axis of the wire, since by Eq. $\vec{J} = g\vec{E}$ this would produce a continual charging of the wire's surface. Thus, the electric field is purely longitudinal.

Although this criticism had already been made by Russell related to the second edition of 1964,[5] the third and fourth editions of this book did not change greatly on this point.

In Jackson's book,[6] the following statement appears in exercise 14.13 (p. 697):

> As an idealization of steady-state currents flowing in a circuit, consider a system of N identical charges q moving with constant *speed* v (but subject to acceleration) in an arbitrary closed path. Successive charges are separated by a constant small interval Δ. Starting with the Lienard–Wiechert fields for each particle, and making no assumptions concerning the speed v relative to the velocity of light show that, in the limit $N \to \infty$, $q \to 0$, and $\Delta \to 0$, but $Nq =$ constant and $q/\Delta =$ constant, no radiation is emitted by the system and the electric and magnetic fields of the system are the usual static value. (Note that for a real circuit the stationary positive ions in the conductors will produce an electric field which just cancels that due to the moving charges.)

Anyone reading this statement, especially the sentence in parentheses, will conclude that Clausius was right. However, we will see here that there is a net electric field different from zero outside a stationary resistive wire carrying a steady current. Despite the words in this exercise, it must be stressed that Jackson himself is aware of this electric field outside wires carrying steady currents (see Ref. 7).

Here are the words of Edwards *et al.*,[8] related to first-order terms, that is, to forces proportional to v_d/c or to the drifting velocity of the moving charges in the wire divided by *c*: "It has long been known that the zero- and first-order forces on a charged object near a charge-neutral, current-carrying conductor at rest in the laboratory are zero in magnitude." Jefimenko's experiment and our calculations show that a normal resistive wire carrying a constant current cannot be charge neutral at all points, although the integrated charge over the wire may be zero. Moreover, it will generate zero-order and first-order forces on a charged object at rest near it, namely, the force due to electrostatic induction F_o and the first-order force F_1 (see below).

One of us also assumed in previous works that a conducting wire is essentially neutral at all points (see Refs. 9, 10, and 11, pp. 85, 161). Here we show in detail that this is not valid for normal resistive wires carrying constant currents.

(B) The second idea leading to the conclusion that a normal resistive current carrying wire generates no electric field outside it arises from the supposition that magnetism is a relativistic effect. A typical representative of this position can be found in *The Feynman Lectures on Physics*[12] (see specifically Section 13-6, "The Relativity of Magnetic and Electric Fields,"[11] p. 13-7; our emphasis):

> We return to our atomic description of a wire carrying a current. *In a normal conductor*, *like copper*, the electric currents come from the motion of some of the negative electrons—called the conduction electron—while the positive nuclear charges and the remainder of the electrons stay fixed in the body of the material. We let the density of the conduction electrons be ρ_- and their velocity in S be $\mathbf{v}$. The density of the charges at rest in S is ρ_+, which must be equal to the negative of ρ_-, since we are considering an uncharged wire. *There is thus no electric field outside the wire*, and the force on the moving particle is just $\mathbf{F} = q\mathbf{v}_0 \times \mathbf{B}$.

In Purcell's *Electricity and Magnetism* we can find the same ideas.[13] In Section 5.9 of that book, which considers magnetism as a relativistic phenomenon, he models a current carrying wire by two strings of charges, positive and negative, moving relative to one another. He then considers two current carrying metallic wires at rest in the frame of the laboratory and says (p. 178), "In a metal, however, only the positive charges remain fixed in the crystal lattice. Two such wires carrying currents in opposite directions are seen in the lab frame in Fig. 5.23a. The wires being neutral, there is no electric force from the opposite wire on the positive ions which are stationary in the lab frame." That is, he believes that there will be no electric field generated by the stationary current-carrying resistive wire in any point outside itself.

Other books present similar statements, so we do not quote them here.

(C) The third kind of idea related to this widespread belief is connected with Weber's electrodynamics. As we shall see, even if a resistive current-carrying wire were neutral at all points in its interior and along its surface, Weber's electrodynamics predicts that it would exert a net force on a point charge at rest outside it. This force is proportional to v_d^2/c^2, where v_d is the drifting velocity of the conduction electrons and $c = 3 \times 10^8\ \mathrm{ms}^{-1}$. Based on the incorrect belief (see below) that this wire exerts no force on a stationary charge nearby, unware even of the larger first-order electric field proportional to v_d, many authors condemned Weber's law as experimentally invalidated.

This goes back at least to Maxwell's *Treatise on Electricity and Magnetism*. He was considering the force between a conducting wire carrying a constant current and another wire carrying no current, both of them at rest in the laboratory. He said (see Ref. 14, Vol. 2, Article 848, p. 482),

> Now we know that by charging the second conducting wire as a whole, we can make $e' + e'_1$ [net charge on the wire without current] either positive or negative. Such a charged wire, even without a current, according to this formula [based on Weber's electrodynamics], would act on the first wire carrying a current in which $v^2 e + v_1^2 e_1$ [sum of the positive and negative charges of the current carrying wire by the square of their drifting velocities] has a value different from zero. Such an action has never been observed.

As with Clausius comment, Maxwell did not quote any experiments which tried to observe this force (and which failed to find the effect), the upper limit of this effect etc.

Writing in 1951, Whittaker criticized Weber's electrodynamics along the same lines (Ref. 15, p. 205; our emphasis):

> The assumption that positive and negative charges move with equal and opposite velocities relative to the matter of the conductor is one to which, for various reasons which will appear later, objection may be taken; but it is an integral part of Weber's theory, and cannot be excised from it. In fact, if this condition were not satisfied, and if the law of force were Weber's, electric currents *would exert* forces on electrostatic charges at rest....

Obviously he is expressing the view that there are no such forces. In consequence, Weber's electrodynamics must be wrong according to Whittaker's view, because we now know that only the negative electrons move in metallic wires. And applying Weber's electrodynamics to this situation (in which a current in a metallic conductor is due to the motion of conduction electrons, while the positive charges of the lattice remain stationary) implies that a conducting wire should exert force on a stationary electric charge nearby. Whittaker could not be aware, at the time, of the experimental fact that *electric currents exert forces on electrostatic charges at rest*, see the experiments by Jefimenko discussed below.

Other examples of this widespread belief are as follows. In 1969 Skinner said, relative to Fig. 2, in which the stationary closed circuit carries a constant current and there is a stationary charge at P (Ref. 16, p. 163): "According to Weber's force law, the current of Fig. 2.39 [our Fig. 2] would exert a force on an electric charge at rest at the point P. ... And yet a charge at P does not experience any force." As with Clausius' and Maxwell's generic statements, Skinner did not quote any specific experiment which tried to find this force. Amazingly the caption of his Fig. 2.39

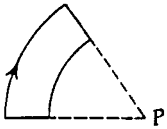

Fig. 2. A constant current flows in the closed wire and there is a point charge at P.

states, "A crucial test of Weber's force law." To most readers sentences like this convey the impression that the experiment had been performed and Weber's law refuted. But the truth is just the opposite.

Pearson and Kilambi, in a paper discussing the analogies between Weber's electrodynamics and nuclear forces, made the same kind of criticisms in a section called "Invalidity of Weber's Electrodynamics."[(17)] They consider a straight wire carrying a constant current. They calculate the force on a stationary charge nearby due to this wire with classical electromagnetism and with Weber's law, supposing the wire to be electrically neutral at all points. According to his calculations, classical electromagnetism does not yield any force on the test charge and he interprets this as (our emphasis): "The vanishing of the force on the stationary charge q corresponds simply to the *fact* that a steady current does not give rise to any induced electric field." With Weber's law he finds a second-order force and interprets this as meaning (our emphasis) "that Weber's electrodynamics give rise to *spurious* induction effects. This is probably the most obvious defect of the theory, and the only way of avoiding it is to suppose that the positive charges in the wire move with an equal velocity in the opposite direction, which of course they do not." As we will see, the *fact* is that a steady current gives rise to an induced electric field, as shown by Jefimenko's experiment.

In this work we argue that all of these statements were misleading. That is, we show the existence of a force on the stationary charge proportional to the current in a resistive stationary wire carrying a constant current. We also compare our calculations with Jefimenko's experiment (see below) which proved the existence of this force.

2. GEOMETRY OF THE PROBLEM

In this work the frame of reference is always the laboratory. The situation considered here is that of a cylindrical conducting resistive wire of length l and radius $a \ll l$ (Fig. 3). The axis of the wire coincides with the

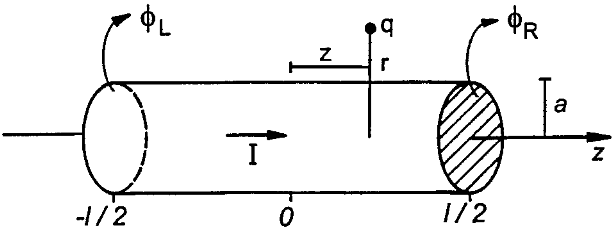

Fig. 3. A cylindrical wire of length l and radius $a \ll l$ carrying a constant current I. A point charge q is at a distance r to the axis of the wire, with a longitudinal component z relative to the center of the wire.

z direction, with $z = 0$ at the center of the wire. A battery maintains constant potentials at the extremities $z = -l/2$ and $z = +l/2$ of the wire, given by ϕ_L and ϕ_R, respectively. The wire carries a constant current I, has a finite conductivity g and is at rest relative to the laboratory. There is air or vacuum outside the wire. At a distance r to the axis of the wire there is a stationary point charge q. We want to know the force exerted by the wire on q in the following approximation:

$$l \gg r > a \qquad \text{and} \qquad l \gg |z| \tag{1}$$

where z is the longitudinal component of the vector position of q. We utilize throughout this paper cylindrical coordinates (r, φ, z) with $r = \sqrt{x^2 + y^2}$ and unit vectors $\hat{r}$, $\hat{\varphi}$, and $\hat{z}$.

This wire must be closed somewhere. The calculations presented here with this approximation should be valid for the circuit in Fig. 4 (square circuit of side l with a wire of radius $a \ll l$, with a point charge close to the middle of one of its sides and far from the battery). That is, the three other sides will not contribute significantly to the potential and field near the center of the fourth side. Alternatively, it should also give approximate results for a circular loop of larger radius $R = l/2\pi$ and smaller radius $a \ll R$ (a ring) if the point charge is at a distance $R + r$ to the center of the wire, such that $a < r \ll R$. It might even be utilized as a first gross approximation for the force on the point charge in Fig. 1 considering a generic circuit of large length and small curvatures (that is, with radii of curvature much larger than the diameter of the wire and also much larger than the distance of the point charge to the wire).

We consider separately three components of the force exerted by the wire on q: that due to the charges induced in the wire by q, that due to the surface charges which exist in resistive current carrying wires (proportional

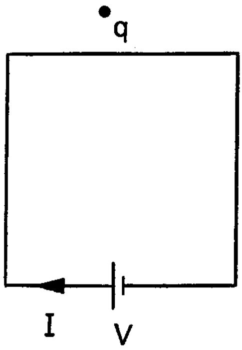

Fig. 4. Square circuit of side l made of a cylindrical wire of radius $a \ll l$, with a point charge close to the middle of one of its sides.

to the current or to the drifting velocity v_d of the electrons), and that due to v_d^2/c^2.

3. FORCE DUE TO ELECTROSTATIC INDUCTION

Consider a neutral conductor carrying no current. If we put a point particle q nearby, it will induce a distribution of charges in the conductor such that the potential anywhere inside it will reach a constant value in equilibrium. The net effect of these induced charges is an attraction between q and the conductor. We can estimate the value of this attraction for the situation in Fig. 3 in the case $l \gg r \gg a$ without any calculation. We also do not need to know the exact value of the equilibrium distribution of surface charges induced in the wire, $\sigma_i(a, \varphi, z)$.

This situation is equivalent to the force between a point charge at a distance r to an infinite conducting line. As there are only one charge and one distance involved in this problem, dimensional analysis requires the force between the point charge and the infinite conducting line to be given by

$$\vec{F}_o = -\alpha_L \frac{q^2}{4\pi\varepsilon_o} \frac{\hat{r}}{r^2}, \qquad 0 < \alpha_L < 1 \tag{2}$$

where $\hat{r}$ is the unit vector pointing away from the line to the charge q and α_L is a positive dimensionless constant of the order of unity. It would be

1 if all the induced charge were located at the origin, that is, at a distance r to q. As part of the induced charge will be distributed along the wire with a linear charge density $\lambda_i(z)$, which means at a distance to q greater than r, we conclude that α_L must be smaller than 1. Although we do not need to know the exact value of α_L or of $\lambda_i(z)$, we know the order of magnitude of the force due to electrostatic induction.

An analogous analysis might be performed for the induction force between a point charge q at a distance r from an infinite plane. As before, there are only one charge and one distance involved in this problem, so that the force must be given by Eq. (2) with a dimensionless constant α_P replacing α_L (as we now have an infinite plane instead of an infinite line, the dimensionless constant does not need to be the same). But in this case we can solve the problem exactly by the method of images. The final solution in this case yields an image charge $-q$ at the other side of the plane, also at a distance r to it. As the distance between q and $-q$ is $2r$, this yields $\alpha_P = 1/4$. This shows that our reasoning without performing any calculation was correct.

Suppose now we have the case in Fig. 3, but with r being of the same order of magnitude as a. As there are only one charge and two distances involved in the problem (considering l going to infinity), the force must be given by $\vec{F}_o = -h(r, a)\, q^2 \hat{r}/4\pi\varepsilon_o$. Here $h(r, a)$ is a function of r and a such that if $r \gg a$, it will be proportional to $1/r^2$ and, if $r \to a$, it diverges to infinity, as this is the general behavior of induction forces (if the charge approaches an infinite plane or the surface of a conducting sphere, the induction force always goes to infinity).

We have then estimated the value of the induction force in the case of Fig. 3, for $l \gg r \gg a$, as given by Eq. (2). This estimate is ours, as we were unable to locate it anywhere in the literature. This force will be there whether or not there is current in the wire. For an order of magnitude, suppose a charge generated by friction of 10^{-9} C, at a distance of 10 cm from a long, thin wire (length, 1 m; diameter, 1 mm). The force F_o due to electrostatic induction it this case should be of the order of 10^{-6} N. The electric field E_o due to electrostatic induction should be of the order 10^3 V/m.

In the sequel we consider the influence of the current on the net force exerted by the wire on q.

4. FORCE PROPORTIONAL TO THE CURRENT

When a constant current flows in a resistive wire connected to a battery, the electric field driving the conduction electrons against the resistive friction of the wire is due to free charges distributed along the surface of the wire. This was first pointed out by Kirchhoff.[18–20] (English translation of Ref. 20 in Ref. 21). We represent this surface charge density by $\sigma_f(a, \varphi, z)$. For dc currents, σ_f is constant in time but varies along the length of the wire (is a function of z). The battery is responsible for this distribution of surface charges due to the chemical forces which maintain its terminals at different potentials but does not generate the electric field in all points along the circuit. These surface charges generate not only the electric field inside the wire but also an electric field outside it.

To see that the battery does not generate the electric field at all points along the wire, consider Fig. 1. We know that the electric field driving the constant current will in general follow the geometry of the wire. When we bend a portion of the wire, the electric field will follow this bending. If something changes inside the battery when we bend the wire, the electric field at points closer to the battery would also change. However, the electric field changes its path or direction only in the portion which was bent, maintaining the previous values and directions in the other points. As the electric field inside the wire changed only in the bent portion, it is something local which created this change in its direction. The geometry of the wire has obviously changed, but as the geometry does not create an electric field, the reason must be sought somewhere else. We then arrive at Kirchhoff's idea that the electric field inside a wire carrying a constant current is due to free charges spread along the surface of the wire. The role of the battery is to maintain this distribution of free charges along the surface of the wire (constant in time for dc currents but variable along the length of the wire). There will be a continuous gradient of surface charges along the length of the wire, being more positive toward the positive terminal of the battery, decreasing in magnitude until reaching a zero value in an intermediary point, and becoming increasingly negative toward the negative terminal. If there were no battery, there would be zero density of charges at all points along the surface of the wire. It is the distribution of these surface charges in space which creates the electric field inside the wire driving the current. When we bend a portion of the wire, the free charges redistribute themselves in space along the surface of the wire, creating the electric field which will follow the new trajectory of the wire. Supposing the wire to be globally neutral, the integration of the surface charges along the whole surface of the wire must always go to zero, although σ_f is not zero at all points along the surface.

However, most authors are not aware of these surface charges and the related electric field outside the wire, as we can see from the quotations above. Fortunately this subject has again been considered in some important works: Heald, Jefimenko, Griffiths, Jackson, and those quoted by them (see Ref. 22, Ref. 3, pp. 299–319, 509–511, Ref. 23, pp. 279, 336, Ref. 7). As none of them considered the geometry in Fig. 3, we decided to analyze it here.

Our approach in this paper is the following: we consider the cylindrical wire carrying the constant current I and calculate the potential ϕ_1 and electric field $\vec{E}_1$ inside and outside the wire due to these surface charges in the absence of the test charge q. When we put the test charge at a distance r from the wire, the force on it due to the surface charges will then be given by $\vec{F}_1 = q\vec{E}_1$, supposing that it is small enough so that it does not disturb the current or the wire (except from the induction charges already considered above). We begin calculating the potential due to the surface charges.

As there is a constant current in the wire, the electric field inside it and driving the current must be constant over the cross section of the wire, neglecting the small radial Hall effect inside the wire due to the poloidal magnetic field generated by the current. This means that the potential and surface charge distribution must be a linear function of z. This was proved in an important paper by Russell,[5] so that we do not go into further detail here. Due to the axial symmetry of the wire, it cannot depend on the poloidal angle either. This means that

$$\sigma_f(a, \varphi, z) = \sigma_A \frac{z}{l} + \sigma_B \tag{3}$$

where σ_A and σ_B are constants.

Before proceeding we wish to discuss this expression. We are assuming the wire to be globally neutral, that is to have no net charge as a whole. When we integrate the free charge density σ_f over the whole surface of the wire, we need to obtain a zero net value. This will happen with Eq. (3) only in the symmetrical case in which $\sigma_B = 0$. This might represent, for instance, the top side in Fig. 4. On the other hand, we perform the calculations with a generic value of σ_B so that the calculation might be applicable, for instance, to the left half of the top side in Fig. 4. The integration of σ_f over this left side (from $z = -l/2$ to 0) will yield a positive value, as it is closer to the positive terminal. This positive charge will be balanced by the negative charge lying on the right half of the top side in Fig. 4 (z going from 0 to $+l/2$). With a generic σ_B we might also consider, for instance, the left side in Fig. 4 with a positive charge, which will be balanced by the negative charge in the right side in Fig. 4. It should be emphasized that the zero of σ_f is specified by the battery. The battery itself also specifies where σ_f will be positive (portions of the wire closer to the positive terminal of the battery) or negative (portions of the wire closer to the negative terminate of the battery).

Due to the axial symmetry of σ_f, we can calculate ϕ at $\varphi = 0$ and then generalize the solution to all φ. The potential inside or outside the wire is then given by

$$\phi_1(r, z) = \frac{1}{4\pi\varepsilon_o} \int_{\varphi_2=0}^{2\pi} \int_{z_2=-l/2}^{l/2} \frac{\sigma_f a \, d\varphi_2 dz_2}{\sqrt{r^2 + a^2 - 2ra\cos\varphi_2 + (z_2 - z)^2}}$$

$$= \frac{1}{4\pi\varepsilon_o} \int_{\varphi_2=0}^{2\pi} \int_{z_2=-l/2}^{l/2} \frac{(\sigma_A z_2/l + \sigma_B) \, d\varphi_2 \, dz_2}{\sqrt{(1 - 2(r/a)\cos\varphi_2 + (r^2/a^2)) + ((z_2 - z)/a)^2}} \tag{4}$$

Defining the dimensionless variables $s^2 \equiv 1 - 2(r/a)\cos\varphi_2 + (r^2/a^2)$ and $u \equiv (z_2 - z)/a$, we are then led to $\phi_1(r, z) = (a/4\pi\varepsilon_o)[(\sigma_A a/l) I_1 + (\sigma_A z/l + \sigma_B) I_2]$, where

$$I_1 \equiv \int_{\varphi_2=0}^{2\pi} \int_{u=-(l/2a+z/a)}^{l/2a-z/a} u \frac{d\varphi_2 \, du}{\sqrt{s^2 + u^2}} \tag{5}$$

and

$$I_2 \equiv \int_{\varphi_2=0}^{2\pi} \int_{u=-(l/2a+z/a)}^{l/2a-z/a} \frac{d\varphi_2 \, du}{\sqrt{s^2 + u^2}} \tag{6}$$

These integrals can be solved with approximation (1), where we now allow r to be smaller or greater than a, yielding (see Appendix)

$$\phi_1(r, \varphi, z) = \frac{a\varphi_f(z)}{\varepsilon_o} \ln\frac{l}{a} = \frac{a(\sigma_A z/l + \sigma_B)}{\varepsilon_o} \ln\frac{l}{a} \qquad \text{if} \quad r \leqslant a \tag{7}$$

$$\phi_1(r, \varphi, z) = \frac{a\varphi_f(z)}{\varepsilon_o} \ln\frac{l}{r} = \frac{a(\sigma_A z/l + \sigma_B)}{\varepsilon_o} \ln\frac{l}{r} \qquad \text{if} \quad r \geqslant a \tag{8}$$

The coulombian force on a test charge q located at (r, φ, z) is then given by (with $\vec{F}_1 = -q\nabla\phi_1$)

$$\vec{F}_1 = -\frac{qa}{\varepsilon_o}\frac{\partial\sigma_f(z)}{\partial z}\left(\ln\frac{l}{a}\right)\hat{z} = -\frac{qa\sigma_A}{l\varepsilon_o}\left(\ln\frac{l}{a}\right)\hat{z} \qquad \text{if} \quad r < a \tag{9}$$

$$\vec{F}_1 = \frac{qa\sigma_f(z)}{\varepsilon_o}\frac{\hat{r}}{r} - \frac{qa}{\varepsilon_o}\frac{\partial\sigma_f(z)}{\partial z}\left(\ln\frac{l}{r}\right)\hat{z}$$

$$= \frac{qa(\sigma_A z/l + \sigma_B)}{\varepsilon_o}\frac{\hat{r}}{r} - \frac{qa\sigma_A}{l\varepsilon_o}\left(\ln\frac{l}{r}\right)\hat{z} \qquad \text{if} \quad r \geqslant a \tag{10}$$

We can relate these expressions with the current I flowing in the wire. Figure 3 and the fact that ϕ_1 is a linear function of z yield

$$\phi_1(r \leqslant a, z) = \frac{\phi_R - \phi_L}{l} z + \frac{\phi_R + \phi_L}{2} \tag{11}$$

Equating this with Eq. (7) and utilizing Ohm's law $\phi_L - \phi_R = RI$, where $R = l/g\pi a^2$ is the resistance of the wire, with g being its conductivity, yields $\sigma_A = -R\varepsilon_o I/a \ln(l/a)$ and $\sigma_B = \varepsilon_o(\phi_R + \phi_L)/2a \ln(l/a) = \varepsilon_o(RI + 2\phi_R)/2a \ln(l/a)$. The density of free charges along the surface of the wire can then be written as

$$\sigma_f(a, \varphi, z) = -\frac{R\varepsilon_o I}{a \ln(l/a)} z + \frac{\varepsilon_o(\phi_R + \phi_L)}{2a \ln(l/a)} \tag{12}$$

This means that the potential and the force on the test charge q are given by

$$\phi_1 = -\frac{RI}{l} z + \frac{\phi_R + \phi_L}{2} \quad \text{if } r \leqslant a \tag{13}$$

$$\phi_1 = -\frac{RI}{l} \frac{\ln(l/r)}{\ln(l/a)} z + \frac{\phi_R + \phi_L}{2} \frac{\ln(l/r)}{\ln(l/a)} \quad \text{if } r \geqslant a \tag{14}$$

$$\vec{F}_1 = q\vec{E}_1 = q\frac{RI}{l}\vec{z} \quad \text{if } r < a \tag{15}$$

$$\vec{F}_1 = q\vec{E}_1 = q\left[\frac{-1}{\ln(l/a)}\left(\frac{RI}{l} z - \frac{RI + 2\phi_R}{2}\right)\frac{\hat{r}}{r} + \frac{RI}{l}\frac{\ln(l/r)}{\ln(l/a)}\hat{z}\right] \quad \text{if } r \geqslant a \tag{16}$$

Now that we have obtained the potential outside the wire, we might also revert the argument. That is, we might solve Laplace's equation $\nabla^2\phi = 0$ in cylindrical coordinates inside and outside the wire (for $a \leqslant r \leqslant l$) by the method of separation of variables imposing the following boundary conditions: finite $\phi(0, \varphi, z)$, $\phi(a, \varphi, z) = (\phi_R - \phi_L) z/l + (\phi_R + \phi_L)/2$ and $\phi(l, \varphi, z) = 0$. The latter condition is not a trivial one and was obtained only after we found the solution in the order presented in this work. The usual boundary condition that the potential goes to zero at infinity does not work in the case of a long cylinder carrying a dc current. By this reverse method we obtain the potential inside and outside the wire, then the electric field by $\vec{E} = -\nabla\phi$ and, finally, the surface charge density by ε_o

times the normal component of the electric field outside the wire in the limit in which $r \to a$. In this way we checked our calculations.

If we put $\phi_L = \phi_R = \phi_o$ or $I = 0$ in Eqs. (13) to (16), we recover the electrostatic solution (long wire charged uniformly with a constant charge density σ_B, with total charge $Q_B = 2\pi a l \sigma_B$, namely,

$$\phi(r \leqslant a) = \phi_o = \frac{a\sigma_B}{\varepsilon_o} \ln \frac{l}{a} \tag{17}$$

$$\phi(r \geqslant a) = \phi_o \frac{\ln(l/r)}{\ln(l/a)} = \frac{a\sigma_B}{\varepsilon_o} \ln \frac{l}{r} \tag{18}$$

$$\vec{E}_1(r < a) = 0 \tag{19}$$

$$\vec{E}_1(r \geqslant a) = \frac{\phi_o}{\ln(l/a)} \frac{\hat{r}}{r} = \frac{a\sigma_B}{\varepsilon_o} \frac{\hat{r}}{r} \tag{20}$$

We can also obtain the capacitance per unit length of this long, thin cylindrical wire as $C/l = (Q_B/\phi(a))/l = 2\pi\varepsilon_o/\ln(l/a)$.

This is the first time in the literature the potentials, (8) and (14), and the forces and electric fields, (10) and 16), outside a cylindrical wire have been calculated. Kirchhoff obtained Eq. (7) but did not consider the fields and forces outside the wire (see Ref. 19, especially the last equation on p. 400). Our analysis confirms and refines the previous work of Coombes and Laue, who in 1981 discussed the limiting case of an infinitely long wire.[24] They arrived at the same uniform electric field both inside and outside the wire. This is correct for an infinitely long wire. In our case we arrived at a uniform electric field inside the wire and at an electric field outside the wire with longitudinal and radial components depending on r, as we were considering a large but finite length l.

These expressions show that this force is proportional to the current in the wire. Moreover, there will be not only a tangential component of the electric field outside the wire but also a radial one. In the symmetric case in which $\phi_L = -\phi_R = RI/2$, the ratio of the radial component of $\vec{F}_1$ to the tangential component is given by $z/(r \ln(l/r))$. For a wire of 1-m length and $z = r = 10$ cm we have this ratio as 0.4, indicating that these two components are of the same order of magnetude.

Schaeffer (cited in Ref. 7), Sommerfeld, Marcus, Griffiths, and Jackson considered the electric field due to a long coaxial cable of length l carrying a constant current along the inner wire of resistivity g and radius a, returning along a hollow cylinder with inner radius b such that $l \gg b > a$ [Ref. 25, pp. 125–130, Eq. (8), Ref. 26, Ref. 23, pp. 336–337, Ref. 7, Eq. (A17)]. In

Sommerfeld's case the return conductor had finite conductivity and an external radius tending to infinity, while in Marcus, Griffiths, and Jackson's case the return conductor was a cylindrical shell of radius b and zero resistivity. For all these authors the potential and electric field went to zero for $r > b$. Their solution in the region $a < r < b$ and considering the zero of the potential at $z = 0$ is given by

$$\phi_{\text{coaxial}} = -\frac{I}{g\pi a^2} \frac{\ln(b/r)}{\ln(b/a)} z \tag{21}$$

We now compare this solution with our Eq. (14) in this particular case, in which $\phi_R + \phi_L = 0$. The main difference is the appearance in our case of $\ln(l/r)/\ln(l/a)$, instead of $\ln(b/r)/\ln(b/a)$. That is, while the potential and electric field outside the resistive current carrying wire (and also the force exerted by this wire on a point charge) depend on the length of the long wire, the same does not happen in the interior region of the coaxial cable near $z = 0$. If we keep a, g, and I constant (and also b for the coaxial cable) and double the length of the wire (coaxial cable), the potential outside the wire will change but not that inside the coaxial cable. The two solutions will only coincide if we fix $b = l$. As this is not the general case, the two solutions are not equivalent to one another in all situations.

In the sequel we consider a force due to the square of the current.

5. FORCE PROPORTIONAL TO THE SQUARE OF THE CURRENT

Up to now we have considered only the force due to electrostatic induction and the force of the surface charges on the stationary test charge. We have not yet taken into account the force of the stationary lattice and mobile conduction electrons on the stationary test charge. We consider it here in this section, analyzing two theoretical models. We first consider Lorentz's law or Liénard–Schwarzschild's force. In this case there are also components of the force exerted by a charge q_2 belonging to the current carrying circuit on q which depend on the square of the velocity of q_2, v_d^2, and on its acceleration. If we have a constant current, the acceleration of q_2 will be its centripetal acceleration due to any curvature in the wire, proportional to v_d^2/r_c, where r_c is the radius of curvature of the wire in each point. This might lead to a force proportional to v_d^2 or to I^2. However, it has been shown that if we have a closed circuit carrying a constant current, there is no net effect of the sum of all these terms on a stationary charge outside the wire. For a proof see Ref. 6, p. 697, exercise 14.13) or Ref. 8. In conclusion, we might say the following: according to Lorentz's force, the stationary lattice creates an electric field which is just balanced by the force due to the free electrons inside the closed wire, even when there is a constant current along the resistive wire. This might be interpreted as considering the wire to be electrically neutral in its interior (the radial Hall effect is considered later).

We now consider Weber's electrodynamics.[11] As stated above, we are neglecting the small radial Hall effect inside the wire due to the poloidal magnetic field generated by the current. This means that the interior of the wire can be considered essentially neutral. Despite this fact, Weber's electrodynamics predicts a force exerted by this neutral wire in a stationary charge nearby, even for closed circuits carrying constant currents. The reason for this effect is that the force exerted by the mobile electrons on the stationary test charge is different from the force exerted by the stationary positive ions of the lattice on the test charge. One of us has already performed these calculations in related situations, so that we present here only the final result. For the calculations see, for instance, Refs. 10 and 11 (Sec. 6.6, pp. 161–168). Once more, we assume (1). For the situation in Fig. 3, with a uniform current density $\vec{J} = (I/\pi a^2)\,\hat{z}$, the force on the test charge is given by

$$\vec{F}_2 = -q\frac{I v_d}{4\pi\varepsilon_o c^2}\frac{\hat{r}}{r} = -\frac{\mu_o}{4\pi^2}\frac{qI^2}{a^2 en}\frac{\hat{r}}{r} \qquad \text{if} \quad r > a \tag{22}$$

where v_d is the drifting velocity of the electrons. We also utilized $\mu_o = 4\pi \times 10^{-7}$ kg m C^{-2}, $c^2 = 1/\varepsilon_o \mu_o$, and $v_d = I/\pi a^2 en$, where $e = 1.6 \times 10^{-19}$ C elementary charge and n is the number of free electrons per unit volume.

This force is proportional to the square of the current. The electric field $\vec{E}_2 = \vec{F}_2/q$ points toward the current, as if the wire had become negatively charged. Sometimes this second-order field is called motional electric field.

If we have a bent wire carrying a constant current, Weber's electrodynamics predicts another component of the force exerted by this current on a stationary charge outside it, proportional to the acceleration of the conduction electrons. As we are supposing a constant current, the relevant acceleration here is the centripetal one proportional to v_d^2/r_c, where r_c is the radius of curvature of the wire at that location. This means that also this component of the force will be proportional to v_d^2 or to I^2. The order of magnitude is the same as the previous one. In Refs. 10 and 11 (Sec. 6.6, pp. 161–168), we calculated the net second-order force on a stationary charge due to a circular closed circuit with Weber's force. We showed that its net value had the order of magnitude of Eq. (22), taking into account the v_d^2 component of the force and also that due to the centripetal acceleration of the conduction electrons. That is, Weber's second order force does not go to zero even for closed circuits, contrary to Lorentz's force.

6. RADIAL HALL EFFECT

Another simple question which might be asked is the following: Is a stationary resistive wire carrying a constant current electrically neutral in its interior and along its surface?

Most authors quoted in Section 1 would answer positively to this question as this was their reason for believing this wire would not generate any electric field outside itself. However, we already showed that there will be a longitudinal distribution of surface charges which will give rise to the longitudinal electric field inside the wire and also to an electric field outside it. Here we show that there will also be a radial electric field inside the wire due to the fact that its interior is negatively charged. As we saw in Section 1, Reitz *et al.* rejected explicitly this charge. But they were not alone in this. See, for instance, Griffiths' statements in Ref. 23 (p. 273)—"Within a material of uniform conductivity, $\nabla \cdot \mathbf{E} = (\nabla \cdot \mathbf{J})/\sigma = 0$ for steady currents (equation $\nabla \cdot \mathbf{J} = 0$), and therefore the charge density is zero. Any unbalanced charge resides on the *surface*"—or Coombes and Laue[24] "For a steady current in a homogeneous conductor, the charge density ρ is zero inside the conductor." The same can be said of Lorrain *et al.* (Ref. 27, p. 287): "A wire that is stationary in reference frame S carries a current density J. The net volume charge density in S is zero: $\rho = \rho_p + \rho_n = 0$."

We here consider the radial Hall effect due to the poloidal magnetic field inside the wire. As is usually considered (Ref. 15, p. 90), we suppose the constant total current I to flow uniformly over the cross section of the cylindrical wire with a current density $J = I/\pi a^2$. With the magnetic circuital law $\oint_C \vec{B} \cdot d\vec{l} = \mu_o I_C$, where C is the circuit of integration and I_C is the current passing through the surface enclosed by C, we obtain that the magnetic field inside and outside the wire is given by

$$\vec{B}(r \leqslant a) = \frac{\mu_o I r}{2\pi a^2} \hat{\varphi} \tag{23}$$

$$\vec{B}(r \geqslant a) = \frac{\mu_o I}{2\pi r} \hat{\varphi} \tag{24}$$

The magnetic force on a conduction electron of charge $q = -e$ inside the wire, at a distance $r < a$ from the center and moving with drifting velocity $\vec{v} = -|v_d|\,\vec{z}$, is given by

$$\vec{F} = q\vec{v} \times \vec{B} = -\frac{|\mu_o e v_d I r|}{2\pi a^2}\hat{r} \tag{25}$$

This radial force pointing inward will create a concentration of negative charges in the body of the conductor. In equilibrium there will be a radial force generated by these charges which will balance the magnetic force, $qE = qvB$. That is, there will be inside the wire, beyond the longitudinal electric field E_1 driving the current, a radial electric field pointing inward given by

$$\vec{E}_r(r \leqslant a) = -\frac{|\mu_o v_d I r|}{2\pi a^2}\hat{r} \tag{26}$$

The longitudinal electric field inside the wire driving the current is given by $E_1 = RI/l$. In order to compare it with the magnitude of the radial electric field E_r due to the Hall effect, we consider the maximum value of this last field very close to the surface of the wire, at $r \to a$: $E_r \to |\mu_o v_d I|/2\pi a$. This means that (with $R = l/g\pi a^2$)

$$\frac{|E_r|}{|E_1|} = \frac{|\mu_o v_d g a|}{2} \tag{27}$$

For a typical copper wire ($v_d \approx 4 \times 10^{-3}$ ms^{-1} and $g = 5.7 \times 10^{7}$ Ω m) with 1-mm diameter, this yields $E_r/E_1 \approx 7 \times 10^{-5}$. This shows that the radial electric field inside the wire is negligible compared to the longitudinal one.

By Gauss' law $\nabla \cdot E = \rho/\varepsilon_o$, we obtain that inside the wire there will be a constant negative charge density ρ_- given by $\rho_- = -|Iv_d|/\pi a^2 c^2$. The total charge inside the wire is compensated by a positive charge spread over the surface of the wire with a constant surface density $\sigma_+ = |\rho_- a/2| = |Iv_d|/2\pi a c^2$. That is, the negative charge inside the wire in a small segment of length dz, $\rho_- \pi a^2\, dz$, is equal and opposite to the positive charge along its surface, $\sigma_+ 2\pi a\, dz$. This means that the radial Hall effect will not generate any electric field outside the wire, only inside it. For this reason it is not relevant to the experiments discussed here. In any event it is important to clarify this effect.

Contrary to the surface density of free charges $\sigma_f(a, z)$, this constant charge density σ_+ does not depend on the longitudinal component z.

In our analysis of the radial Hall effect, we are not considering the motional electric field discussed above, as it is not yet completely clear whether it exists or not.

In conclusion, we may say that the total surface charge density along the wire, not taking into account the motional electric field and the induction of charges in the conductor due to external charges, is given by the constant σ_+ added to the σ_f given by Eq. (12).

We now compare all three components of the electric field outside the wire with one another and discuss an important experiment related to this subject.

7. DISCUSSION AND CONCLUSIONS

Although many authors forget about the force due to electrostatic induction when dealing with a current carrying wire interacting with an external charge, there is no doubt it exists. Comparing the three forces above, it is the only one which diverges as we approach the wire. If we are far away from the wire, it falls as $1/r^2$, while the radial component of F_1 and F_2 fall as $1/r$.

We now compare the three components of this force in a particular example: copper wire ($g = 5.7\times10^7\ \Omega$ m, $n = 8.5\times10^{28}$ m^{-3}) with a length $l = 1$ m and a diameter of 1 mm ($a = 5\times10^{-4}$ m) The resistance of the wire is then given by $R = l/g\pi a^2 = 0.022\ \Omega$. With a potential difference between its extremities of $\phi_L - \phi_R = 1$ V, this yields a current of $I = 44.8$ A. The drifting velocity in this case amounts to $v_d = I/\pi a^2 en = 4\times10^{-3}$ ms^{-1}. We suppose, moreover, the symmetrical case in which $\phi_R = -\phi_L = -0.5$ V. The test charge will be a typical one generated by friction, $q = 10^{-9}$ C, at a distance of $r = 10$ cm $= 0.1$ m to the wire. The magnitude of each one of the forces and their ratios are then given by (considering only the radial component of $\vec{F}_1$ and $z = r = 10$ cm): $F_o \approx 10^{-6}$ N, $F_1 \approx 10^{-10}$ N, $F_2 \approx 10^{-16}$ N (in terms of electric field: $E_o \approx 10^3$ V/m, $E_1 \approx 10^{-1}$ V/m, and $E_2 \approx 10^{-7}$ V/m), so that $F_o/F_1 \approx 10^4$, $F_o/F_2 \approx 10^{10}$, and $F_1/F_2 \approx 10^6$. This means that, in this case, $F_o \gg F_1 \gg F_2$ or $E_o \gg E_1 \gg E_2$.

Despite this fact, the force $\vec{F}_1$ has already been observed in the laboratory by Jefimenko. He had an ingenious idea of utilizing grass seeds as test particles near current carrying wires. They are electrically neutral in normal state so that they do not induce any charges in the conductor. On the other hand, they are easily polarized in the presence of an electric field, aligning themselves with it. The lines of electric field are then observed in analogy with iron fillings generating the lines of magnetic field. What we consider here is the result of his experiment as presented in Plate 6 of Ref. 3

(see also his Section 9–6, "Electric Field Outside a Current-Carrying Conductor," pp. 299–305) and Fig. 1 of Ref. 2. The current was flowing in a circuit like that in our Fig. 3, with symmetrical potentials: $\phi_R = -\phi_L$. He performed the experiment but did not make the calculations for this case. These calculations have been presented here. In order to compare our results with his experiments, we need to obtain the lines of electric field. We obtain this in the plane xz ($y = 0$). Any plane containing the z axis will yield a similar solution. We are looking for a function $\xi(r, z)$ such that

$$\nabla\xi(r, z)\,\nabla\phi(r, z) = 0 \tag{28}$$

For $r < a$ we have ϕ as a linear function of z, such that ξ can be found proportional to r. We write it as $\xi(r < a, z) = -Alr$, with A as a constant. The equipotential lines $\phi(r, z) = constant$ can be written as $z_1(r) = K_1$, where K_1 is a constant (for each constant we have a different equipotential line). Analogously the lines of electric force will be given by $z_2(r) = K_2$, where K_2 is another constant (for each K_2 we have a different line of electric force). From Eq. (28) we get $dz_2/dr = -1/(dz_1/dr) = (\partial\phi/\partial z)/(\partial\phi/\partial r)$. Integrating this equation we can obtain $\xi(r, z)$. With Eq. (8) this yields the solution for $r > a$. We are then led to

$$\xi(r, z) = -Alr, \qquad \text{if} \quad r < a \tag{29}$$

$$\xi(r, z) = Ar^2 \ln\frac{r}{l} - A\frac{r^2}{2} - Az^2 - 2Bz, \qquad \text{if} \quad r > a \tag{30}$$

where $A = (\phi_R - \phi_L)/l = -I/\pi g a^2$ and $B = (\phi_R + \phi_L)/2$. From these equations we can easily verify Eq. (28).

In order to compare these results with Jefimenko's experiment, we need essentially the value of l/a. From his Plate 6 we get $l/a \approx 40/3$. The plot of the equipotentials between $z = -l/2$ and $l/2$ given by Eqs. (7) and (8) is given in Fig. 5. A plot of the lines of electric force given by Eqs. (29) and (30) is given in Fig. 6. This is extremely similar to Jefimenko's experiment (Plate 6 of Ref. 3 or Fig. 1 of Ref. 2), showing the correctness of our approach.

There is also an interesting experiment by Sansbury in which he detected directly a force between a charged metal foil and a current-carrying conductor by means of a torsion balance.[28] He placed a neutral silver foil which was at the extremity of a torsion balance close to a U-shaped neutral conductor without current. When he charged the foil with a charge of approximately 0.5×10^{-9} C, he observed an attraction between the vane and the wire. This was due to the force of electrostatic induction F_o

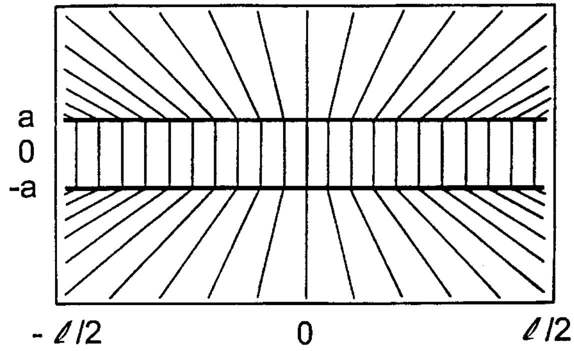

Fig. 5. Equipotentials as given by Eqs. (7) and (8) with Jefimenko's value $l/a \approx 40/3$ and with $\sigma_B = 0$ (or $\phi_R = -\phi_L$).

discussed above. He then passed a dc current of 900 A in the wire and observed an extra attraction or repulsion between the charged foil and the wire, depending on the sign of the charge in the foil. This force was of the order of 10^{-7} N, although he was not able to make precise measurements. Although he analyzed the possibility that this extra force might be the force F_1 discussed here, he only considered the longitudinal electric field outside the wire. He then concluded that this force would be three orders of magnitude smaller than the effect he measured. However, he was not aware of the radial component of $\vec{E}_1$, which can be larger than the longitudinal component, as we showed here. Moreover, his U-shaped wire was bent close to the foil and the approximation of a long straight wire may not use applicable. Close to a corner the electric field outside the wire is even larger than the longitudinal one inside it.[29] Maybe what Sansbury detected

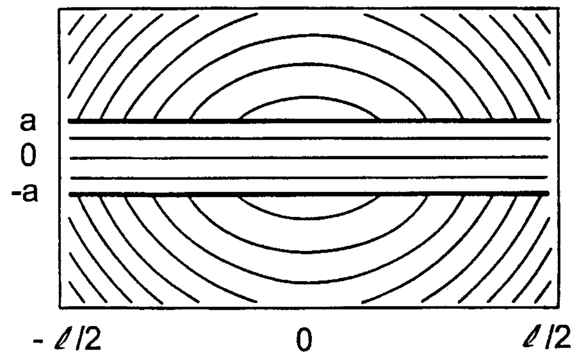

Fig. 6. Lines of electric force as given by Eqs. (29) and (30) with $\phi_R = -\phi_L$.

directly was the force F_1 discussed here. It would be important to repeat his experiment carefully taking this into account. Further discussions of his experiment with different approaches can be found in Refs. 30–32 and Ref. 33 (Sec. 6.10).

The example discussed here is important to show clearly the existence of the electric field outside a resistive wire carrying a constant current. It does not depend on a variable current (longitudinal acceleration of the electrons along the wire) or on a centripetal acceleration of the electrons (due to any curvature in the wire). That is, this electric field will be there even if there is not any acceleration of the conduction electrons. In the case of a coaxial cable discussed by Sommerfeld and many others (see above), they have found an electric field only in the region between the cables, but not outside the return conductor. The reason for this is that they were considering a return conductor of infinite area (Sommerfeld) or of zero resistivity (Marcus, Griffiths, and Jackson). For this reason it may not have been clear to many people that usually any current carrying resistive wire should generate an electric field outside it. We hope that the calculations presented in this paper, coupled with Jefimenko's experiments, will make people aware of this electric field.

As regards those who consider magnetism as a relativistic effect, we have shown here that a resistive current carrying wire generates not only a magnetic field but also an electric field. As Jackson has shown, it is impossible to derive magnetic fields from Coulomb's law and the kinematics of special relativity without additional assumption (Ref. 6, pp. 578–581, and Ref. 34).

It should also be mentioned that the magnetic field in this case is the usual poloidal field in the direction $\hat{\varphi}$, proportional to r for $r \leqslant a$ and to $1/r$ for $r \geqslant a$. It is orthogonal to $\vec{E}_1$ at all points in space. This means that Poynting's vector $\vec{S} = \vec{E} \times \vec{B}/\mu_o$ will follow the equipotential lines represented in Fig. 5 when $\phi_R = -\phi_L$. This general behavior of the lines of Poynting's vector was pointed out by Heald.[22] As we can see from Fig. 5, just outside the wire $\vec{S}$ is orthogonal to it only at $z = 0$. At all other points it is inclined relative to the z axis, at an angle θ with a tangent given by the ratio of the radial and longitudinal components of $\vec{E}_1$. As we have seen, just outside the wire this is given by $\tan\theta = z/(a\ln(\ell/a))$. Many textbooks consider an electric field outside the current-carrying wire only when discussing boundary conditions. As the longitudinal component of $\vec{E}$ is continuous at a boundary and must exist inside a resistive wire carrying a current, it must also exist just outside the wire. These authors then present Poynting's vector pointing radially inward toward the wire (see, e.g., Ref. 35, pp. 180–181, and Ref. 12, p. 27–8). This goes back to Poynting himself in 1885, as pointed out by Marcus.[26] There are two main things to comment on here.

First, these drawings and statements suggest that this electric field should exist only close to the wire, while as a matter of fact it exists at all points in space. Second, they indicate that these authors are not aware of the surface charges generating the field. As we have seen, it is only at one point that $\vec{S}$ will be orthogonal to the wire just outside it. This point is an exception and not the rule. The rule is that there will be a radial component which may be larger than the longitudinal one, pointing toward the wire or away from it. One of the effects of this radial component is that $\vec{S}$ will usually be inclined just outside the wire and not orthogonal to it.

The verification of the existence or not of the second-order electric field is much more difficult due to its small order of magnitude (compared with E_o and E_1). However, if the resistance of the wire goes to zero, σ_A also goes to zero. This means that in a superconductor there should not be an external electric field proportional to the current. Avoiding also the induction force, there remains in this case only the second-order electric field. This was the approach utilized by Edwards *et al.* in their experiment,[8] which is the best one known to us analyzing this effect. They found an electric field proportional to I^2, independent of the direction of the current, pointing toward the wire and with an order of magnitude compatible with that predicted by Weber's law. Despite this positive evidence, more research is necessary before a final conclusion may be drawn related to this second-order electric field (Ref. 36 and Ref. 11, Sec. 6.6, pp. 161–168).

As we have seen, usually $F_o \gg F_1 \gg F_2$. Moreover, F_o and F_1 have been shown to exist experimentally. We can then disregard the criticisms of Maxwell, Whittaker, and Skinner presented above against Weber's electrodynamics. That is, there is a force between the wire and q proportional to the current I, contrary to their statements. Their argument against Weber's electrodynamics is then invalid. It is much more difficult to determine if there is or is not a second-order component of this force proportional to v_d^2/c^2. Only future experiments taking into account all of these effects as F_o and F_1 can decide this matter.

In conclusion, we can say that despite the widespread belief that a stationary resistive wire carrying a constant current exerts no force on a stationary charge, there will certainly be a component of this force due to the induced charges and another one proportional to the current in the wire, as proved by these calculations and Jefimenko's experiment. The existence or not of a second-order force still needs to be confirmed.

ACKNOWLEDGMENTS

The authors wish to thank Prof. Mark A. Heald for calling their attention to Ref. 24 and for suggestions in related problems. They also thank

Profs. O. D. Jefimenko, J. D. Jackson, T. E. Phipps, Jr., J. P. Wesley, J. Fukai, J. I. Cisneros, J. A. Hernandes, and J. E. Lamesa and the referee for comments regarding the first version of this paper. A.J.M. wishes to thank FAPESP-Brazil for financial support.

APPENDIX

We now show how to calculate integrals (5) and (6).

Applying approximation (1) in the limits of integration of I_1 and integrating it in u yields a zero value (as it is an odd function integrated between symmetric limits).

Integrating I_2 in u yields, applying (1) in its limits of integration

$$I_2 = \int_0^{2\pi} d\varphi_2 \ln \frac{\sqrt{s^2 + (\ell/2a)^2} + (\ell/2a)}{\sqrt{s^2 + (\ell/2a)^2} - (\ell/2a)} \tag{31}$$

Once more with approximation (1) this can be written

$$I_2 = \int_0^{2\pi} d\varphi_2 \ln \frac{(\ell/a)^2}{s^2} = 4\pi \ln \frac{\ell}{a} - \int_0^{2\pi} \left[\ln \left(1 - 2\frac{r}{a} \cos \varphi_2 + \frac{r^2}{a^2} \right) \right] d\varphi_2 \tag{32}$$

This last integral is equal to zero if $r \leqslant a$. If $r > a$, we can put r^2/a^2 in evidence and utilize this result once more to solve the last integral, namely,

$$\int_0^{2\pi} \left[\ln \left(1 - 2\frac{r}{a} \cos \varphi_2 + \frac{r^2}{a^2} \right) \right] d\varphi_2 = 0, \qquad \text{if} \quad r \leqslant a \tag{33}$$

$$\int_0^{2\pi} \left[\ln \left(1 - 2\frac{r}{a} \cos \varphi_2 + \frac{r^2}{a^2} \right) \right] d\varphi_2 = 2\pi \ln \frac{r^2}{a^2}, \qquad \text{if} \quad r \geqslant a \tag{34}$$

This means that the final value of I_2 is found to be

$$I_2 = 4\pi \ln \frac{\ell}{a} \qquad \text{if} \quad r \leqslant a, \tag{35}$$

$$I_2 = 4\pi \ln \frac{\ell}{r} \qquad \text{if} \quad r \geqslant a, \tag{36}$$

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