Cosmological Parameters from Quasar Clustering

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Contents

Basics of cosmology

Cosmological model

- geometry of the curved space (metric)
- physical content of the matter
  (determines the evolution of geometry)

- simplicity
- concordance with observations

Observational foundations

- the universe is expanding
- cosmic microwave background
- light elements abundance
- dark matter

- anisotropies in the CMB
- supernovae IA

dark energy

The cosmological constant

- **1917 - Einstein introduces the cosmological term in his GR field equations.**

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 8\pi G \, T_{\mu\nu} + \Lambda g_{\mu\nu}$$

  - ✓ main motivation: observational
  - ✓ Λ acts like a repulsive force

- **1922 - Friedman obtained solutions to the E.E. without Λ and that were expansionist.**

- **1929 - Hubble announces the discovery that the real U is expanding.**

- **1931 - Einstein excludes the Λ-term from his equations**
  ``The greatest blunder of my life''

The cosmological constant

- **Lemaitre believed that we should keep the Λ-term in E.E. and observations should give the last word.**
- **In the 80's Λ is introduced in the context of the inflationary universe.**
- **A sufficiently long period of inflation** $\boxed{\Omega_T = 1}$ $\Omega_i = \dfrac{\rho_i}{\rho_c}\ ; \quad \rho_c = \dfrac{3H^2}{8\pi G}$
- **observations** $\Omega_{m0} = 0.3 \pm 0.1$ $H_0 = 100\, h\, km/s\, Mpc^{-1} ; h = 0.65 \pm 0.1$

for matter that clumps on scales of clusters (Peebles (1988) - Carlberg et al.(1997))

❖ **How to reconcile inflation with observations?**

★ **Smooth component with Ω_{smooth} ~ 0.7**

$$\boxed{\Omega_T = \Omega_{m0} + \Omega_{smooth} = 1}$$

Peebles Ap.J. 284, 439 (1984);
Turner, Steigman& Krauss PRL52, 2090,(1984)

Dark energy candidates

DARK ENERGY

- **Cosmological constant**
- **Evolving scalar field (quintessence)**
- **X-fluid**

$$\Omega_{Total} = \Omega_{matter} + \Omega_{radiation} + \Omega_{DEnergy} = 1$$

Negative Pressure!!!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

if

$$\rho + 3P < 0 \quad \Rightarrow \quad \ddot{a} > 0$$

Acceleration!!

Sne Ia as a tool for cosmology

SneIa ➡ **Direct evidence for DARK ENERGY with negative pressure**

- Supernovae Cosmology Project – Perlmutter et al; Ap.J 517, 565, 1999
- The High-z Supernova team – Riess et al. 116, 1009,1998; 117, 707, 1999

  Garnavich et al., Ap. J. 509, 74, 1998.

- **basic idea: use type Ia supernovae as standard candles for the classic magnitude-redshift test**

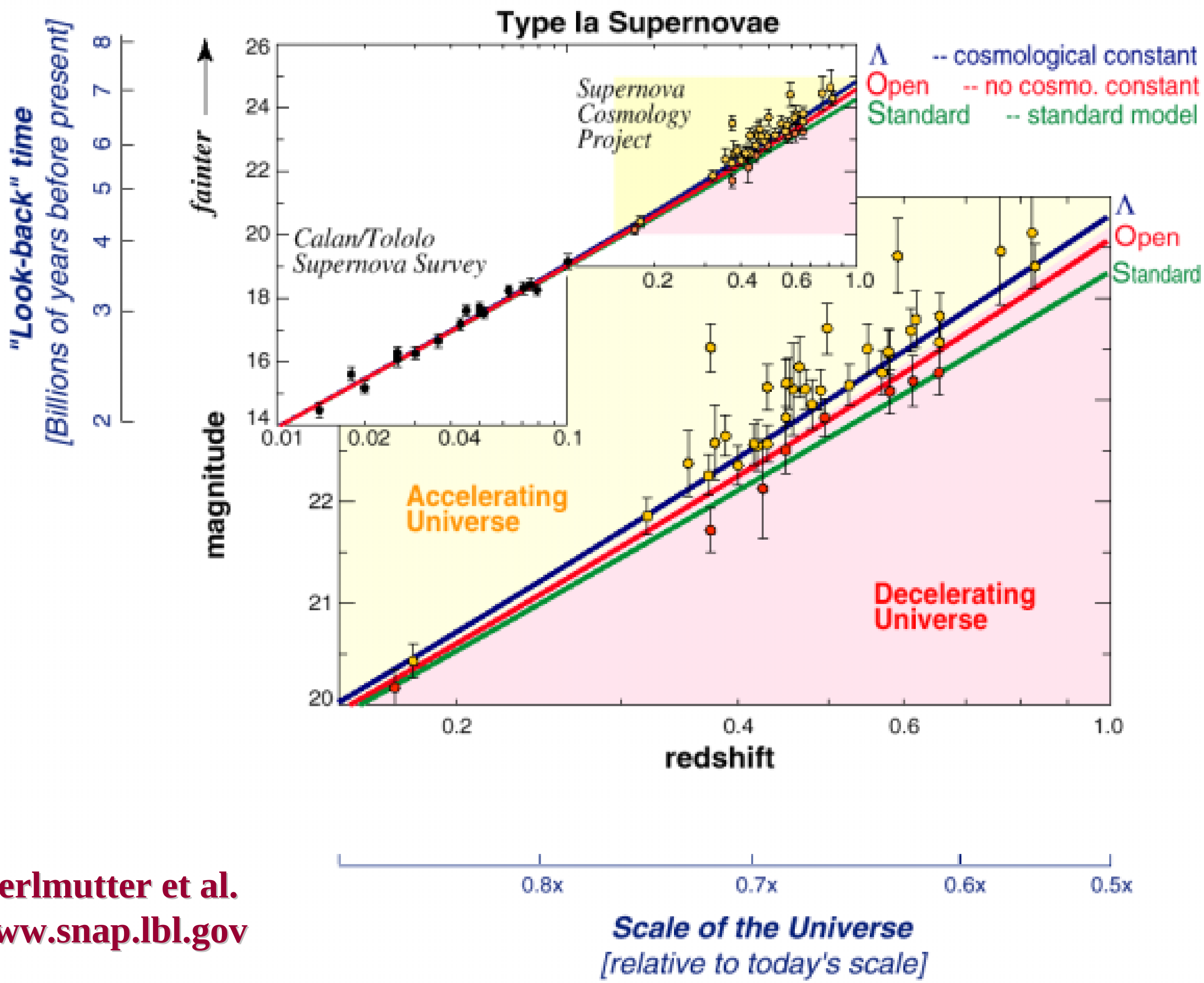
Advantages

- Luminous (10^9 - 10^{10} L_o).
- small dispersion (<~ 0.3 mag)
- little evolution expected.

Problems

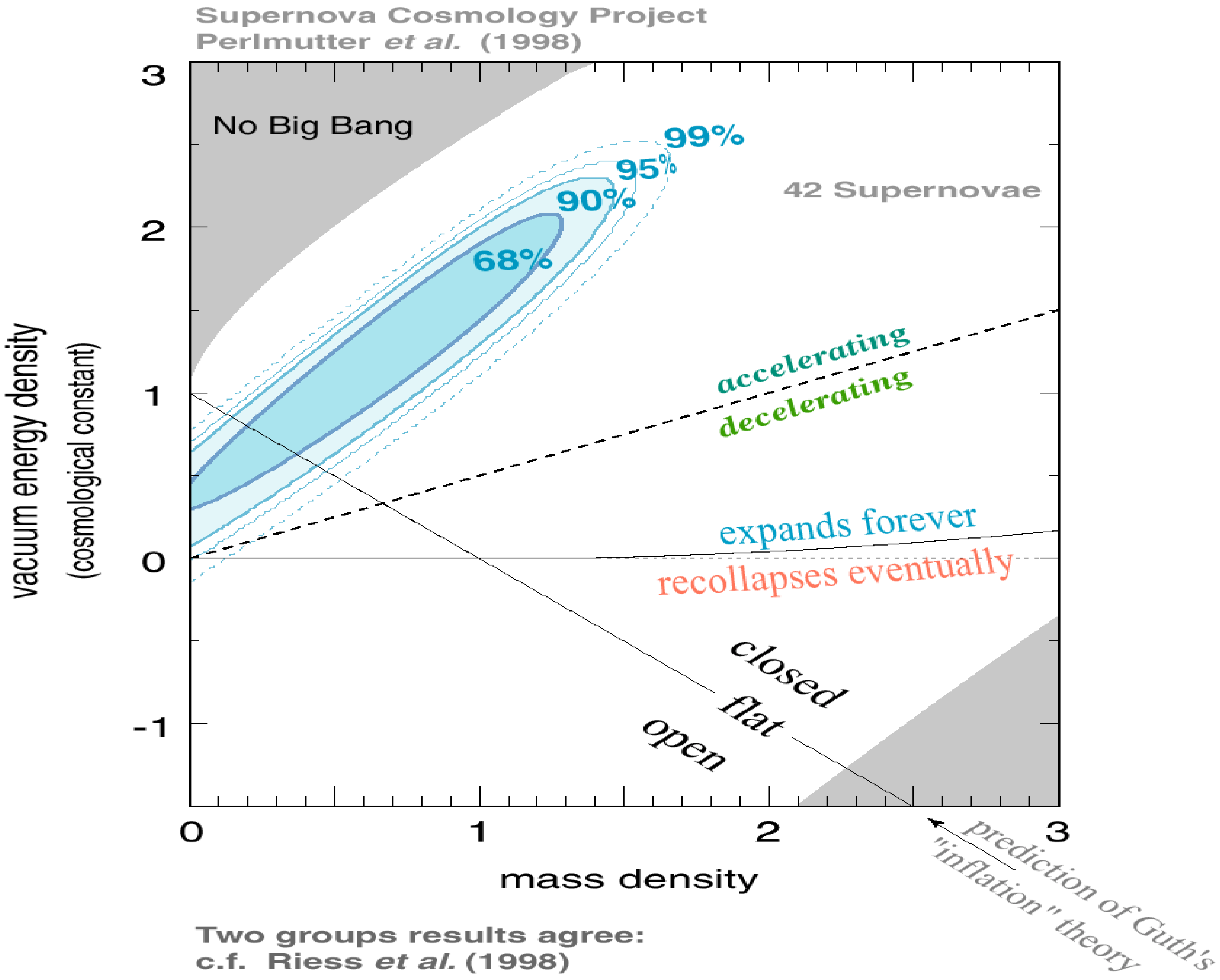
- Rare, ~ 1/500 years/ galaxy.
- random, can't schedule telescope time.
- fast, difficult to catch before maximum light.
  - Sne Ia rises to maximum light within a few days
  - At high z it fades below the largest telescope's limits within a month or two

Supernovae results

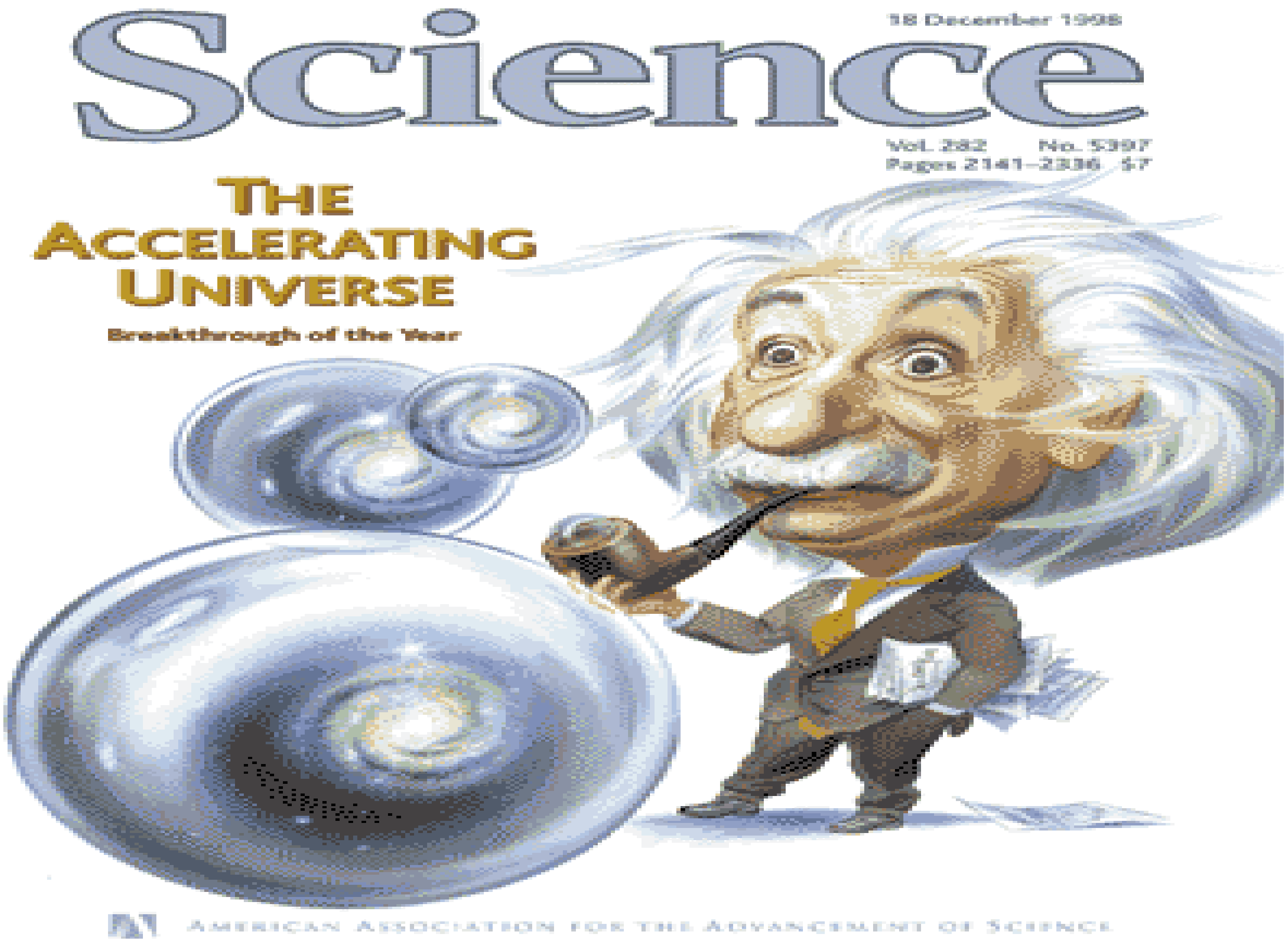


Perlmutter et al.
www.snap.lbl.gov

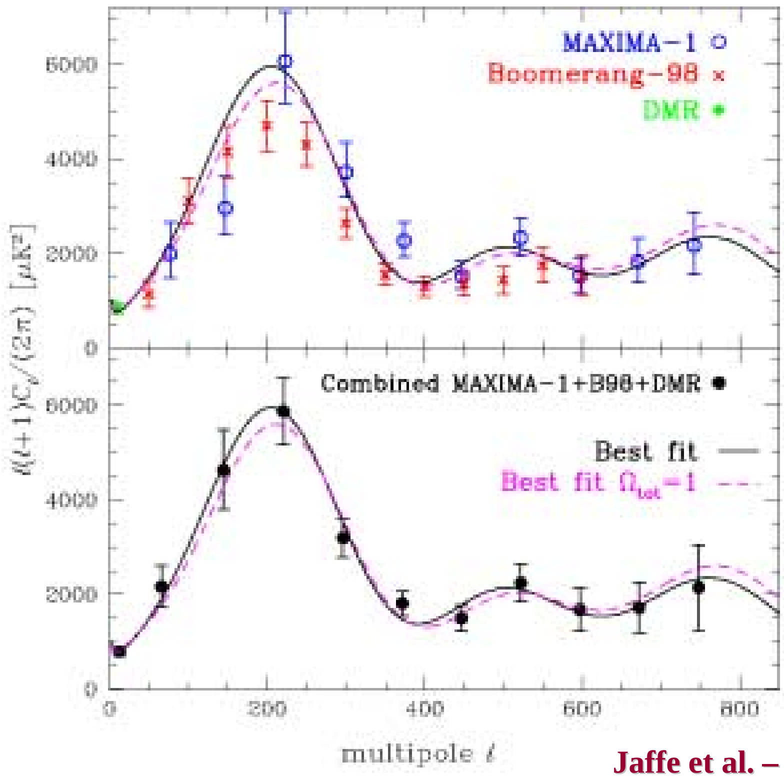
Supernovae results

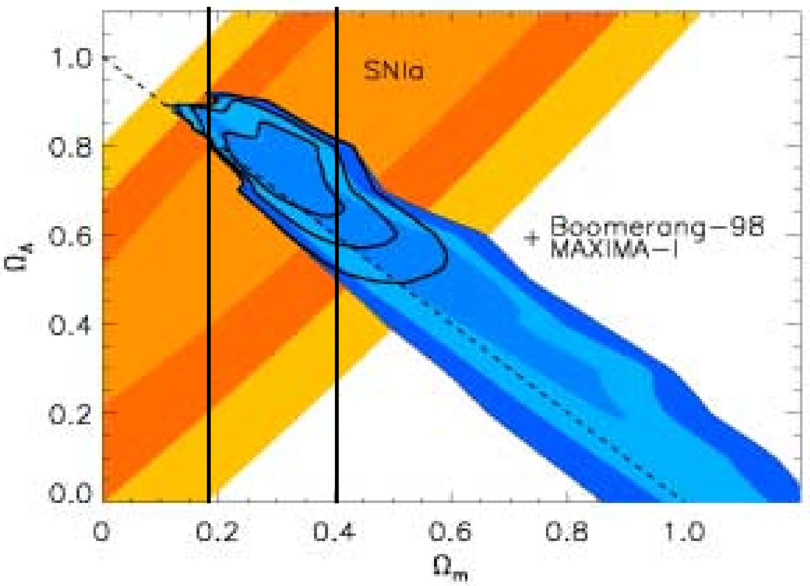


Science's cover



CMB + Ω_m Indirect evidence for DARK ENERGY





Jaffe et al. – astroph/0007333

The temperature correlation function is defined by :

$$C(\alpha) = \left\langle \frac{\Delta T}{T}(\hat{n}) \frac{\Delta T}{T}(\hat{m}) \right\rangle_{\hat{n}.\hat{m}=\cos\alpha}$$

$\langle ... \rangle$ denotes an average over positions $\vec{x}$ and all directions $\hat{n}, \hat{m}$ separated by an angle α.

Expanding $C(\alpha)$ using Legendre polynomials, we get

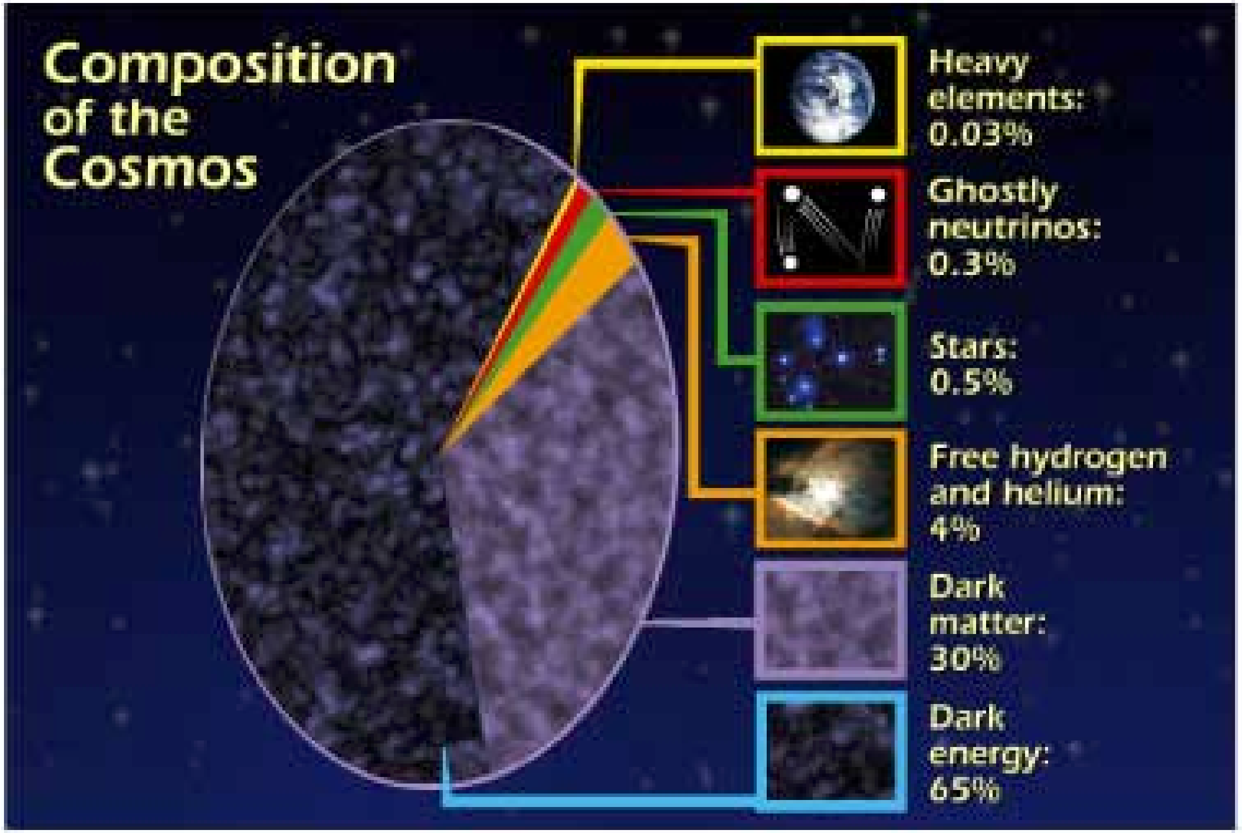
$$C(\alpha) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} P_l(\cos\alpha)\; C_l$$

$$C_l = 2\pi \int_{-1}^{1} d\cos\theta\;\; P_l(\cos\theta)\; d\theta$$

Concordance of direct and indirect evidences

$$\Omega_X \approx 0.7 \pm 0.2$$

Composition



Credit STScI

the Alcock-Paczynski test

- precise measurements of cosmological parameters
- other tests/measurements to determine the densities and to study the equation of state are wellcome!
- the AP test applied to quasar clustering.

M. O. Calvão, JTMN, I. Waga,
Phys. Rev. Letters, 88, 091302-1, 2002

Background model

Geometry

$$ds^2 = dt^2 - a^2(t)\left[\frac{dR^2}{1-kR^2} + R^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Matter Content

- **nonrelativistic matter + *x*-fluid with equation of state**

$$P_x = w\rho_x, \quad w = const\ (<0)$$

The fluids interact only trough gravity and are separately conserved.

Dynamics

$$H^2 = H_0^2 \left[\Omega_{m0} (1+z)^3 + \Omega_{x0} (1+z)^{3(w+1)} + \Omega_{k0} (1+z)^2 \right]$$

$$\ddot{a} = -\frac{8\pi G}{6} \left(\rho_m + \rho_x + 3P_x \right) a$$

where

$$H = \frac{\dot{a}}{a}, \qquad \Omega_m = \frac{\rho_m}{\rho_{cr}},$$

$$\Omega_x = \frac{\rho_x}{\rho_{cr}}, \qquad \Omega_k = -\frac{k}{a^2 H^2},$$

$$\rho_{cr} = \frac{3H^2}{8\pi G}$$

$$\Omega_m + \Omega_x + \Omega_k = 1$$

$$q = \frac{1}{2} \left[\Omega_m + (1+3w) \Omega_x \right]$$

$$\Omega_m \; ; \; \Omega_x \; ; \; w$$

$$Sin_K \chi = \begin{cases} \chi & if \; k = 0 \\ Sinh\chi & if \; k < 0 \\ Sin\chi & if \; k > 0 \end{cases}$$

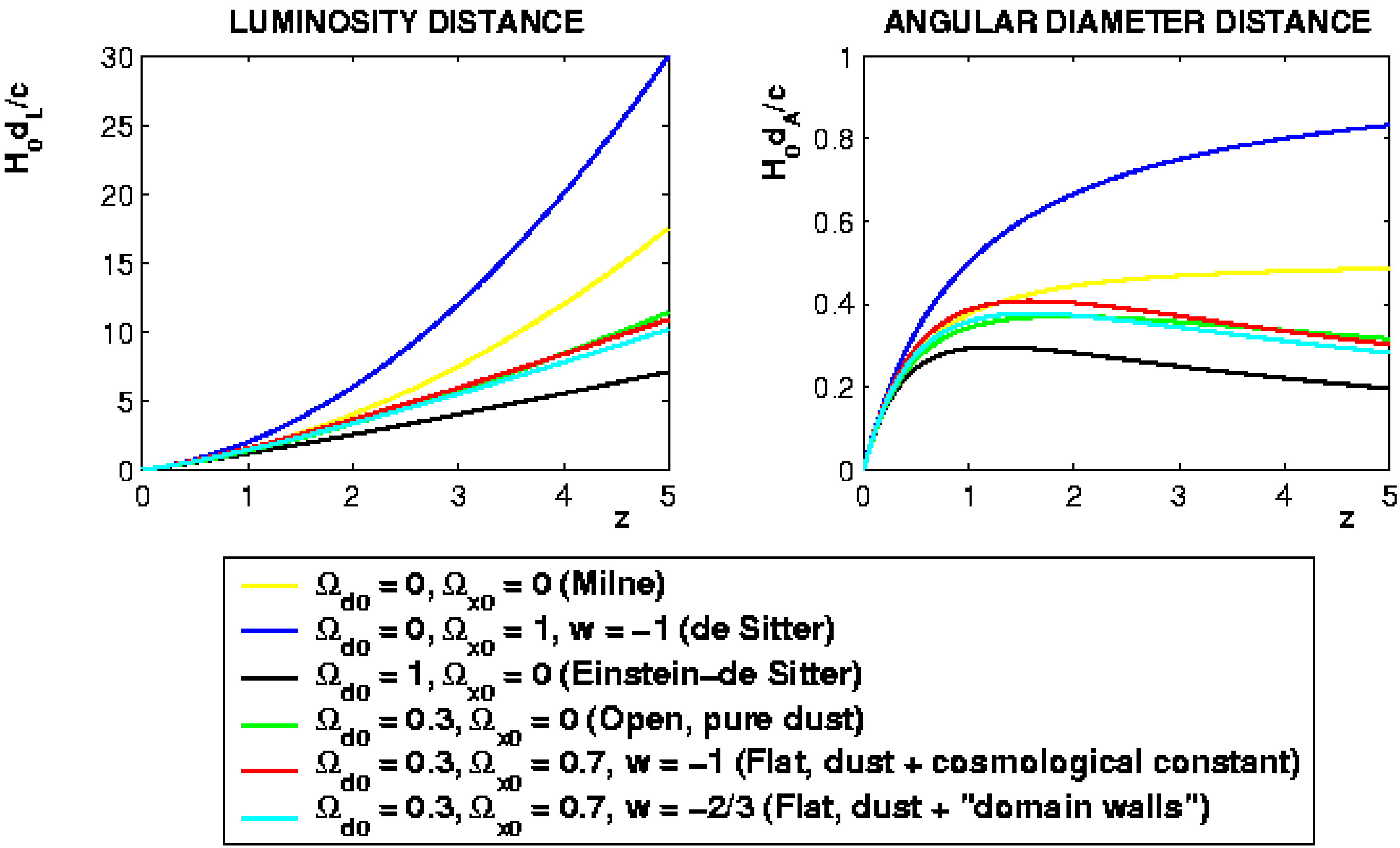
$$a_0 R = \frac{1}{H_0 \sqrt{\Omega_{k0}}} \mathrm{Sin}_K \left(\sqrt{\Omega_{k0}} \int_0^z \frac{H_0 dz'}{H(z')} \right)$$

$$= f(z)$$

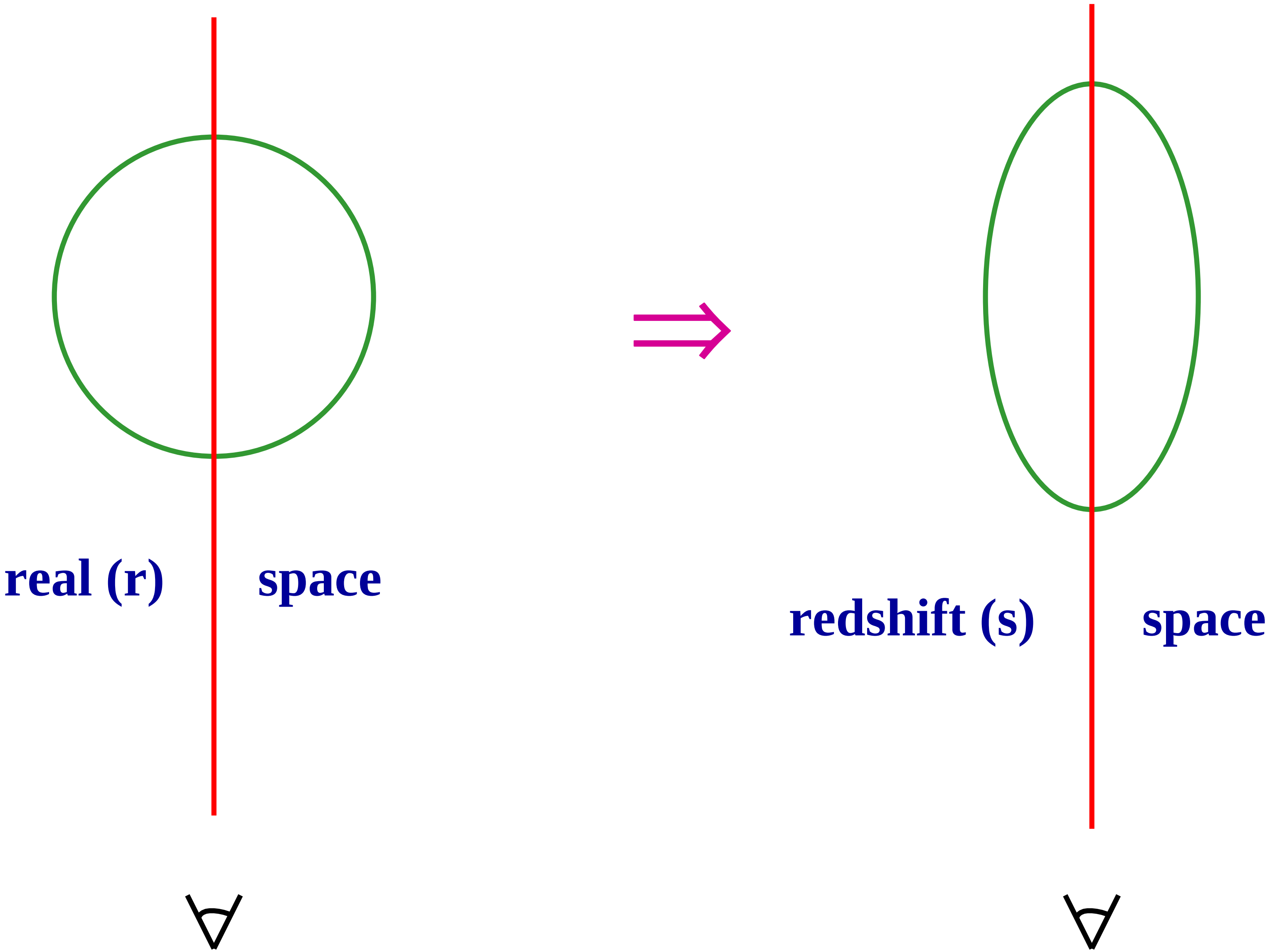
Proper motion distance → $$= f(z; \Omega_{m0}, \Omega_{x0}, w; H_0)$$

Luminosity and angular diameter distances

$$(1+z)\,f = d_L = (1+z)^2 d_A$$



geometrical interpretation



Real and redshift spaces

Real space

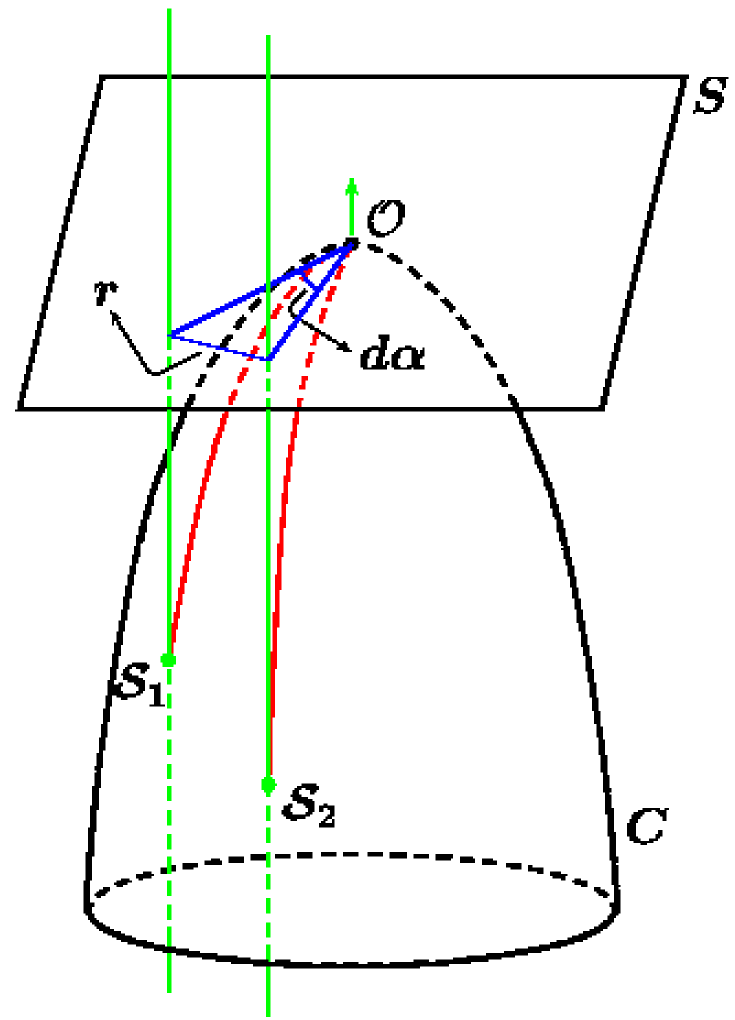
Coordinate distance

$$r^2 = r_{\parallel}^2 + r_{\perp}^2$$

$$r_{\perp} = f(z)d\alpha,$$

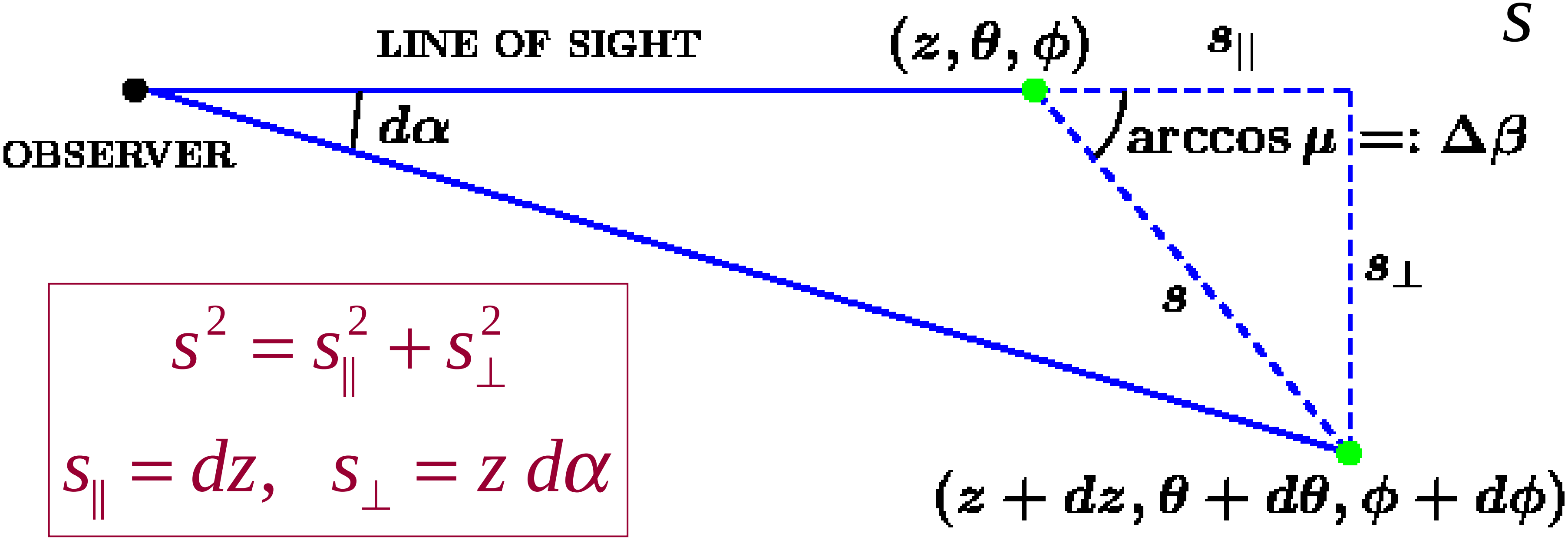
$$r_{\parallel} = \frac{(df/dz)dz}{\sqrt{1-kR^2}} = g(z)dz$$

$$g(z) = g(z;\Omega_{m0},\Omega_{x0},w;H_0) = \frac{1}{H(z)}$$



Redshift space

$$\mu \equiv \frac{s_{\|}}{s}$$



$$s^2 = s_{\|}^2 + s_{\perp}^2$$

$$s_{\|} = dz, \quad s_{\perp} = z\, d\alpha$$

$(\Delta\beta = 0, \pi \Leftrightarrow \mu = \pm 1)$ $\Leftrightarrow$ **longitudinal ("parallel")**

$(\Delta\beta = \pi/2 \Leftrightarrow \mu = 0)$ $\Leftrightarrow$ **transverse ("perpendicular")**

Alcock-Paczynski test

$$\frac{r_\perp}{r_\parallel} = j(z)\frac{z\,d\alpha}{dz} = j(z)\frac{s_\perp}{s_\parallel}$$

where the distortion or anisotropy parameter is given by

$$j(z) = j(z;\Omega_{m0},\Omega_{x0},w) = \frac{f}{gz}$$

$$r^2 = r_\perp{}^2 + r_\parallel{}^2$$

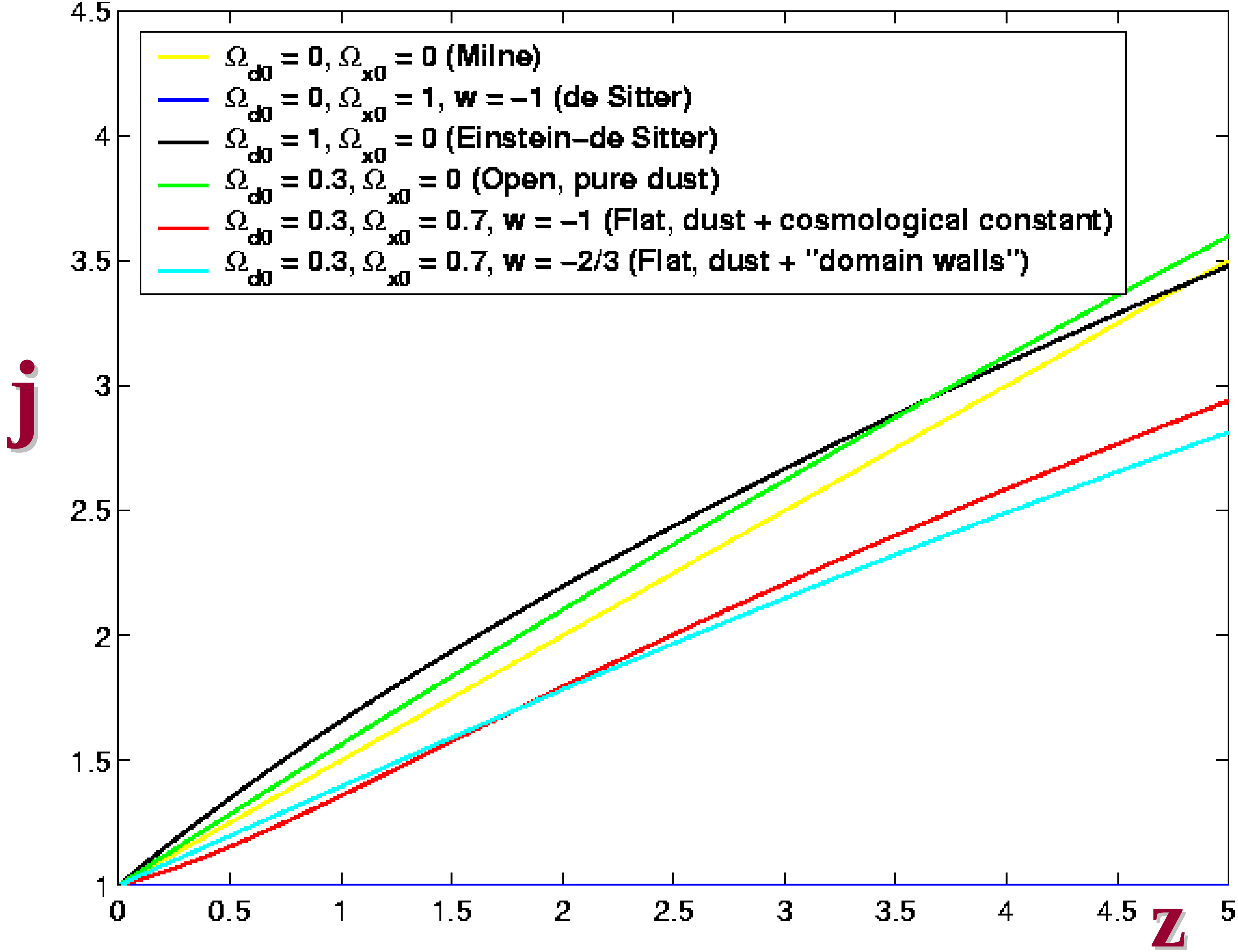
$$r_\perp = f(z)\,d\alpha,$$

$$r_\parallel = \frac{(df/dz)\,dz}{\sqrt{1-kR^2}} = g(z)\,dz$$

$$s^2 = s_\parallel^2 + s_\perp^2$$

$$s_\parallel = dz, \quad s_\perp = z\,d\alpha$$

distortion parameter (function)



original test

Main advantages of the original test:

- **quite discriminatory between open (Λ= 0) and flat ($\Omega_m + \Omega_\Lambda$=1) models**
- **evolution-independent**

Main disadvantage of the original test:

- **non-existence of *spherical* galaxy clusters**

- **voids: Ryden and Mellott (1995, 1996)**
- **Lyman-α forest : Hui, Stebbins & Burles (1998)**
- **quasar pairs: Phillipps (1994); Popowski *et al* (1998)**

Quasar clustering

Two-point correlation function $\xi(\vec{r})$

- **conditional probability:**

$$dP(r) = n\left[1 + \xi(r)\right]dV$$

existence of a mean

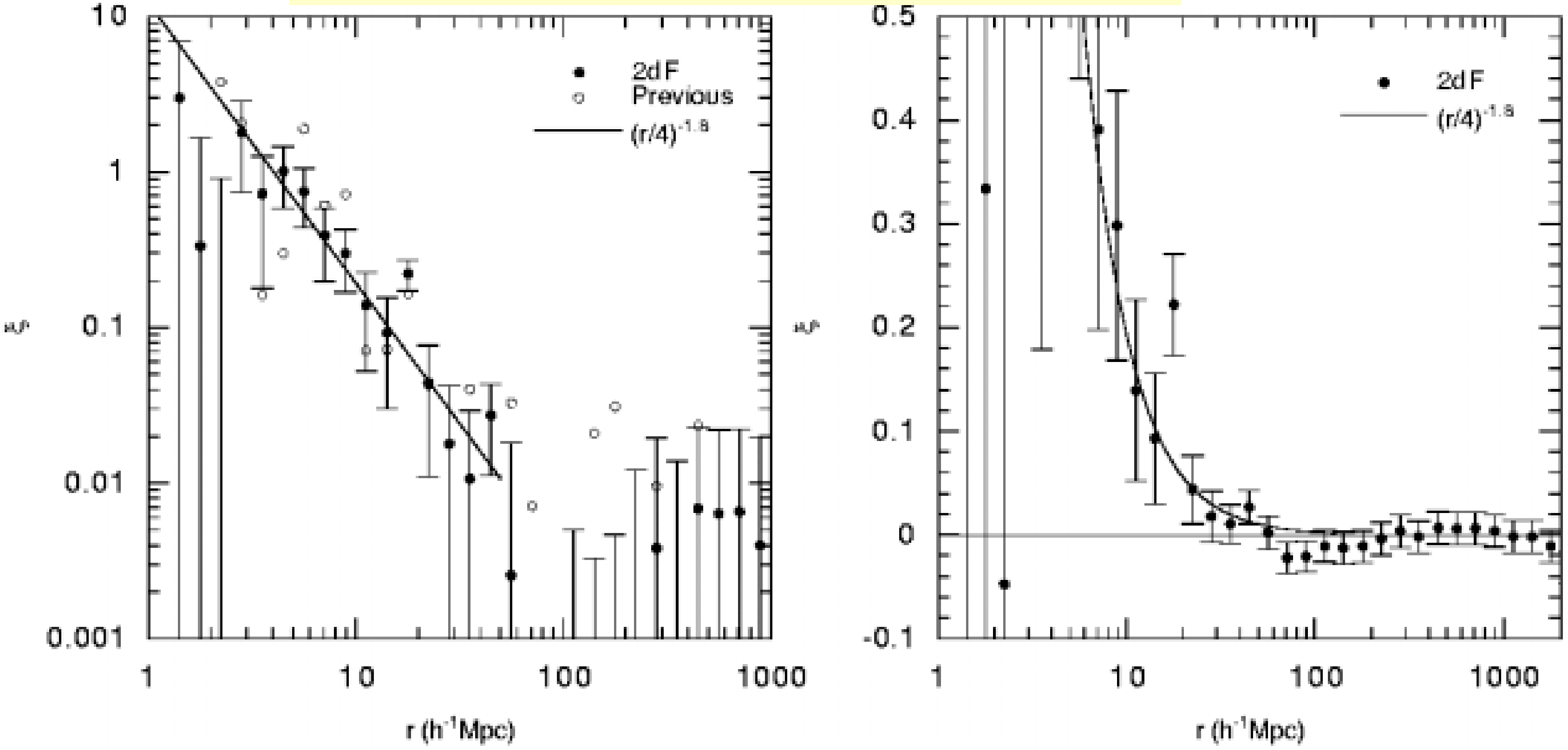
random Poisson distribution

statistical isotropy

2DF QSO Redshift Survey

astro-ph/0003206

Authors: T.Shanks, B.J. Boyle, S.M. Croom, N. Loaring, L. Miller, R.J. Smith



quasar correlation function

In general

$$\frac{r}{s} = g\sqrt{j^2 - (j^2 - 1)\mu^2}$$

distortion or anisotropy

- **real space:**

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}; \quad r_0 \cong 4\,h^{-1}\,\mathrm{Mpc}, \;\; \gamma \cong 1.8$$

- **redshift space:**

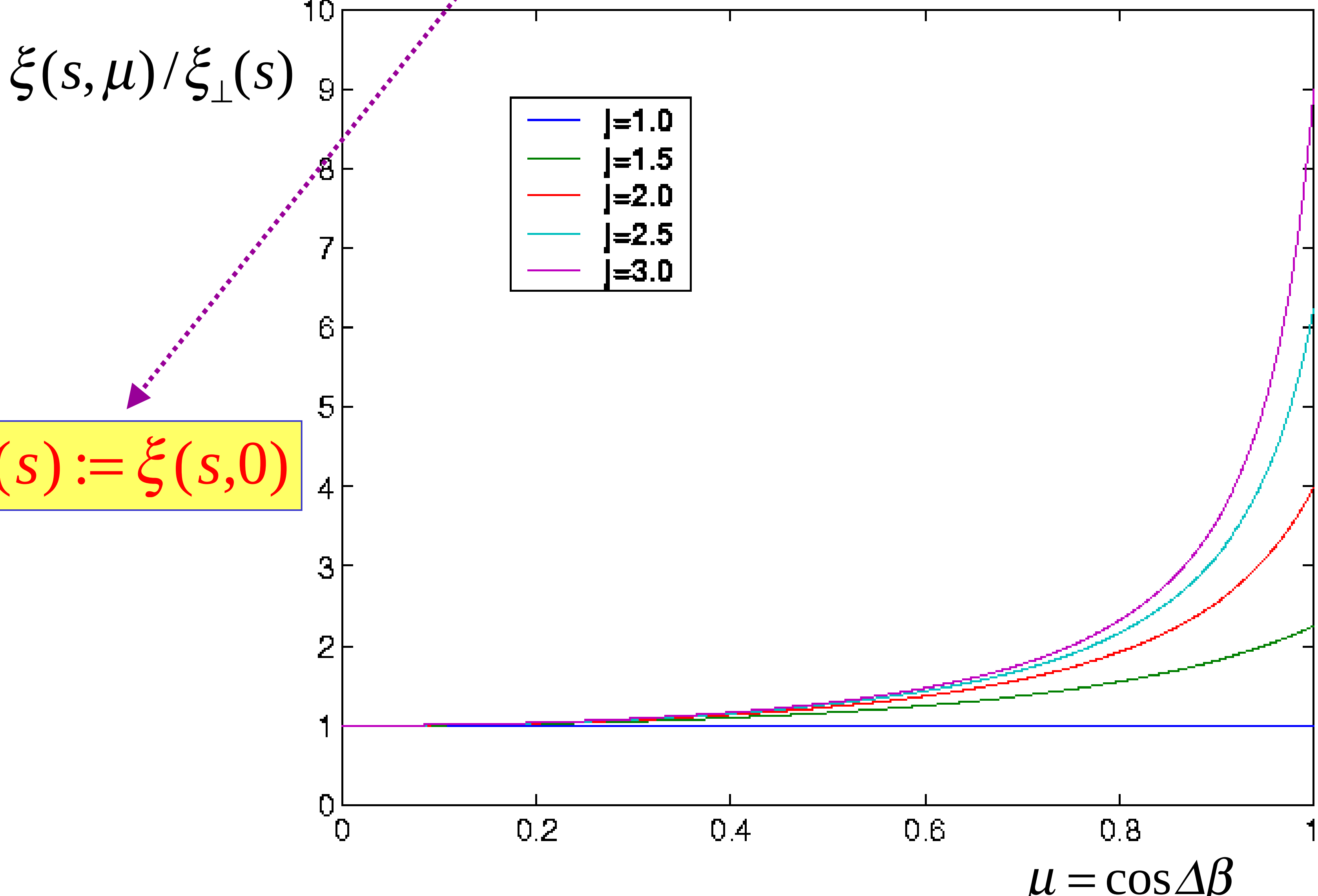
$$\xi(s,\mu) = \xi(z; s,\mu; r_0, \gamma; \Omega_{m0}, \Omega_{x0}, w)$$

$$= \left(\frac{s}{s_0}\right)^{-\gamma} \left[j^2 - (j^2 - 1)\mu^2\right]^{-\gamma/2}, \quad s_0 := \frac{r_0}{g}$$

anisotropy

Dependence of "normalized" apparent correlation function on the direction in redshift space

$$\xi_\perp(s) := \xi(s, 0)$$



anisotropy

In general:

$$\xi(s,\mu) > \xi_{\perp}(s) \quad \Rightarrow \quad \xi_{\parallel}(s) > \xi_{\perp}(s)$$

$$\Updownarrow$$

Amplification **of the "apparent" correlation function for pairs *along the line of sight* with respect to the one for pairs perpendicular to that line: *anisotropy*.**

quasar surveys

Quasars

- star-like objects (originally radio-loud)
- time-variable continuum non-thermal fluxes
- ultra-violet excess ("blueness"): $U - B < 0 \Leftrightarrow F_U > F_B$

$$B - V < 0 \Leftrightarrow F_B > F_V$$

- broad emission lines
- large redshifts
- large luminosities ($\geq 100 L_{\odot}$)

generic features of a survey

(a) limiting apparent magnitude: $m_{B,\max} = B_{\max}$

(b) area (solid angle): A

(c) expected number of detected sources: N_Q

$\Downarrow$ **(b) + (c)**

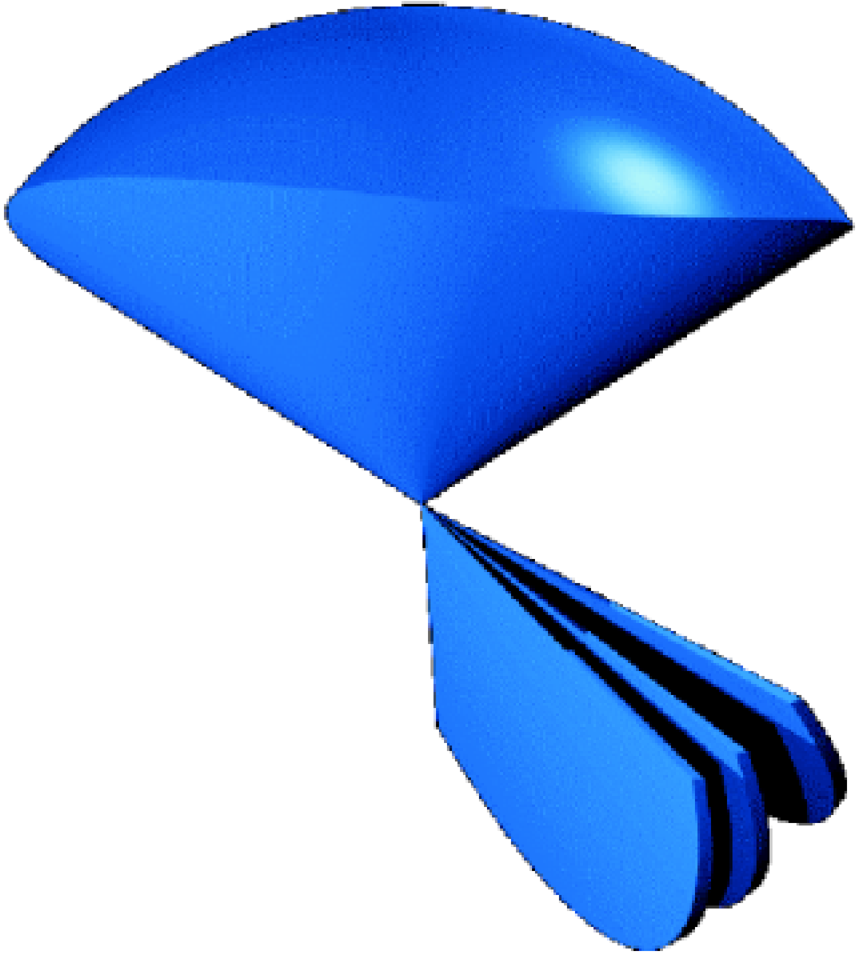
(d) (surface) density: $\Sigma := N_Q / A$

(e) "distribution function": $$F(z)dz := \frac{dN(z)}{N_Q}$$

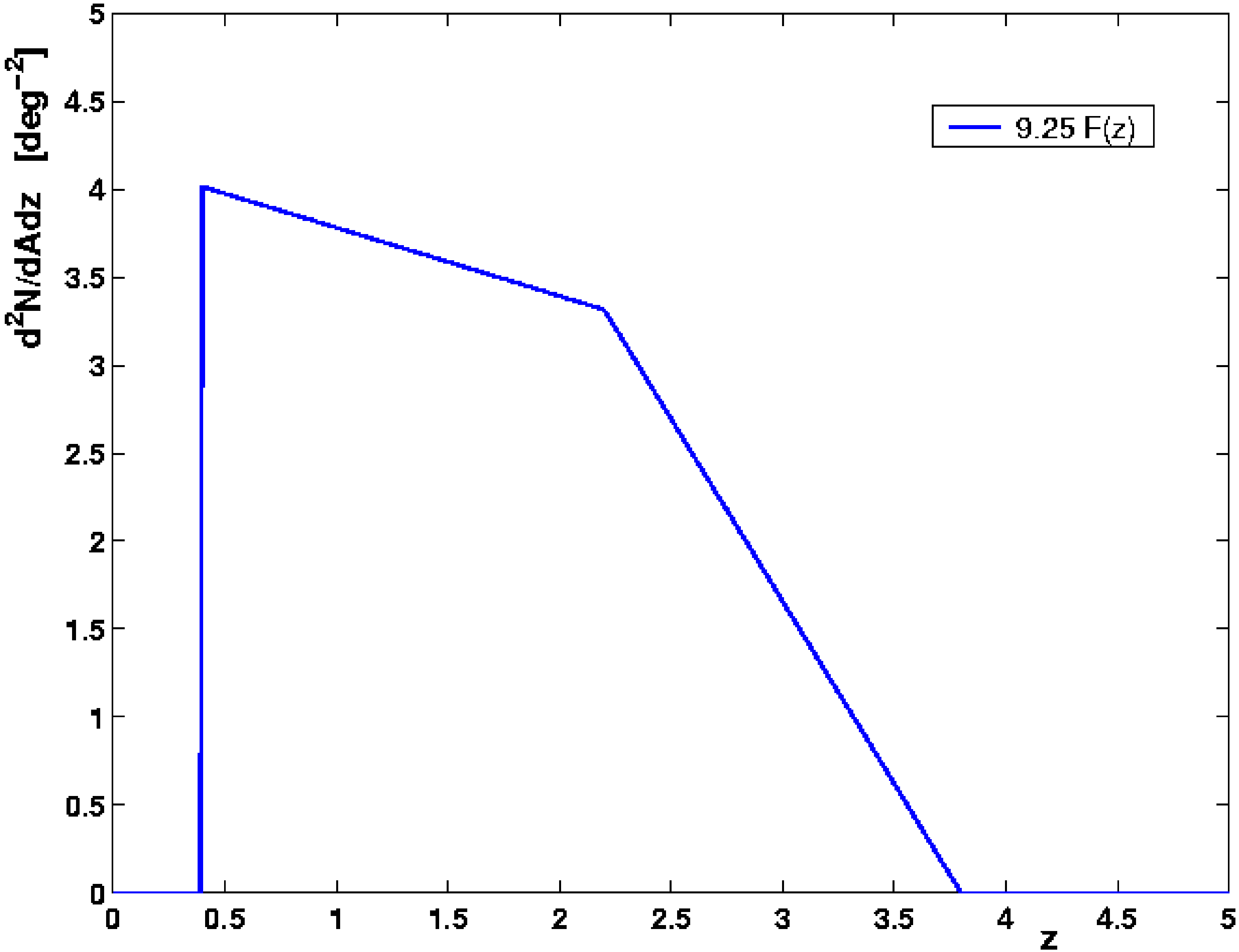
Sloan Digital Sky Survey (SDSS)

$$N_Q \approx 80000, \; A \approx 5000 \deg^2 \Rightarrow \Sigma \approx 16 \deg^{-2}$$

$$B_{\max} \approx 19.6$$



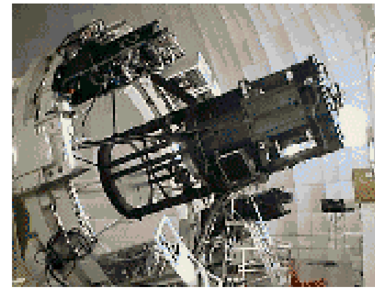
template distribution for SDDS

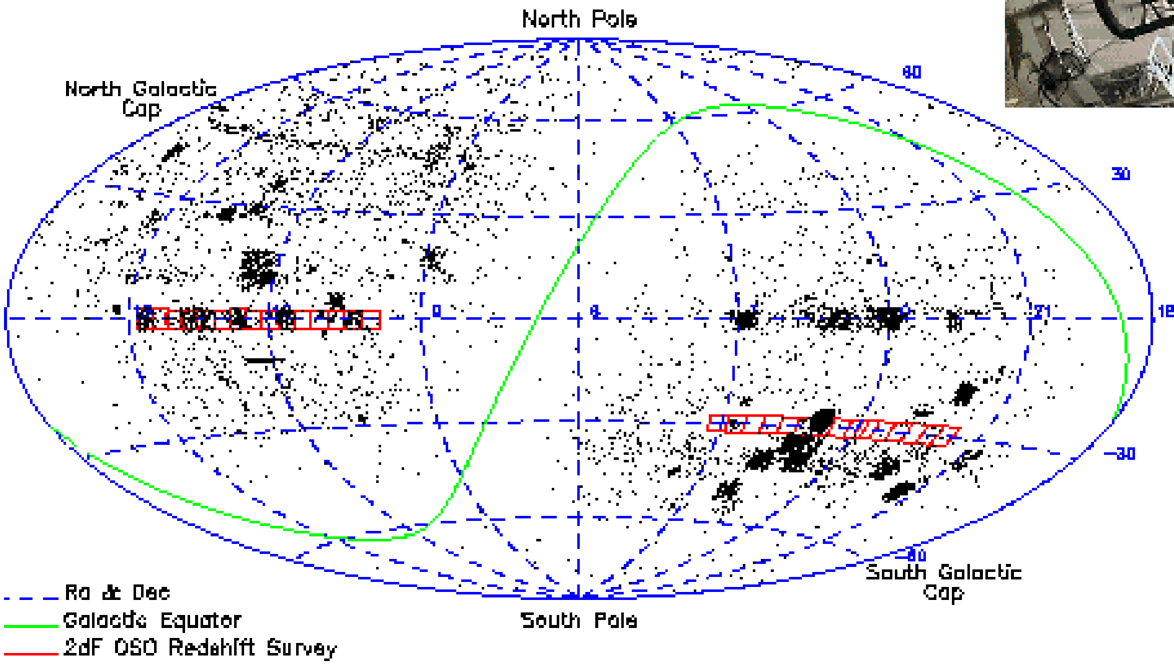


Two-degree Field Survey (2dF)

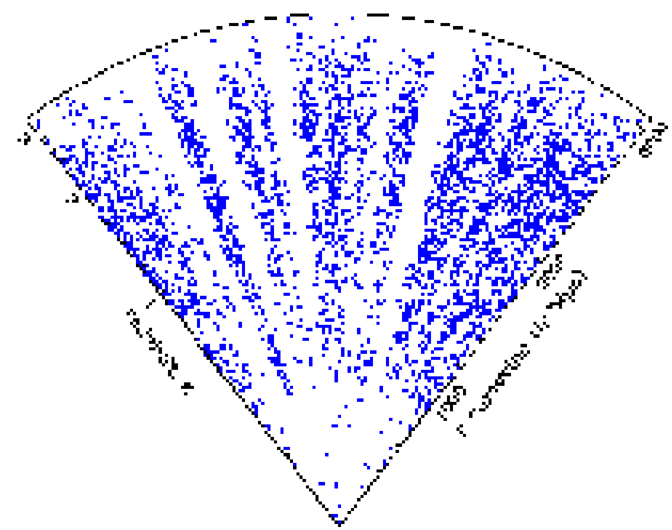
$$N_Q \approx 30000, \;\; A \approx 750\,\mathrm{deg}^2 \Rightarrow \Sigma \approx 40\,\mathrm{deg}^{-2}$$

$$B_{\mathrm{max}} \approx 21.4$$

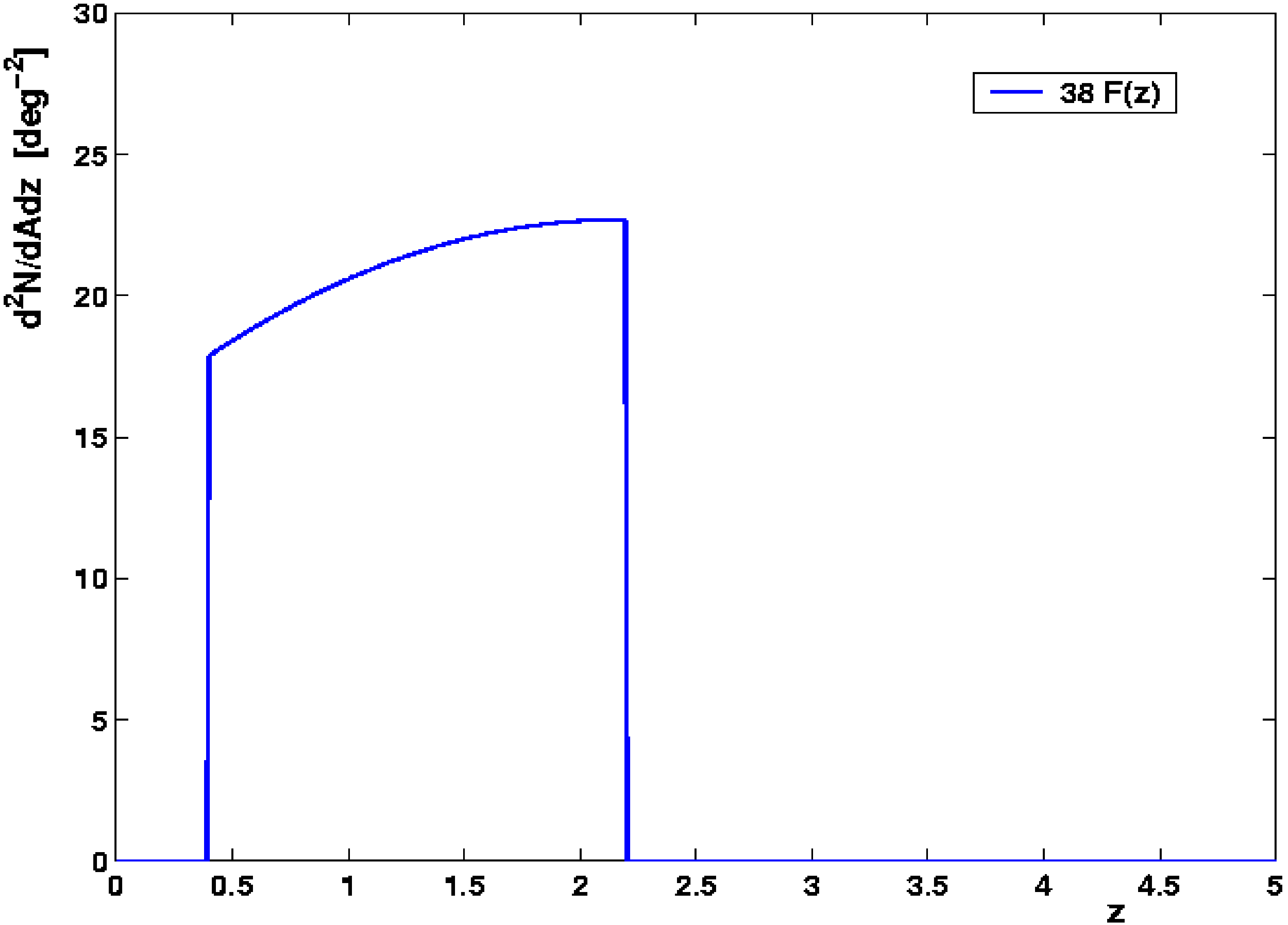




[Boyle *et alii* (1998)]



template distribution of 2DF



Poisson limit regime

Number density

$$n(z) = \frac{dN(z)}{dz}\frac{dz}{dV}, \qquad dV = g\, f^2 d\Omega$$

$$n(z) = \frac{(180/2\pi)^2 \Sigma F(z)}{gf^2}$$

- N_{cl} ⟶ **mean number of *clustered* neighbors per quasar:**

cut-off parameter

We used $\lambda = 2$

$$N_{cl} = \int_0^{\lambda r_0} n\xi(r)\, 4\pi r^2 dr,$$

$$\Downarrow$$

$$N_{cl} = \frac{4\pi\lambda^{3-\gamma}}{3-\gamma} n r_0^3$$

In our case $N_{cl} < 1$

'Sparse sampling' or Poisson limit regime

number of pairs in a bin

- a physical model specifies the correlation function in real space (r-space).
- a redshift survey provides data in redshift space (s-space).

$$(dV)_r = g^3 j^2 \left(-2\pi s^2 \, ds \, d\mu\right) = g^3 j^2 (dV)_s$$

$$n_s (dV)_s \equiv n(z) (dV)_r$$

$$n_s(s,\mu) = g^3 j^2 n(z)$$

The number of pairs expected in a small bin in (s, μ) within (z , z + Δz) is:

$$dN_{pairs} = N(z, z+\Delta z)\, dP = N(z, z+\Delta z)\, n_s \left[1+\xi(s,\mu)\right]\left(-2\pi s^2 \, ds \, d\mu\right)$$

$$N(z, z+\Delta z) = N_Q \int_z^{z+\Delta z} F(z)\, dz$$

is the number of observed quasars in the redshift range

number of clustered quasar pairs

$$dN_{pairs} = -2\pi \left(\frac{180}{\pi} \right)^2 \frac{N_Q^2}{A} \frac{F^2(z)}{z^2} [1 + \xi(s,\mu)] s^2 dz\, ds\, d\mu$$

2DF - Total of 1200 bins

20 linear bins in **s** (from $0.1s_0$ to $2s_0$) ;
5 linear bins in **|μ|** (from 0 to 1) ;
12 linear bins in **z** from (0.4 to 2.2)

maximum likelihood estimation

Suppose the data (real or simulated) consists of pairs counts N_i in **i** bins. We have a model M for the correlation function **M(s_0, γ, Ω_m, Ω_Λ)** .The model predicts a number of pairs A_i in each bin. In the Poisson limit ($N_c < 1$), the probability of detecting N_i pairs in bin **i** when A_i are expected is:

$$P(N_i \mid A_i) = \frac{e^{-A_i} A_i^{N_i}}{N_i!}$$

The probability for separate bins are independent, so the likelihood **L** of obtaining the data given the model is:

$$\mathrm{L} \equiv P(D \mid \mathrm{M}) = \Pi_i \frac{e^{-A_i} A_i^{N_i}}{N_i!}$$

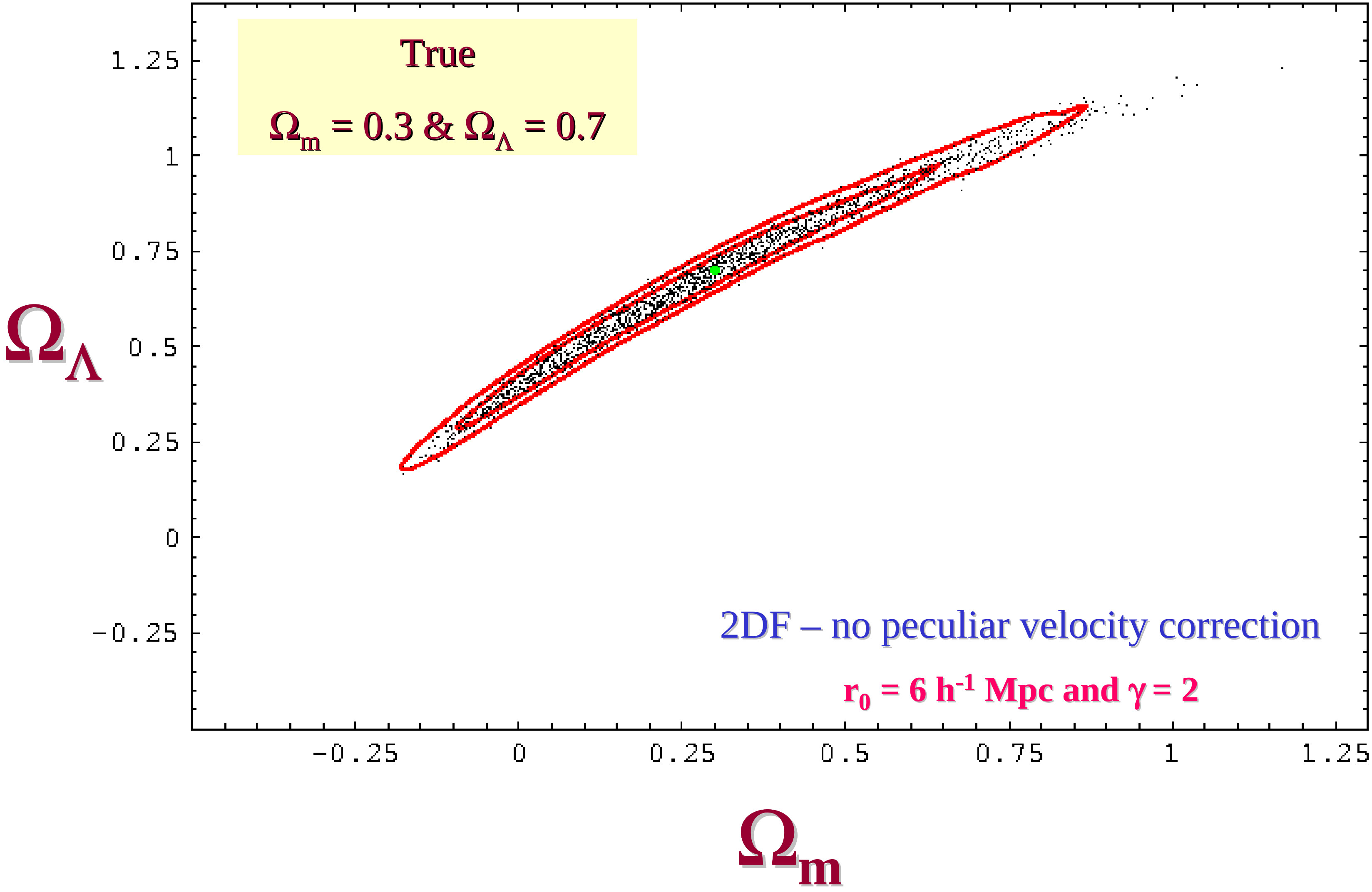
So,

$$Ln\,\mathrm{L} = \sum_i \left(-A_i + N_i\,Ln\,A_i - Ln\,N_i!\right)$$

Since N_i are independent of the model parameters we find the maximum likelihood model by maximizing the quantity:

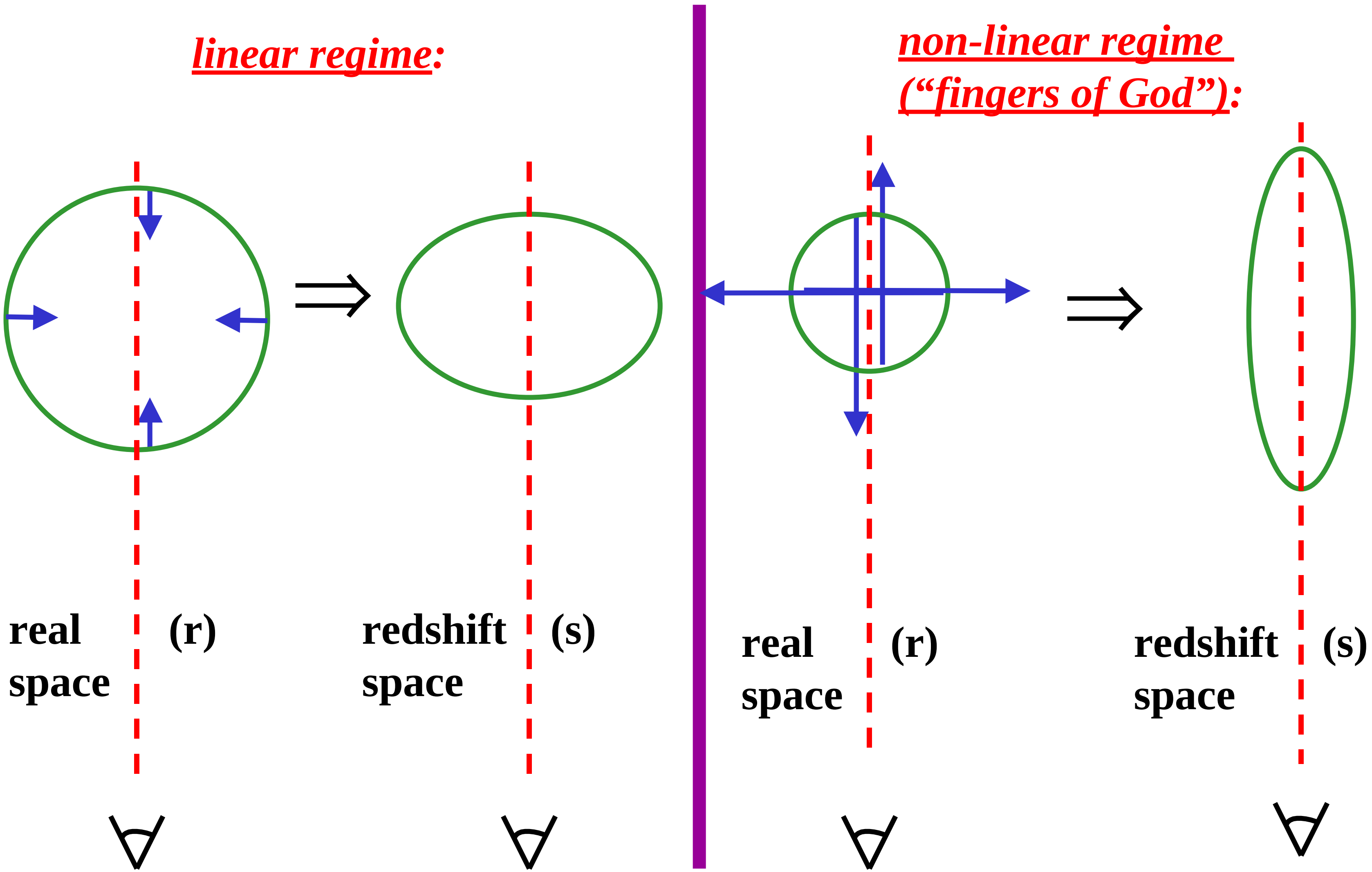
$$Ln\mathrm{L}' = \sum_i \left(-A_i + N_i\,Ln\,A_i\right)$$

contours


True
Ωm = 0.3 & ΩΛ = 0.7
1.25
1
0.75
0.5
0.25
0
−0.25
−0.25
0
0.25
0.5
0.75
1
1.25
ΩΛ
Ωm
2DF – no peculiar velocity correction
r0 = 6 h-1 Mpc and γ = 2

peculiar velocities

- **consideration of collapsing motions:**



$$\xi_{cor}(s,\mu) = \xi_0 P_0(\mu) + \xi_2 P_2(\mu) + \xi_4 P_4(\mu)$$

Hamilton ApJ , 385, L5 (1992)

Where P_i are the Legendre polinomials and

$$\xi_0 = -\left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right)\frac{\gamma - 3}{3}\xi(s,\mu)$$

$$\xi_2 = -\left(\frac{4\beta}{3} + \frac{4\beta^2}{7}\right)\frac{\gamma}{3}\xi(s,\mu)$$

$$\xi_4 = \frac{8\beta^2}{35}\frac{\gamma(\gamma+2)}{3(5-\gamma)}\xi(s,\mu)$$

$$\beta = \frac{f}{b}$$

$$f = \frac{d\ln\Delta_+}{d\ln a}$$

b is the bias parameter and Δ_+ is the growing mode of the matter density contrast. We use b=2 in our simulations.

$$\Delta_+ = \frac{5}{2}\Omega_{m0} H \int_z^\infty \frac{1+x}{H^3}dx \quad ; \; w = -1 \; ; \; H = H(z, \Omega_{m0}, \Omega_{\Lambda 0})$$

Heath MNRAS, 179, 351, (1977)

$$\Delta_+ \cong \Omega_m^{4/7}(z) + \frac{\Omega_\Lambda(z)}{70}\left(1 + \frac{\Omega_m(z)}{2}\right)$$

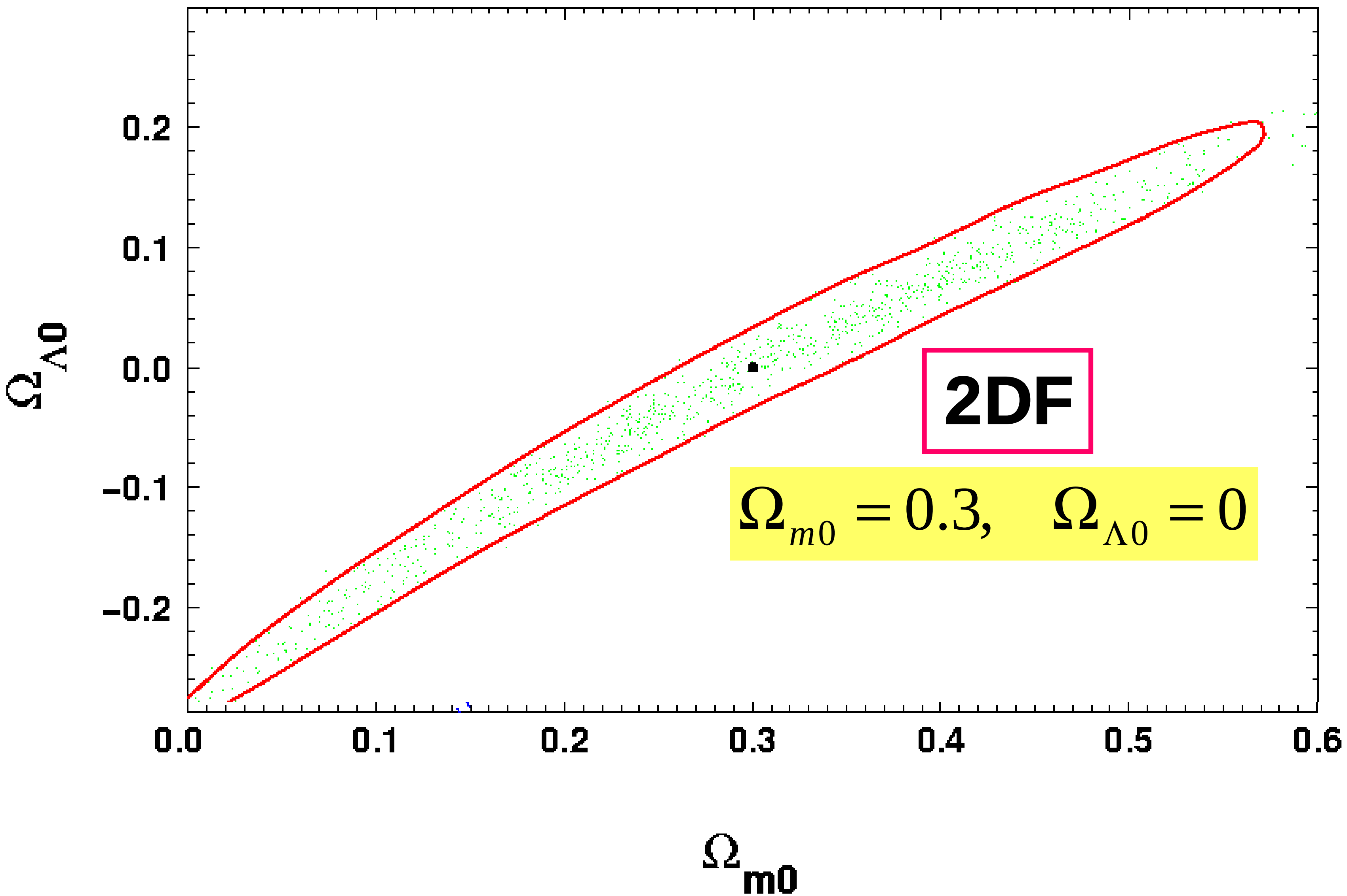
Lahav, Lilje, Primack & Rees MNRAS, 251, 136, (1991)

for the flat case (k = 0) and any w we have,

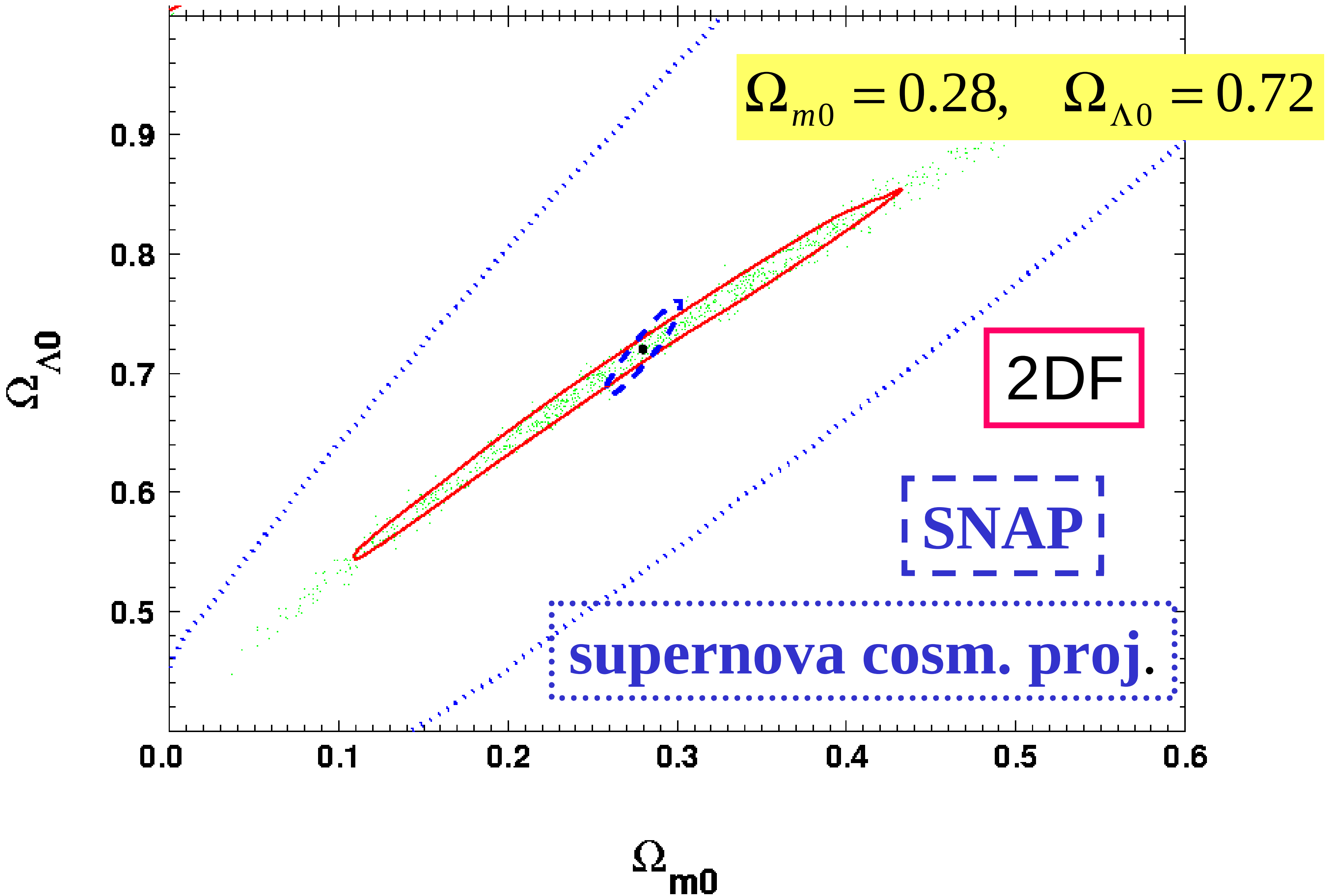
Silveira & Waga PRD 50, 4890, (1994).

$$\Delta_+ = \left(\frac{1}{1+z}\right) \text{Hypergeometric2F1}\left[-\frac{1}{3w}, \frac{w-1}{2}, 1 - \frac{5}{6w}; -\frac{1-\Omega_{m0}}{\Omega_{m0}}\left(\frac{1}{1+z}\right)^{3-m}\right]$$

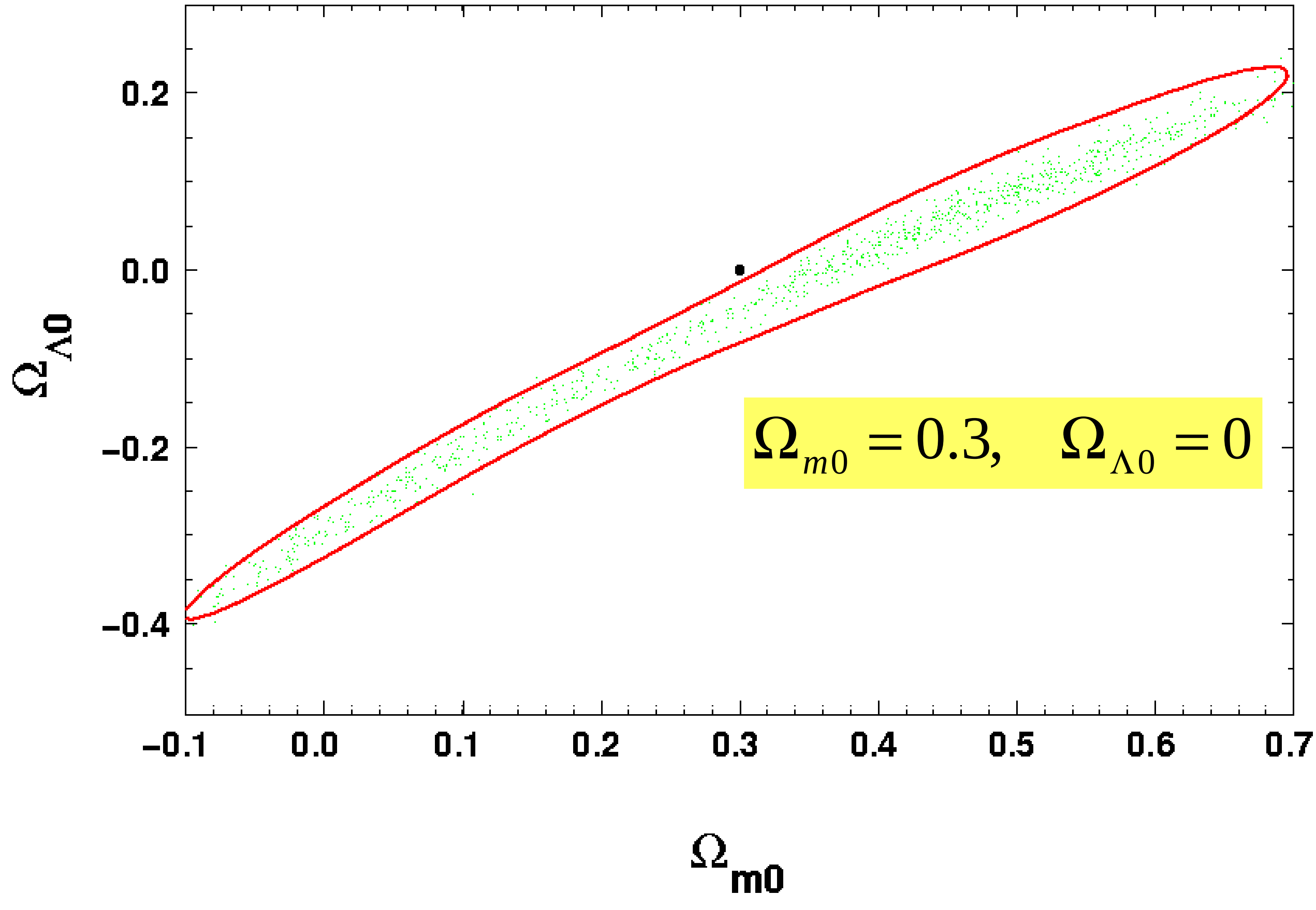
two sigmas contour (w=-1)



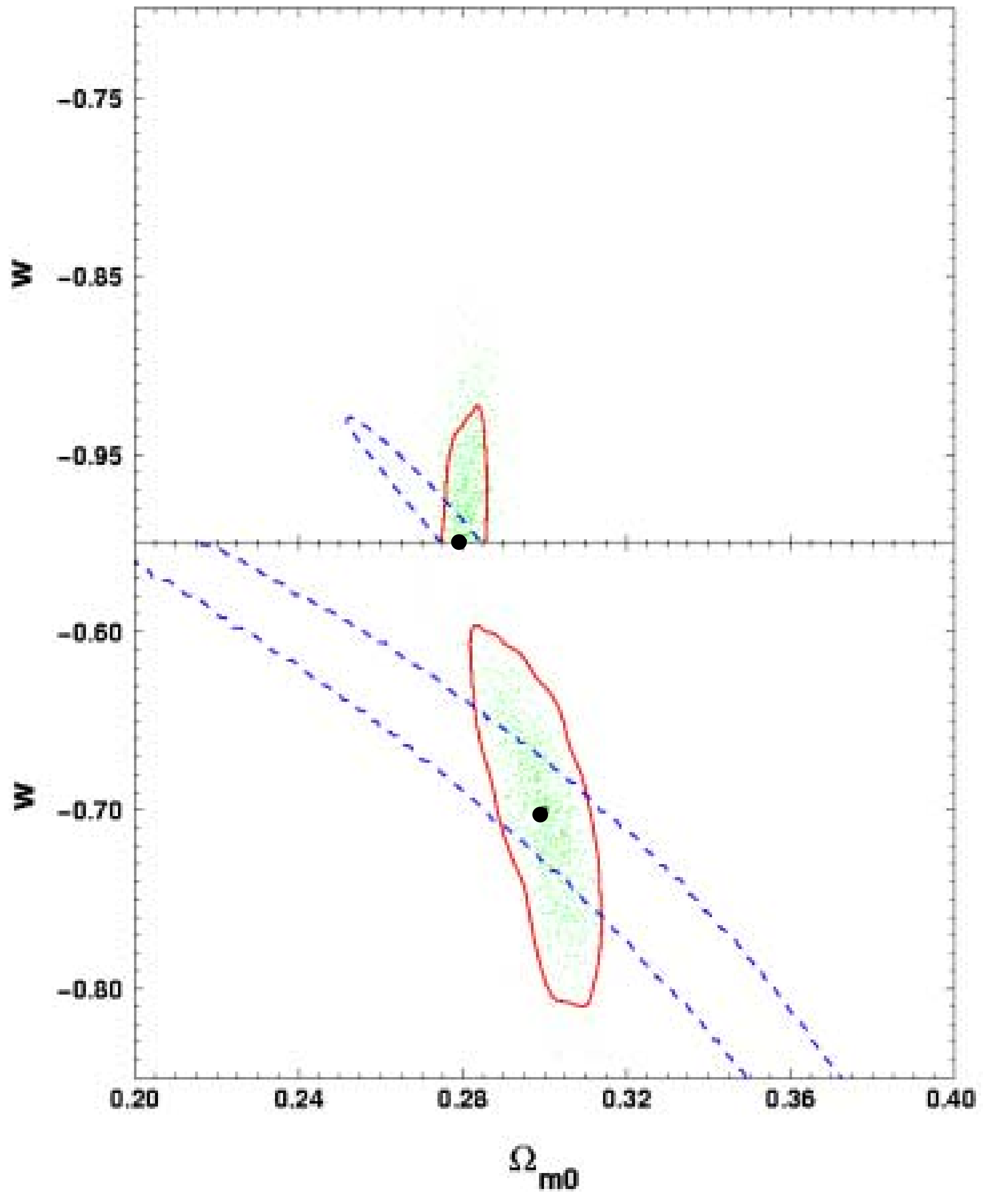
one sigma contours (w=-1)



effect of peculiar velocities



equation of state (flat models)



$$\Omega_{m0} = 0.28, \quad w = -1$$

SNAP mission

$$\Omega_{m0} = 0.3, \quad w = -0.7$$

DEEP survey

Ongoing work

- the role of the parameters r_0 and γ
  evolution & marginalization
- SDDS and 2DF more realistically
- the role of binning in dz, ds and $d\mu$
- extension to other cosmological models
  (e. g., scalar field cosmology)