Quantum noise reduction in two-photon oscillators

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We present an exact nonlinear derivation of the spectrum of fluctuations in the difference of output intensities for the signal and idler beams generated by intracavity two-photon gain, when the cavity decay rates are the same for the two modes, and taking into account the atomic fluctuations. For the field inside the cavity, we show that the variance in the difference of intensities may be up to 50% below the shot-noise value. We also consider the case of different decay rates, and show that the noise in the output-intensity difference vanishes at zero frequency, for single-ported cavities, and as long as the only source of field decay is the transmission through the mirror. Within this general approach, we show that for the two-photon laser there is a strong correlation between signal and idler output beams when the relay-level detuning is larger than the power broadening. We calculate, in lowest order, the correction to the noise spectrum of the difference of intensities due to the population of the relay level.

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I. INTRODUCTION

Quantum-noise reduction (squeezing) has been observed in many systems [1]. In the optical parametric oscillator (OPO), a nonlinear medium placed inside a cavity down-converts an injected pump beam of frequency ω_0 into two beams (conventionally termed signal and idler beams) at the cavity eigenfrequencies ω_1 and ω_2 (where $\omega_0=\omega_1+\omega_2$). A large phase-quadrature squeezing of the transmitted signal mode occurs in the degenerate OPO ($\omega_1=\omega_2$) near the threshold of oscillation [2]. Experimentally, Wu, Xiao, and Kimble [3] achieved a noise reduction of 63% below the vacuum (classical) level in a balanced homodyne detection of the (nearly) degenerate transmitted-signal beam. Changing the relative phase between the local oscillator and the pump fields, the two quadratures of the output field were measured to demonstrate the strongly phase-sensitive noise reduction.

In the nondegenerate OPO, the main concern has been the generation of "twin photon beams." It was first shown by Reynaud and co-workers [4,5] that the fluctuations in the intensity difference between the transmitted signal and idler beams are reduced well below the classical level. Since then, this technique has been studied both theoretically [6–8] and experimentally [9]. Recently, a noise-reduction factor of 86% below the classical level was achieved [10]. Many applications have been developed, including generation of sub-Poissonian light using active-control techniques [11] and enhancement of the sensitivity of absorption and polarization-rotation measurements [12].

The underlying nonclassical property, not found in the degenerate case, is the simultaneous creation of the "twin" signal and idler photons in the parametric down-conversion. In a lossless cavity, the intracavity field would thus increase in such a way that the number of signal and idler photons would be exactly equal (zero fluctuation). Since the cavity dissipation is a random process acting independently on each mode, at steady state the field inside any real lossy cavity has finite fluctuations in the intensity difference. However, if all the removed photons are detected (this will be the case for ideal detectors provided the dissipation in the cavity and in the mirrors is negligible), the lossless zero-noise photon configuration is, after a large time interval, exactly reproduced at the photodetectors, resulting in a noiseless difference between the photocurrents for the signal and idler beams when the detection time is long enough. In the frequency domain, it means that the noise spectrum vanishes at zero frequency.

Since a single feature of the parametric down-conversion is involved in the underlying principle of the noise reduction—namely, the simultaneous twin-photon generation—we would expect the result to be valid for any oscillator sustained by a genuine nondegenerate two-photon process (as opposed to cascade transitions, in which an intermediate state gets populated, and there is a delay in the emission of the second photon). For instance, the same noise reduction was found for intracavity four-wave mixing [13] in the case of equal signal and idler cavity decay rates (balanced case).

In Ref. [4] Reynaud derived the noise spectrum of the difference in the transmitted intensities for the OPO, in the case where the output mirror is the only source of the decay rates of signal and idler beams, assumed to be equal (balanced single-ported case). In fact, the only physical property used was the conservation of the intensity difference operator I_1-I_2 by the parametric interaction

$$[H,I_1-I_2]=0\ , \tag{1.1}$$

where H is the parametric Hamiltonian.

Using the same single property, Graham, in an earlier paper [14], obtained general (and exact) identities for the steady-state intracavity field fluctuations generated in a parametric process. The correlation between signal and idler modes was shown to lie halfway between the maximum classical and quantum allowed values.

A similar nonclassical correlation was found, under more special assumptions, for a nondegenerate two-photon laser operating far above the threshold of oscillation [15]. For two-photon lasers, due to the presence of a relay level which mediates the transition between the excited and final atomic states, equality (1.1) does not hold anymore. Recently [16], however, we have recovered Graham's result using an approach suited to two-photon lasers and masers, which takes into account the pumping contribution in these devices.

In this paper, we generalize the treatment in Ref. [16], by developing a method, applicable also to two-photon lasers and masers, which allows one to show the connection between the above-mentioned general results for the intracavity field fluctuations and the noise spectrum of the output field, calculated by Reynaud in the balanced single-ported case.

Furthermore, we derive a new identity for the transmitted field in the more general case of different decay rates. We use a general and exact result for the intracavity field fluctuations and the input-output theory of Gardiner and Collet [17] to show that the noise spectrum of the transmitted field vanishes at zero frequency whenever the cavity is single-ported, and as long as the only source of field decay is the transmission through the mirror (that is, as long as dissipation in the cavity and in the mirror can be neglected). This derivation is the mathematical counterpart of the physical picture discussed above. Lane, Reid, and Walls [6] obtained the same result for the nondegenerate OPO, by linearizing the equations of motion about their steady-state values. We obtain instead an exact result valid not only for the OPO, but for any oscillator sustained by a genuine nondegenerate two-photon process, which includes resonant processes in which atomic levels may be appreciably populated.

The literature about twin-photon generation refers almost exclusively to parametric processes (although the recent work by Blockley and Walls [18] is an exception). In the light of our results, however, it becomes clear that processes involving transfer of population, such as in the resonant interaction with excited atoms, may also be used to generate (eventually more powerful) correlated photon beams. Furthermore, our results confirm that the role of a pump field in the noise spectrum of the intensity difference is of minor importance (especially in the balanced case, see, e.g., Ref. [8]) for there is no need of a phase reference in phase-insensitive measurements.

To have a deeper insight into this point, we consider in detail a two-photon laser whose active medium consists of an ensemble of three-level atoms in a ladder configuration. When excited to the upper state, the atoms interact with two cavity modes, resulting in the configuration shown in Fig. 1. The intermediate level is

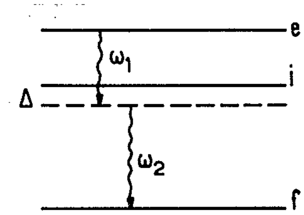

FIG. 1. Atomic levels relevant for the two-photon laser.

detuned so as to avoid a two-photon cascade process, whereas the upper and lower levels are resonantly coupled to the cavity.

This system has been recently studied by Boone and Swain [19]. They derived the master equation for the field density matrix, which we shall use to compute the field fluctuations.

We show that population effects of the finite-lifetime upper state do *not* degrade the signal-idler correlation. So long as the intensity difference is concerned, the only important atomic parameters are the detuning (Δ) and lifetime of the *intermediate level*. In the limit of large Δ, the gain is a genuine two-photon process and the noise spectrum in the balanced single-ported case is thus the one calculated by Reynaud for the OPO. More generally, for the nonbalanced single-ported case, but with negligible intracavity and mirror dissipation, we show that the noise vanishes at zero frequency in this limit. We also calculate the first-order correction in the inverse quadratic power of the detuning Δ, which represents the degradation of the twin-photon correlation due to the population of the intermediate state.

One should remark that many other schemes of generating sub-Poissonian light with active systems have been presented in the literature, some of them having already led to experimental demonstration. They involve usually the control of one of the sources of quantum noise in these devices, namely the pumping noise [20], the spontaneous-emission noise [20,21], or the vacuum fluctuations coming into the cavity through the coupling mirrors [22]. Noise reduction may also be achieved in one- and two-photon correlated-emission lasers and masers [23]. The twin-photon-generation method has, over these other procedures, the advantage of automatically canceling out the fluctuations in the gain, since they are the same for both beams. It does that, however, at the expense of having to subtract two beams of about the same intensity, thus leading in general to low-intensity squeezed light.

This paper is organized as follows. In Sec. II we show that the intracavity intensity-difference noise may be reduced 50% below the classical lower bound. We pay special attention to the role of the intermediate state in two-photon lasers, thus assessing the limits of validity in this case of the results concerning the output field, obtained in Sec. III (for the balanced case) and Sec. IV (for the single-ported case). In Sec. V, we study the corrections to the previous result due to the population of the relay level in a two-photon laser. We summarize our results in Sec. VI.

II. INTRACAVITY FIELD FLUCTUATIONS

In any intracavity light generator, the dynamics of the field is governed by two distinct processes: (1) a gain mechanism, based either on a parametric process, or on an interaction with an excited (active) medium; (2) a loss mechanism, associated with intracavity and mirror losses, and with the transmission of the field through the mirror, which provides the output field to be measured.

In Secs. II–IV we consider the intracavity generation of two field modes, termed signal and idler. We calculate, in a general and exact way, the fluctuations in the difference between signal and idler photon numbers, n_2-n_1, for a class of systems satisfying

$$\frac{d}{dt}\langle (n_2-n_1)^k\rangle|_{\text{gain}}=0\ ,\quad k=1,2 \tag{2.1}$$

that is, systems in which the gain process is (exclusively) based on the simultaneous generation of pairs of signal and idler photons ("twin" photons). Formally, it means that the gain does not couple the sum with the difference of photon numbers. We analyze the consequences of Eq. (2.1) for the intracavity field in this section, and for the output field in Secs. III and IV.

Even though Eq. (2.1) may be trivially derived from (1.1), it applies, however, for system for which (1.1) is not valid anymore, as we shall see in this section.

The field-density-matrix operator ρ obeys a master equation of the general form

$$\frac{d\rho}{dt}=\frac{d\rho}{dt}\bigg|_{\text{gain}}+\mathcal{L}\rho\ , \tag{2.2}$$

where $\mathcal{L}\rho$ is the loss contribution, coming from the interaction of the field with its reservoir, which is assumed to be in the vacuum state. This means that we do not consider inputs at the coupling mirror M (see Fig. 2) other than the vacuum field. Our derivation in this section is also valid for masers operating at very low temperatures [16].

The loss contribution in Eq. (2.2) is thus

$$\mathcal{L}\rho=\sum_{k=1,2}\frac{\gamma_k}{2}(2a_k\rho a_k^\dagger-a_k^\dagger a_k\rho-\rho a_k^\dagger a_k)\ , \tag{2.3}$$

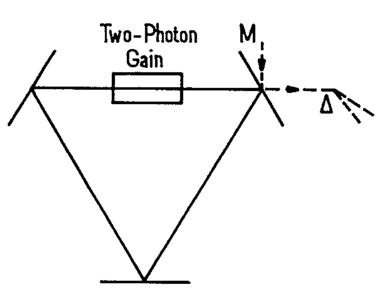

FIG. 2. Optical cavity for the two-photon oscillator. The two output beams are separated and independently measured.

where γ_k is the cavity decay rate for mode k, and $a_k,a_k^\dagger$ are the boson annihilation and creation operators for mode k ($n_k=a_k^\dagger a_k$ is the photon-number operator), respectively.

From Eqs. (2.2) and (2.3), the equation of motion for the populations

$$P_{n_1n_2}(t)=\langle n_1n_2|\rho(t)|n_1n_2\rangle$$

is

$$\frac{dP_{n_1n_2}(t)}{dt}=\frac{dP_{n_1n_2}}{dt}\bigg|_{\text{gain}}-(\gamma_1n_1+\gamma_2n_2)P_{n_1n_2}+\gamma_1(n_1+1)P_{n_1+1,n_2}+\gamma_2(n_2+1)P_{n_1,n_2+1}\ . \tag{2.4}$$

Using our basic assumption about twin-photon generation, Eq. (2.1), we find from Eq. (2.4)

$$\frac{d}{dt}\langle n_2-n_1\rangle=-\gamma_2\langle n_2\rangle+\gamma_1\langle n_1\rangle\ , \tag{2.5}$$

$$\frac{d}{dt}\langle (n_2-n_1)^2\rangle=\gamma_1\langle n_1\rangle+\gamma_2\langle n_2\rangle+2\gamma_1\langle n_1(n_2-n_1)\rangle-2\gamma_2\langle n_2(n_2-n_1)\rangle\ . \tag{2.6}$$

In the case of equal cavity decay rates (balanced case), the dynamics of the average difference between photon numbers is independent of the sum (and thus of the gain process), as may be seen directly from (2.5) and (2.6). The intracavity steady-state moments are, in such case, given by

$$\langle n_1\rangle=\langle n_2\rangle\ , \tag{2.7}$$

$$\langle (n_2-n_1)^2\rangle=\frac{\langle n_1+n_2\rangle}{2}\ . \tag{2.8}$$

On the other hand, for a classical field distribution, we must have

$$\langle (n_2-n_1)^2\rangle-\langle n_2-n_1\rangle^2\geq\langle n_1+n_2\rangle\ . \tag{2.9}$$

We thus have a squeezing factor of 50% for the intracavity field in the balanced case, either above or below the threshold of oscillation.

This is the first striking consequence of Eq. (2.1). Before going further, we make our study less abstract, considering how the twin-photon condition, Eq. (2.1), appears in some specific classes of systems.

The parametric process generating two nondegenerate modes is described by a Hamiltonian operator H such that

$$[H,n_1-n_2]=0\ . \tag{2.10}$$

From the corresponding master equation

$$\frac{d\rho}{dt}=\frac{1}{i\hbar}[H,\rho]+\mathcal{L}\rho\ , \tag{2.11}$$

one gets trivially Eq. (2.1), which leads to the results (2.5)–(2.8). This was the procedure followed by Graham [14], thus showing that the noise-reduction effect arises from the intrinsic two-photon nature of the parametric interaction, expressed by Eq. (2.10).

On the other hand, the twin-photon effect in two-photon lasers and micromasers has a more subtle origin. The atom-field Hamiltonian corresponding to the configuration shown in Fig. 1 is, in this case,

$$H=H_{\rm at}+H_F +(\hbar\Omega_{ei}a_1|e\rangle\langle i|+\hbar\Omega_{if}a_2|i\rangle\langle f|+\text{H.c.})\;, \qquad (2.12)$$

where $H_{\rm at}$ and H_F are, respectively, the Hamiltonian for the three-level atom and for the signal and idler modes; Ω_{ei} and Ω_{if} are the $e\to i$ and $i\to f$ atom-field couplings, in units of frequency. We have neglected, in Eq. (2.12), the couplings between the upper mode (of frequency ω_1) and the $i\to f$ transition, and between the lower mode (of frequency ω_2) and the $e\to i$ transition, corresponding to the limit

$$|\omega_2-\omega_1|\gg\Delta\;. \qquad (2.13)$$

As pointed out elsewhere [16], these crossed couplings may degrade the twin-photon correlation.

The Hamiltonian H in Eq. (2.12) does not commute with n_1-n_2, even in the large detuning limit. In fact, the twin-photon effect appears here as the result of the association of the large-Δ atom-field interaction *and* the atomic excitation scheme, which must not provide atoms in the intermediate state $|i\rangle$.

Accordingly, we cannot obtain the field master equation for the two-photon laser by replacing the atom-field Hamiltonian, Eq. (2.12), into Eq. (2.11), but we would need instead to consider a Hamiltonian describing the whole dynamics involved in the problem, including the excitation. Alternatively, we may calculate the change of the reduced density matrix of the field due to one atom, and then write down the master equation by summing up the contributions of the successive excited atoms introduced into the laser cavity [24].

In many papers about two-photon lasers, the excitation to the upper state $|e\rangle$ was incorporated in the form of an effective two-photon Hamiltonian [25], satisfying Eq. (2.10). However, this approach has been recently shown to fail even in the large-detuning limit [19,26].

In order to analyze in detail the meaning of the large detuning limit [i.e., the limit in which Eq. (2.1) holds], we take the master equation derived by Boone and Swain [19] from the microscopic Hamiltonian given by Eq. (2.12) within the usual Scully-Lamb approach [24]. Assuming that the atoms are excited to the upper state $|e\rangle$ with a rate R and according to Poisson statistics one finds

$$\frac{d}{dt}P^{(t)}_{n_1n_2}\Big|_{\rm gain}=R[-(g_{n_1n_2}+f_{n_1n_2})P_{n_1n_2} +g_{n_1-1,n_2-1}P_{n_1-1,n_2-1} +f_{n_1-1,n_2}P_{n_1-1,n_2}]\;, \qquad (2.14)$$

where

$$g_{n_1n_2}=\frac{2\frac{g^4}{\Gamma^4}(n_1+1)(n_2+1)\left[3+\frac{\Delta^2}{\Gamma^2}+\frac{3g^2}{\Gamma^2}(n_1+n_2+2)\right]}{\left[1+\frac{4g^2}{\Gamma^2}(n_1+n_2+2)+\frac{\Delta^2}{\Gamma^2}\right]\left[1+\frac{g^4}{\Gamma^4}(n_1+n_2+2)^2+\frac{2g^2}{\Gamma^2}(n_1+n_2+2)+\frac{\Delta^2}{\Gamma^2}\right]}\;, \qquad (2.15)$$

and

$$f_{n_1n_2}=\frac{2\frac{g^2}{\Gamma^2}(n_1+1)}{1+\frac{\Delta^2}{\Gamma^2}+\frac{4g^2}{\Gamma^2}(n_1+n_2+2)} \qquad (2.16)$$

represent both the $e\to f$ and the $e\to i$ transition rates (when the field is in the Fock state $|n_1n_2\rangle$). The constant g stands for the atom-field coupling ($g=\Omega_{ei}=\Omega_{if}$) and Γ is the atomic decay rate.

In systems without any phase reference, the field coherencies $\langle n_1n_2|\rho|n_1'n_2'\rangle$, $n_1'\neq n_1$ or $n_2'\neq n_2$, are not coupled to the populations, as we see for instance in Eq. (2.14) above. In such a case, it is easily seen that Eq. (2.1) holds if and only if the gain part of the master equation has the form

$$\frac{dP_{n_1n_2}}{dt}\Big|_{\rm gain}=-A(n_1n_2)P_{n_1n_2} +A(n_1-1,n_2-1)P_{n_1-1,n_2-1}\;, \qquad (2.17)$$

where $A(n_1n_2)$ is any function.

Comparing (2.17) with (2.14), we see that, for the two-photon laser, Eq. (2.1) is satisfied when $f_{n_1n_2}\ll g_{n_1n_2}$. Since the two-photon Rabi angles are large even near the threshold of oscillation (because the nonsaturated gain is proportional to the squared intensity), that is $g^2(\bar{n}_1+\bar{n}_2)/\Delta\Gamma\gtrsim1$, the sufficient condition for Eq. (2.1) is

$$\frac{g^2(\bar{n}_1+\bar{n}_2)}{\Delta}\ll\Delta\;, \qquad (2.18)$$

as can be seen from Eqs. (2.15) and (2.16). Therefore, the twin-photon effect occurs when the intermediate-level detuning is much larger than the power-broadening linewidth of the atomic transitions, given by the left-hand side of Eq. (2.18). This is then the condition for the 50% reduction in the intensity-difference noise to be attained.

In summary, we have shown that, for a large variety of oscillators, the noise in the intensity difference between signal and idler photon numbers may be reduced down to 50% below the classical lower bound. The important feature of the gain process is, in any case, the simultaneous creation of twin signal and idler photons. On the other hand, it is clear that excitation to the intermediate state may degrade that noise reduction. This will be discussed in Sec. V. Before that, we show in the next section that the 50% noise reduction in the intracavity field may result in complete elimination of noise at zero frequency for the difference of output intensities.

III. NOISE SPECTRUM OF THE OUTPUT FIELD

In this section we apply the input-output theory of Gardiner and Collet [17] to derive an expression for the noise spectrum $S(\omega)$ of the output signal-idler intensity difference in terms of the intracavity correlation functions. From the results of Sec. II we calculate $S(\omega)$ for a two-photon oscillator in the balanced case (i.e., when the cavity damping times are equal), in a way which remains independent of the particular details of the model, and which reproduces the results for the nondegenerate OPO [8,10].

The intracavity boson operators $a_k(t)$ $(k=1,2)$ obey the usual commutation rules

$$[a_k(t), a_{k'}(t)]=0 \;, \tag{3.1a}$$

$$[a_k(t), a_{k'}^\dagger(t)]=\delta_{k,k'} \;, \tag{3.1b}$$

whereas the commutators between the operators at different times involve the dynamics of the field interaction with the active medium.

The output signal and idler operators are $b_k(t)$, $k=1,2$. We choose the normalization such that

$$I_k = b_k^\dagger b_k$$

is the number of transmitted photons per unit time. This is the important physical quantity, since the measured photocurrent will ultimately display the photocounting field statistics [27]. We thus define the intensity operator of the transmitted signal (idler) beam to be $I_{1(2)}$. According to this normalization, and since there is no causal relationship between the output fields at different times, the commutation rules are

$$[b_k(t), b_{k'}(t')]=0 \;, \tag{3.2a}$$

$$[b_k(t), b_{k'}^\dagger(t')]=\delta_{k,k'}\delta(t-t') \;. \tag{3.2b}$$

The spectrum $S(\omega)$ of the fluctuations in the intensity difference of the signal and idler beams is given by

$$S(\omega)=\int_{-\infty}^{+\infty} e^{i\omega t}[\langle (I_1-I_2)(t)(I_1-I_2)(0)\rangle - \langle I_1 - I_2\rangle^2]dt \;, \tag{3.3}$$

where the average is taken over the steady-state distribution.

In order to calculate $S(\omega)$ as a function of the intracavity field, it is useful to write Eq. (3.3) in normal ordering. Using Eqs. (3.2) and (3.3), we find

$$S(\omega)=\langle I_1 + I_2\rangle + \int_{-\infty}^{+\infty} e^{i\omega t}[\langle :(I_1-I_2)(t)(I_1-I_2)(0):\rangle - \langle I_1 - I_2\rangle^2]dt \;. \tag{3.4}$$

The signal and idler cavity decay rates associated with the transmission through the output mirror M (see Fig. 2) are γ_{M_1} and γ_{M_2}, respectively, whereas γ_1 and γ_2 are the *total* signal and idler cavity decay rates ($\gamma_k \ge \gamma_{M_k}$, $k=1,2$), according to the definition used in Sec. II. They correspond to losses due to transmission through the mirrors, as well as intracavity and mirrors absorption.

When the incoming field at the output mirror M is in the vacuum state, the normally time-ordered correlation functions obey the relation [17]

$$\langle b_k^\dagger(t_1)\cdots b_k^\dagger(t_n) b_k(t_{n+1})\cdots b_k(t_m)\rangle = \gamma_{M_k}^{m/2}\langle \tilde{T}[a_k^\dagger(t_1)\cdots a_k^\dagger(t_n)] T[a_k(t_{n+1})\cdots a_k(t_m)]\rangle \;, \quad k=1,2 \tag{3.5}$$

where $T(\tilde{T})$ is the time (anti)-ordering operator

$$T(O(t_1)O(t_2))=O(t_>)O(t_<) \;,$$

$t_>$ $(t_<)$ is the largest (smallest) time.

From Eqs. (3.4) and (3.5), we may express $S(\omega)$ as a function of the intracavity field moments

$$S(\omega)=\gamma_{M_1}\langle n_1\rangle + \gamma_{M_2}\langle n_2\rangle + 2\int_0^\infty dt\, \cos(\omega t) G(t) \;, \tag{3.6}$$

where $G(t)$ is the normally time-ordered autocorrelation function in the steady state

$$\begin{aligned} G(t) &= \langle :[(\gamma_{M_2}n_2-\gamma_{M_1}n_1)(t) - \langle \gamma_{M_2}n_2-\gamma_{M_1}n_1\rangle][(\gamma_{M_2}n_2-\gamma_{M_1}n_1)(0) - \langle \gamma_{M_2}n_2-\gamma_{M_1}n_1\rangle]:\rangle \\ &= \sum_{k=1,2} (-)^k \gamma_{M_k}\langle a_k^\dagger(0)[(\gamma_{M_2}n_2-\gamma_{M_1}n_1)(t) - \langle \gamma_{M_2}n_2-\gamma_{M_1}n_1\rangle] a_k(0)\rangle \;. \end{aligned} \tag{3.7}$$

The first two terms on the right-hand side of (3.6) correspond to the shot-noise contribution. It is clear that the noise spectrum will be below the shot-noise level whenever the Fourier transform of $G(t)$ becomes negative.

In general, one needs a detailed dynamic analysis of each particular two-photon oscillator under study in order to compute the autocorrelation in Eq. (3.7). This is hardly surprising, since the noise spectrum contains information about the decay rate of the fluctuations which is not found in the steady-state statistical moments obtained in Sec. II for the intracavity field. Nevertheless, in the balanced case, we can still calculate $S(\omega)$ in a general and exact way, for two reasons. First, because the dynamics of the photon-number difference n_1-n_2 is completely independent of the gain process (that is, of n_1+n_2) when $\gamma_1=\gamma_2$, as we saw in Sec. II. Second, when $\gamma_{M_1}=\gamma_{M_2}$, the output intensity difference I_1-I_2 is coupled only to the intracavity difference between photon numbers, so that the correlation function in Eq. (3.7) becomes independent of n_1+n_2. We thus assume for the moment that

$$\gamma_{M_1}=\gamma_{M_2}=\gamma_M , \tag{3.8}$$

$$\gamma_1=\gamma_2=\gamma , \tag{3.9}$$

which, using also Eq. (2.7), leads to

$$G(t)=\gamma_M^2 \sum_{k=1,2} (-)^k \langle a_k^\dagger(0)(n_2-n_1)(t)a_k(0)\rangle . \tag{3.10}$$

This expression may now be explicitly calculated by using the quantum regression theorem [28], which for our purposes may be summarized in the following way:

$$\frac{d}{dt}\langle B(t)\rangle=\langle b(t)\rangle \Longrightarrow \frac{d}{dt}\langle A(0)B(t)C(0)\rangle$$
$$=\langle A(0)b(t)C(0)\rangle , \quad t>0 \tag{3.11}$$

where A, B, b, and C are operators undergoing a Markoffian time evolution. It follows then from (2.5), (3.9), (3.10), and (3.11) that

$$\frac{d}{dt}G(t)=-\gamma G(t) , \tag{3.12}$$

so that

$$G(t)=G(0)e^{-\gamma t} . \tag{3.13}$$

Therefore, using the commutation rules (3.1),

$$G(t)=\gamma_M^2 e^{-\gamma t}[\langle (n_2-n_1)^2\rangle - \langle n_1+n_2\rangle] . \tag{3.14}$$

Since $\langle (n_1-n_2)^2\rangle=\frac{1}{2}\langle n_1+n_2\rangle$ [Eq. (2.8)], we finally obtain the noise spectrum from Eqs. (3.6), (3.7) and (3.14),

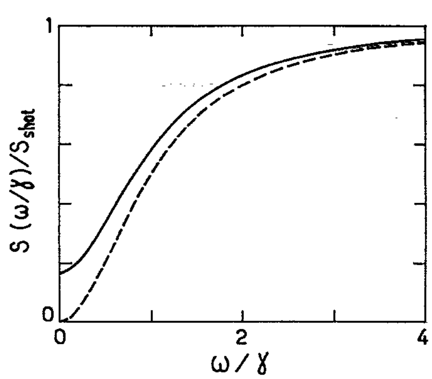

FIG. 3. Spectrum of the fluctuations in the intensity difference, in the balanced case. Full line: $\gamma/\gamma_M=1.2$; dashed line: $\gamma/\gamma_M=1$.

$$S(\omega)=\gamma_M\langle n_1+n_2\rangle \frac{\omega^2+\gamma^2-\gamma\gamma_M}{\omega^2+\gamma^2} . \tag{3.15}$$

In Fig. 3 we plot the spectrum $S(\omega)$ for $\gamma/\gamma_M=1.2$ and $\gamma/\gamma_M=1$. We see that $S(\omega)$ approaches the shot-noise level

$$S_{\rm shot}=\gamma_M\langle n_1+n_2\rangle \tag{3.16}$$

when $\omega \gg \gamma,\gamma_M$.

There is a dip in the spectrum for $\omega \lesssim \gamma,\gamma_M$, representing the nonclassical noise reduction discussed earlier. As pointed out in the literature [6,8] about the nondegenerate OPO, this noise reduction is degraded by any loss not caused by the output mirror M_1. In particular, the maximum noise-reduction factor (which occurs at $\omega=0$) given by

$$\frac{S_{\rm shot}-S(0)}{S_{\rm shot}}=\frac{\gamma_M}{\gamma} , \tag{3.17}$$

decreases from one down to zero when γ/γ_m is increased. This can be simply understood following the reasoning outlined in the Introduction. After a long counting time, many twin-photon pairs leave the cavity, but a fraction of them (equal to γ_M/γ in the case of ideal detectors) will not be detected. Therefore, at $\omega=0$, extra losses introduce noise exactly as does a detector of quantum efficiency $\eta=\gamma_M/\gamma$. One should notice, however, that the two effects are not precisely equivalent, for the presence of extra losses affects the intracavity field, while nonperfect detectors simply change the final outcome of the measurement. In the next section we consider the unbalanced single-ported case, which also leads to exact results in the dissipationless case.

IV. QUANTUM-NOISE QUENCHING AT ZERO FREQUENCY FOR UNEQUAL DECAY RATES

When the cavity decay and/or transmission rates are different, the output intensity difference couples to the intracavity sum of the signal and idler photon numbers. In such cases, the spectrum of fluctuations in the intensity difference $S(\omega)$ depends on the details of the particular gain process involved in the oscillation. In the nondegenerate OPO, for example, the spectrum is sensitive to the pump fluctuations when signal and idler channels are unbalanced.

Nevertheless, we show in this section that the intensity-difference noise is suppressed at zero frequency provided that the output mirror is the unique source of intracavity field damping. As before, our proof is based on general features of two-photon oscillators. In this section we assume thus that

$$\gamma_{M_1}=\gamma_1 , \quad \gamma_{M_2}=\gamma_2 . \tag{4.1}$$

The first consequence of Eq. (4.1) is easily found using Eq. (2.5), which yields the steady-state mean intensities in the most general case and Eq. (3.5),

$$I_1=\gamma_1\langle n_1\rangle=\gamma_2\langle n_2\rangle=I_2 \ . \quad (4.2)$$

The output signal and idler intensities are equal, despite the asymmetry of the cavity. This equation admits an easy physical explanation. Since the active medium generates equal numbers of signal and idler photons, the number of removed photons is also equal in the steady-state regime. Therefore, it is not surprising that Eq. (4.2) holds when all the removed photons are delivered to the output beam.

Next, we show that the spectrum of fluctuations vanishes at zero frequency. Using Eqs. (3.6) and (4.1), we find

$$S(0)=\gamma_1\langle n_1\rangle+\gamma_2\langle n_2\rangle+2\int_0^\infty dt \sum_{k=1,2}(-)^k\gamma_k\langle a_k^\dagger(0)[\gamma_2 n_2(t)-\gamma_1 n_1(t)]a_k(0)\rangle \ . \quad (4.3)$$

Although the correlation function appearing in Eq. (3.6) involves the whole dynamics of each particular system, its dc Fourier component still does not depend on the two-photon gain process. Indeed, using again (2.5) and the quantum regression theorem (3.11), we see that

$$S(0)=\gamma_1\langle n_1\rangle+\gamma_2\langle n_2\rangle-2\int_0^\infty dt \sum_{k=1,2}(-)^k\gamma_k\frac{d}{dt}\langle a_k^\dagger(0)[n_2(t)-n_1(t)]a_k(0)\rangle \ , \quad (4.4)$$

and therefore

$$S(0)=\gamma_1\langle n_1\rangle+\gamma_2\langle n_2\rangle +2\sum_k(-)^k\gamma_k[\langle a_k^\dagger(0)(n_2-n_1)(0)a_k(0)\rangle -\langle n_k\rangle\langle n_{2-}n_1\rangle] \ . \quad (4.5)$$

In this equation, we used that

$$\langle a_1^\dagger(0)(n_1-n_2)(\infty)a_1(0)\rangle=\langle n_1\rangle\langle n_1-n_2\rangle \ ,$$

since correlations go to zero for infinite time intervals.

Finally, using Eqs. (2.5), (2.6) and (3.1), we find the desired result,

$$S(0)=0 \ . \quad (4.6)$$

In the nondegenerate OPO, $S(0)$ was shown to be strongly sensitive to asymmetrical extra losses [6]. Unfortunately, we cannot go beyond the model of a single-ported cavity without considering in detail the particularities of the system (except in the balanced case, as shown in the last section). This is done in the following section, where detailed consideration is given to the role of the intermediate state in two-photon lasers.

V. QUANTUM NOISE IN TWO-PHOTON LASERS

Up to now we have avoided considering any particular model of a two-photon oscillator. By doing that, we have been able to derive general (and exact) identities concerning twin-photon generation in two-photon oscillators. In this section we change our approach in order to study the two-photon laser in detail.

In Sec. II we showed that, in the limit of large intermediate-level detuning [Eq. (2.17)], such a system behaves indeed as a "genuine" twin-photon generator. We now analyze the effect of a finite detuning Δ on both the intracavity and output fields. We calculate the first-order correction in the inverse quadratic power of Δ to the general two-photon-like noise spectrum derived in Sec. III, in the balanced case. We base our treatment on the linear-noise approximation [29], instead of the exact nonlinear calculations of the preceding sections.

Since the atomic decay is an incoherent random process which introduces extra field fluctuations, it is usually assumed that the fundamental condition for twin-photon generation is

$$\Delta\gg\Gamma \quad (5.1)$$

instead of Eq. (2.18), where Γ is the atomic decay rate. In fact, we will show in a forthcoming paper that the fluctuations in the intensity difference become sensitive to the decay of the *intermediate level* when it is tuned closer to resonance. However, the atomic decay is irrelevant to the intensity difference provided that the intermediate state is not populated. This is the *sufficient* condition expressed by Eq. (2.18): we can show that Eq. (5.1) holds in this limit provided that the two-photon laser or maser is not well below the threshold of oscillation (when the field is essentially in the vacuum state). On the other hand, as will be shown in this section, the noise reduction in the intensity difference is degraded when Eq. (2.18) is violated even for very small values of Γ.

We consider a highly saturated two-photon transition, which corresponds to large two-photon Rabi angles,

$$\theta_R=\frac{g^2(\bar{n}_1+\bar{n}_2)}{\Delta\Gamma}\gg 1 \ , \quad (5.2)$$

where g is the atom-field coupling constant (assumed to be the same for the $e\to i$ and $i\to f$ transitions). The condition of small atomic decay rate [Eq. (5.1)] follows from Eqs. (2.18) and (5.2). We neglect terms of order $1/\theta_R$ (and thus of order Γ/Δ) in the master equation (2.14), while still taking into account intermediate-state population effects in its lowest order. We thus write Eq. (2.14), up to first order in $g^2(n_1+n_2)/\Delta^2$, and in the balanced case ($\gamma_1=\gamma_2=\gamma$) as

$$\dot{P}_{n_1n_2}=R[-(g_{n_1n_2}+f_{n_1n_2})P_{n_1n_2}+f_{n_1-1,n_2}P_{n_1-1,n_2} +g_{n_1-1,n_2-1}P_{n_1-1,n_2-1}] \ , -\gamma[(n_1+n_2)P_{n_1n_2}-(n_1+1)P_{n_1+1,n_2} -(n_2+1)P_{n_1,n_2+1}] \ , \quad (5.3)$$

where

$$g_{n_1n_2}=\frac{2(n_1+1)(n_2+1)}{(n_1+n_2+2)^2}\left[1-\frac{g^2}{\Delta^2}(n_1+n_2+2)\right], \quad (5.4)$$

$$f_{n_1n_2}=\frac{2g^2(n_1+1)}{\Delta^2} . \quad (5.5)$$

In the saturated low-decay-rate regime of oscillation, each excited atom has a non-negligible probability of leaving a pair of twin photons before decaying, resulting in a photon generation rate of the order of the atomic excitation rate R. The steady-state mean intensities are thus of order

$$\bar{n}_1,\bar{n}_2\sim R/\gamma . \quad (5.6)$$

Far above the threshold of oscillation ($R/\gamma \gg 1$), we expect, based on general grounds, the photon number distribution to be sharply peaked—with a width of order $(R/\gamma)^{1/2}$—around the average value. Following Van Kampen's approach [29], we expand the master equation (5.3) in powers of $(R/\gamma)^{1/2}$ to obtain the semiclassical laser equation (macroscopic law) and a linear Fokker-Planck equation for the fluctuations. We define the noise operators ξ_1 and ξ_2 and the averages ϕ_1 and ϕ_2 in the following way:

$$n_1=\left[\frac{R}{\gamma}\right]\phi_1+\left[\frac{R}{\gamma}\right]^{1/2}\xi_1 , \quad (5.7)$$

$$n_2=\left[\frac{R}{\gamma}\right]\phi_2+\left[\frac{R}{\gamma}\right]^{1/2}\xi_2 , \quad (5.8)$$

where, guided by the above considerations, we assume that

$$\phi_1,\phi_2,\xi_1,\xi_2\sim 1 . \quad (5.9)$$

We outline the basic steps of the expansion in Appendix A. Corresponding to the zero-order terms in the width of the photon distribution, the semiclassical equations are (see also Ref. [19])

$$\frac{d\phi_1}{d\tau}=\frac{2\phi_1\phi_2}{(\phi_1+\phi_2)^2}[1-\beta(\phi_1+\phi_2)]-\phi_1+2\beta\phi_1 , \quad (5.10)$$

$$\frac{d\phi_2}{d\tau}=\frac{2\phi_1\phi_2}{(\phi_1+\phi_2)^2}[1-\beta(\phi_1+\phi_2)]-\phi_2 , \quad (5.11)$$

where $\tau=\gamma t$ and $\beta=g^2R/\Delta^2\gamma$.

The steady-state solution of Eqs. (5.10) and (5.11) is, to first order in β,

$$\phi_1=\tfrac{1}{2}(1+\beta) , \quad (5.12)$$

$$\phi_2=\tfrac{1}{2}(1-\beta) . \quad (5.13)$$

The resulting linear Fokker-Planck equation for the probability distribution $\pi(\xi_1,\xi_2,\tau)=P_{n_1,n_2}(t)$ is

$$\frac{\partial\pi(\xi_1,\xi_2,\tau)}{\partial\tau}=-\sum_{ij}A_{ij}\frac{\partial}{\partial\xi_i}(\xi_j\pi)+\tfrac{1}{2}\sum_{ij}B_{ij}\frac{\partial^2\pi}{\partial\xi_i\partial\xi_j} , \quad (5.14)$$

where

$$A=-1-\frac{\beta}{2}\begin{bmatrix}1 & 1\\ -3 & 1\end{bmatrix} , \quad (5.15)$$

and

$$B=\begin{bmatrix}1 & \frac{1}{2}\\ \frac{1}{2} & 1\end{bmatrix}+\beta\begin{bmatrix}1 & -\frac{1}{2}\\ -\frac{1}{2} & -1\end{bmatrix} . \quad (5.16)$$

From (5.14) we find, up to first order in β, the steady-state moments

$$\langle\xi_1\rangle=\langle\xi_2\rangle=0 , \quad (5.17)$$

$$\langle\xi_1^2\rangle=\tfrac{1}{2}+\tfrac{7}{8}\beta , \quad (5.18a)$$

$$\langle\xi_2^2\rangle=\tfrac{1}{2}-\tfrac{5}{8}\beta , \quad (5.18b)$$

and

$$\langle\xi_1\xi_2\rangle=\tfrac{1}{4}-\tfrac{3}{8}\beta . \quad (5.18c)$$

The results displayed in Eqs. (5.12), (5.13), and (5.17)–(5.18c) justify, *a posteriori*, the ansatz [Eqs. (5.7)–(5.9)] which underlies the expansion of the master equation.

From Eqs. (5.7), (5.8), (5.12), (5.13), (5.18a), and (5.18b), we derive the following results for the intracavity fluctuations:

$$\frac{\langle(n_1-\langle n_1\rangle)^2\rangle}{\langle n_1\rangle}=1+\tfrac{3}{4}\beta , \quad (5.19a)$$

$$\frac{\langle(n_2-\langle n_2\rangle)^2\rangle}{\langle n_2\rangle}=1-\tfrac{1}{4}\beta , \quad (5.19b)$$

displaying two interesting features of the saturated far-above-threshold two-photon laser.

(i) In the large detuning limit [Eq. (2.18)], the photon-number dispersion for each mode follows a Poissonian statistics, like the usual "one-photon" laser far above threshold.

(ii) For a finite detuning, although the effective gain in the upper mode increases [at the expense of the lower one, see Eqs. (5.12) and (5.13)], the corresponding photon-number statistics becomes super-Poissonian. Besides, there is a small nonclassical noise reduction for the lower mode.

We are mainly interested in the noise in the difference of photon numbers. From Eqs. (5.7), (5.8), (5.12), (5.13), and (5.18), we find

$$\frac{\langle(n_1-n_2-\langle n_1-n_2\rangle)^2\rangle}{\langle n_1+n_2\rangle}=\tfrac{1}{2}+\beta . \quad (5.20)$$

As expected, the noise grows from the large detuning value obtained in Sec. II, as the intermediate state is tuned closer to resonance.

A similar effect occurs in the spectrum of the fluctuations in the output intensity difference, which is calculated within the linear-noise approximation and up to first order in β in Appendix B.

The resulting noise spectrum is

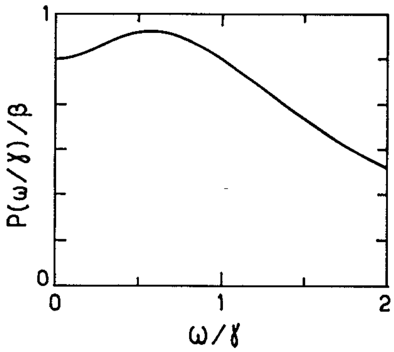

FIG. 4. First-order correction to the spectrum of the fluctuations, when $\gamma/\gamma_M=1.2$.

$$S(\omega)=\gamma_M\langle n_1+n_2\rangle\left[1-\frac{\gamma\gamma_M}{\gamma^2+\omega^2}+P(\omega)\right], \quad (5.21)$$

where $P(\omega)$ is the first order correction to the two-photon spectrum,

$$P(\omega)=\beta\frac{\gamma\gamma_M(\gamma^2+3\omega^2)}{(\gamma^2+\omega^2)^2} . \quad (5.22)$$

The function $P(\omega)/\beta$ is plotted in Fig. 4 as a function of ω/γ, for the case $\gamma/\gamma_M=1.2$. It displays a nearly constant noise level within the cavity bandwidth.

VI. CONCLUSION

When the cavity damping times are equal, the dynamics of the intensity difference in two-photon oscillators becomes completely independent of the gain mechanism, which allows one to derive its noise spectrum in a general and exact way. Therefore, it is not surprising that such different systems as the two-photon laser and the nondegenerate OPO may generate output fields with identical intensity-difference noise spectra.

Furthermore, we have shown that the twin-photon effect in two-photon lasers is not degraded by the population (and incoherent decay) of the upper *resonant* level. This fact may in principle open a prospect for the generation of intense correlated laser beams.

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APPENDIX A: VAN KAMPEN EXPANSION OF THE MASTER EQUATION FOR THE TWO-PHOTON LASER

We expand Eq. (5.3) around the average photon numbers to derive a linear Fokker-Planck equation for the fluctuations defined in Eqs. (5.7) and (5.8). Our derivation is based on the general method developed by Van Kampen for treating nonlinear stochastic processes [29].

Equation (5.3) may be written as

$$\begin{aligned}\frac{\dot{P}_{n_1n_2}}{\gamma}=\frac{R}{\gamma}[&(E_1^{-1}E_2^{-1}-1)g_{n_1n_2}P_{n_1n_2}\\&+(E_1^{-1}-1)f_{n_1n_2}P_{n_1n_2}]\\&+(E_1-1)n_1P_{n_1n_2}+(E_2-1)n_2P_{n_1n_2} ,\end{aligned} \quad (A1)$$

where E_k is the operator changing n_k into n_k+1. In terms of the new continuous variables ξ_1 and ξ_2, E_k may be written as a differential operator given by

$$E_k=1+\left[\frac{R}{\gamma}\right]^{-1/2}\frac{\partial}{\partial\xi_k}+\frac{1}{2}\left[\frac{R}{\gamma}\right]^{-1}\frac{\partial^2}{\partial\xi_k^2}+\cdots . \quad (A2)$$

It follows that

$$\begin{aligned}E_1^{-1}E_2^{-1}-1=&-\left[\frac{R}{\gamma}\right]^{-1/2}\left[\frac{\partial}{\partial\xi_1}+\frac{\partial}{\partial\xi_2}\right]\\&+\frac{1}{2}\left[\frac{R}{\gamma}\right]^{-1}\left[\frac{\partial}{\partial\xi_1}+\frac{\partial}{\partial\xi_2}\right]^2\\&+O((R/\gamma)^{-3/2}) .\end{aligned} \quad (A3)$$

Since n_k depends on the time through ϕ_k [see Eqs. (5.7) and (5.8)], the relation between the time derivatives of $\pi(\xi_1,\xi_2,\tau)=P_{n_1n_2}(t)$ and of $P_{n_1n_2}$ itself is given by

$$\frac{\dot{P}_{n_1n_2}}{\gamma}=\frac{\partial\pi}{\partial\tau}-\left[\frac{R}{\gamma}\right]^{1/2}\left[\frac{d\phi_1}{d\tau}\frac{\partial\pi}{\partial\xi_1}+\frac{d\phi_2}{d\tau}\frac{\partial\pi}{\partial\xi_2}\right], \quad (A4)$$

where $\tau=\gamma t$.

We now expand the coefficients in Eq. (A1) to obtain, using Eqs. (5.4)–(5.8),

$$g_{n_1n_2}=\frac{2\phi_1\phi_2}{(\phi_1+\phi_2)^2}\left\{1-\beta(\phi_1+\phi_2)+\left[\frac{R}{\gamma}\right]^{-1/2}\left[\left[\frac{\xi_1}{\phi_2}+\frac{\xi_2}{\phi_2}-2\frac{\xi_1+\xi_2}{\phi_1+\phi_2}\right][1-\beta(\phi_1+\phi_2)]-\beta(\xi_1+\xi_2)\right]\right\}+O((R/\gamma)^{-1}) , \quad (A5a)$$

and

$$f_{n_1n_2}=2\beta[\phi_1+(R/\gamma)^{-1/2}\xi_1]+O((R/\gamma)^{-1}) , \quad (A5b)$$

where

$$\beta=\frac{g^2}{\Delta^2}\frac{R}{\gamma} . \quad (A5c)$$

Using the definitions in Eqs. (5.7) and (5.8) and substituting the expansions in Eqs. (A2)–(A5) into Eq. (A1), we find

$$\frac{\partial\pi}{\partial\tau}-\left[\frac{R}{\gamma}\right]^{1/2}\left[\frac{\partial\pi}{\partial\xi_1}\frac{d\phi_1}{d\tau}+\frac{\partial\pi}{\partial\xi_2}\frac{d\phi_2}{d\tau}\right]$$
$$=-\left[\frac{R}{\gamma}\right]^{1/2}\left[\frac{2\phi_1\phi_2}{(\phi_1+\phi_2)^2}[1-\beta(\phi_1+\phi_2)]\left[\frac{\partial}{\partial\xi_1}+\frac{\partial}{\partial\xi_2}\right]\pi+2\beta\phi_1\frac{\partial\pi}{\partial\xi_1}-\left[\phi_1\frac{\partial\pi}{\partial\xi_1}+\phi_2\frac{\partial\pi}{\partial\xi_2}\right]\right]$$
$$-\frac{2\phi_1\phi_2}{(\phi_1+\phi_2)^2}[1-\beta(\phi_1+\phi_2)]\left[\frac{\partial}{\partial\xi_1}+\frac{\partial}{\partial\xi_2}\right]\left[\frac{\xi_1}{\phi_1}+\frac{\xi_2}{\phi_2}-2\frac{\xi_1+\xi_2}{\phi_1+\phi_2}\right]\pi$$
$$+\frac{\phi_1\phi_2}{(\phi_1+\phi_2)^2}[1-\beta(\phi_1+\phi_2)]\left[\frac{\partial}{\partial\xi_1}+\frac{\partial}{\partial\xi_2}\right]^2\pi-2\beta\frac{\partial}{\partial\xi_1}(\xi_1\pi)+\beta\phi_1\frac{\partial^2\pi}{\partial\xi_1^2}+\sum_{k=1,2}\frac{\partial}{\partial\xi_k}(\xi_k\pi)+\frac{1}{2}\sum_{k=1,2}\phi_k\frac{\partial^2\pi}{\partial\xi_k^2}\ . \tag{A6}$$

Collecting the terms of order $(R/\gamma)^{1/2}$ and R/γ we find, respectively, the semiclassical equations (5.10) and (5.11) and the Fokker-Planck equation, whose final form, Eqs. (5.14)–(5.16), is obtained by replacing the steady-state semiclassical solutions, ϕ_1 and ϕ_2, into the coefficients of Eq. (A6).

APPENDIX B: OUTPUT-NOISE SPECTRUM OF THE TWO-PHOTON LASER WITH A FINITE DETUNING

We calculate the spectrum of the fluctuations in the output intensity difference for the two-photon laser, up to first order in $\beta=(g^2/\Delta^2)(R/\gamma)$. In the balanced case, the normally time-ordered correlation function of Eq. (3.7) becomes

$$G(t)=\gamma_M^2\sum_{k=1,2}(-)^k\langle a_k^\dagger(0)[(n_2-n_1)(t)-\langle n_2-n_1\rangle_S]a_k(0)\rangle\ , \tag{B1}$$

where the average is taken over the steady-state photon distribution, whose width is of order $(R/\gamma)^{1/2}$. We thus need to consider only the values of $(n_2-n_1)(t)$ differing from the steady-state mean $\langle n_2-n_1\rangle_S$ by an amount of order $(R/\gamma)^{1/2}$. Using the noise operator defined in Eqs. (5.7) and (5.8), we thus set

$$(n_2-n_1)(t)=\langle n_2-n_1\rangle_S+\left[\frac{R}{\gamma}\right]^{1/2}(\xi_2-\xi_1)(t)$$

in Eq. (B1), so that

$$G(t)=G_1(t)-G_2(t)\ , \tag{B2}$$

where

$$G_k(t)=\gamma_M^2(R/\gamma)^{1/2}[\langle a_1^\dagger(0)\xi_k(t)a_1(0)\rangle-\langle a_2^\dagger(0)\xi_k(t)a_2(0)\rangle]\ . \tag{B3}$$

As in Secs. III and IV, we may now use the regression theorem in order to calculate $G(t)$. From the Fokker-Planck equation (5.14)–(5.16), we have

$$\frac{d}{d(\gamma t)}\begin{bmatrix}\langle\xi_1\rangle\\ \langle\xi_2\rangle\end{bmatrix}=\left[-1-\frac{\beta}{2}\begin{bmatrix}1 & 1\\ -3 & 1\end{bmatrix}\right]\begin{bmatrix}\langle\xi_1\rangle\\ \langle\xi_2\rangle\end{bmatrix}\ . \tag{B4}$$

Therefore, by the quantum regression theorem, we find from Eqs. (B3) and (B4)

$$\frac{d}{d(\gamma t)}\begin{bmatrix}G_1(t)\\ G_2(t)\end{bmatrix}=\left[-1-\frac{\beta}{2}\begin{bmatrix}1 & 1\\ -3 & 1\end{bmatrix}\right]\begin{bmatrix}G_1(t)\\ G_2(t)\end{bmatrix}\ . \tag{B5}$$

Integrating Eq. (B5) up to first order in β, we find

$$\begin{bmatrix}G_1(t)\\ G_2(t)\end{bmatrix}=e^{-\gamma t}\left[1-\frac{\beta}{2}\gamma t\begin{bmatrix}1 & 1\\ -3 & 1\end{bmatrix}\right]\begin{bmatrix}G_1(0)\\ G_2(0)\end{bmatrix}\ , \tag{B6}$$

so that

$$G_1(t)-G_2(t)=e^{-\gamma t}[G_1(0)-G_2(0)-2\beta\gamma tG_1(0)]\ . \tag{B7}$$

From Eqs. (B2), (B3), and (B7), and using the commutation rules given by Eq. (3.1), which in terms of the noise operators may be written as

$$\left[a_k^\dagger(t),\left[\frac{R}{\gamma}\right]^{1/2}\xi_{k'}(t)\right]=[a_k^\dagger(t),n_{k'}(t)]$$
$$=-\delta_{k,k'}a_k^\dagger(t)\ , \tag{B8}$$

we finally get

$$G(t)=-\gamma_M^2\frac{R}{2\gamma}(1-2\beta+\beta\gamma t)e^{-\gamma t}\ . \tag{B9}$$

Computing the Fourier transform of the above function, and substituting into the general expression (3.6), we obtain the final result for the noise spectrum $S(\omega)$, displayed in Eqs. (5.21) and (5.22).

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[1] For recent reviews see R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987); S. Reynaud, Ann. Phys. (Paris) **15**, 63 (1990).

[2] B. Yurke, Phys. Rev. A **29**, 408 (1984); M. J. Collet and C. W. Gardiner, *ibid*. **30**, 1386 (1984); M. J. Collet and D. F. Walls, *ibid*. **32**, 2887 (1985).

[3] L. -A. Wu, M. Xiao, and H. J. Kimble, J. Opt. Soc. Am. B **4**, 1465 (1987).

[4] S. Reynaud, Europhys. Lett. **4**, 427 (1987).

[5] S. Reynaud, C. Fabre, and E. Giacobino, J. Opt. Soc. Am. B **4**, 1520 (1987).

[6] A. S. Lane, M. D. Reid, and D. F. Walls, Phys. Rev. A **38**, 788 (1988).

[7] G. Björk and Y. Yamamoto, Phys. Rev. A **37**, 125 (1988).

[8] C. Fabre, E. Giacobino, A. Heidmann, and S. Reynaud, J. Phys. (Paris) **50**, 1209 (1989).

[9] A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and C. Camy, Phys. Rev. Lett. **59**, 2555 (1987); T. Debuisschert, S. Reynaud, A. Heidmann, E. Giacobino, and C. Fabre, Quantum Opt. **1**, 3 (1989); C. D. Nabors and R. M. Shelby, Phys. Rev. A **42**, 556 (1990).

[10] J. Mertz, T. Debuisschert, A. Heidmann, C. Fabre, and E. Giacobino, Opt. Lett. **16**, 1234 (1991).

[11] J. Mertz, A. Heidmann, C. Fabre, E. Giacobino, and S. Reynaud, Phys. Rev. Lett. **64**, 2897 (1990).

[12] J. J. Snyder, E. Giacobino, C. Fabre, A. Heidmann, and M. Ducloy, J. Opt. Soc. Am. B **7**, 2132 (1990); P. R. Tapster, S. F. Seward, and J. G. Rarity, Phys. Rev. A **44**, 3266 (1991).

[13] R. J. Horowicz, M. Pinard, and S. Reynaud, Opt. Commun. **61**, 142 (1987); M. Brambilla, F. Castelli, L. A. Lugiato, F. Prati, and G. Strini, *ibid*. **83**, 367 (1991).

[14] R. Graham, Phys. Rev. Lett. **52**, 117 (1984).

[15] M. S. Zubairy, Phys. Lett. **87A**, 162 (1982).

[16] P. A. Maia Neto, L. Davidovich, and J. M. Raimond, Phys. Rev. A **43**, 5073 (1991).

[17] C. W. Gardiner and M. J. Collet, Phys. Rev. A **31**, 3761 (1985).

[18] C. A. Blockley and D. F. Walls, Phys. Rev. A **43**, 5049 (1991).

[19] A. W. Boone and S. Swain, Quantum Opt. **1**, 27 (1989).

[20] M. Golubev and I. V. Sokolov, Zh. Eksp. Teor. Fiz. **87**, 408 (1984) [Sov. Phys.—JETP **60**, 234 (1984)]; M. C. Teich and B. E. A. Saleh, J. Opt. Soc. Am. B **2**, 275 (1985); Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **34**, 4025 (1986); S. Machida, Y. Yamamoto, and Y. Itaya, Phys. Rev. Lett. **58**, 1000 (1987); Y. Yamamoto and S. Machida, Phys. Rev. A **35**, 5114 (1987); M. A. Marte, H. Ritsch, and D. F. Walls, Phys. Rev. Lett. **61**, 1093 (1988); J. Bergou, L. Davidovich, M. Orszag, C. Benkert, M. Hillery, and M. O. Scully, Phys. Rev. A **40**, 5073 (1989); F. Haake, S. M. Tan, and D. F. Walls, *ibid*. **40**, 7121 (1989); C. Benkert, M. O. Scully, J. Bergou, L. Davidovich, M. Hillery, and M. Orszag, *ibid*. **41**, 2756 (1990).

[21] D. Meschede, H. Walther, and G. Mueller, Phys. Rev. Lett. **54**, 551 (1985); G. Rempe, H. Walther, and N. Klein, *ibid*. **58**, 353 (1987); P. Filipowicz, J. Javanainen and P. Meystre, Phys. Rev. A **34**, 3077 (1986); M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, Phys. Rev. Lett. **59**, 1899 (1987); L. Davidovich, J. M. Raimond, M. Brune, and S. Haroche, Phys. Rev. A **36**, 3771 (1987); G. Rempe, F. Schmidt-Kaler, and H. Walther, Phys. Rev. Lett. **64**, 2783 (1990).

[22] J. Gea-Banacloche, Phys. Rev. Lett. **59**, 543 (1987).

[23] M. O. Scully and M. S. Zubairy, Phys. Rev. A **35**, 752 (1987); J. Bergou, M. Orszag, and M. O. Scully, *ibid*. **38**, 768 (1988); J. Krause and M. O. Scully, *ibid*. **36**, 1771 (1987); M. O. Scully, K. Wódkiewicz, M. S. Zubairy, J. Bergou, N. Lu, and J. Meyer ter Vehn, Phys. Rev. Lett. **60**, 1832 (1988); J. Bergou, C. Benkert, L. Davidovich, M. O. Scully, S. -Y. Zhu, and M. S. Zubairy, Phys. Rev. A **42**, 5544 (1990); C. Benkert, M. O. Scully, and M. Orszag, *ibid*. **42**, 1487 (1990); for experimental results see M. Winters, J. Hall, and P. Toschek, Phys. Rev. Lett. **65**, 3116 (1990).

[24] M. Sargent III, M. O. Scully, and W. E. Lamb, *Laser Physics* (Addison-Wesley, Reading, MA, 1974).

[25] See, for example, Z. C. Wang and H. Haken, Z. Phys. **55**, 361 (1984); **56**, 77 (1984); **56**, 83 (1984).

[26] Shi-Yao Zhu and Xiao-Shen-Li, Phys. Rev. A **36**, 3889 (1987); Shi-Yao Zhu and M. O. Scully, *ibid*. **38**, 5433 (1988).

[27] H. J. Carmichael, J. Opt. Soc. Am. B **4**, 1588 (1987).

[28] M. Lax, Phys. Rev. **157**, 213 (1967); **172**, 350 (1968); see also C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1983).

[29] N. G Van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1981).