Comment on "Multiphoton-ionization transition amplitudes and the Keldysh approximation"

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We show that the approach of Trombetta, Basile, and Ferrante [Phys. Rev. A **40**, 2774 (1989)] does not establish the Keldysh ansatz as a well-defined approximation. In particular, it in effect replaces a vanishing contribution to the ionization rate by a nonvanishing one.

Several analyses have led to the conclusion that the Keldysh approximation in strong-field ionization is best regarded as an ansatz rather than a rigorously justified approximation.[1–3] Trombetta, Basile, and Ferrante claim to establish the Keldysh ansatz "as a well-defined theoretical model" by using "consistently the hypothesis under which the approximation should be valid."[4] We wish to focus attention here on the crucial step that leads Trombetta, Basile, and Ferrante to this conclusion, and to show that their claims are unjustified.

The crucial step in Ref. 4 is the replacement of expression (17) by (18). This step amounts to replacing the free evolution of the initial state by the bound-state evolution, i.e.,

$$\begin{aligned} U_0(t)|i\rangle &= \exp(-ip^2t/2m\hbar)|i\rangle \\ &\to \exp[-i(p^2/2m+V)t/\hbar]|i\rangle \\ &= \exp[iI_0t/\hbar]|i\rangle \ , \end{aligned} \tag{1}$$

where I_0 is the ionization potential of the initial bound state. This approximation is based on the assumption that "the initial state is very weakly bound."[4] However, a weakly bound initial state does not mean that V is small compared with $p^2/2m$, as assumed in Ref. 4, but simply that V may be small compared with the atom-field interaction.

In fact, the replacement (1) is a very poor approximation, regardless of how small V may be. To see this, consider the long-time limit in which a transition rate is defined. Note that, *whereas Eq. (18) of Ref. 4 gives a finite ionization rate (the Keldysh result), the expression (17) yields exactly zero for that rate.* Indeed, as the authors themselves observe, A_{fi} as given by their Eq. (17) is identical to $\langle f|U_F^\dagger(t)|i\rangle$; this term, being bounded for all times, does not contribute to the transition rate, defined as $\lim_{t\to\infty}|A_{fi}(t)|^2/t$. (See, for instance, Ref. 2.) Only for *finite* terms can such a term play an important role, as shown in Ref. 5.

We therefore believe the claims in Ref. 4 to be unfounded. We wish to remark also that, contrary to what is stated in Ref. 4, there is no arbitrariness in "calling after Keldysh" the first two terms of the expansion in the binding potential V (and not just the first, which gives a zero transition rate in the long-time limit), as done in Refs. 1 and 2. In fact, those terms yield exactly the Keldysh ansatz, with the addition to the final state of corrections due to the binding potential. This should actually yield an improved ansatz. In any case, it is easy to see that the derivations in Refs. 1 and 2 would be equally valid if the final state were taken to be a plane wave instead of an eigenstate of the atomic Hamiltonian. This procedure is actually followed in Ref. 3.

We fully agree that the Keldysh ansatz is a valuable benchmark in the theory of strong-field interactions. However, serious problems concerning its applicability and theoretical justification have been raised.[1–3,5,6] We believe the paper of Trombetta, Basile, and Ferrante[4] has not met its objective of establishing the Keldysh ansatz as a well-defined theoretical model, and that the justification of that ansatz remains an open problem.

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