

Probing entanglement in phase space: signature of GHZ states in the Wigner function

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Abstract

We show that the value at the origin of phase space of the Wigner function of entangled GHZ states, involving three modes of the electromagnetic field either in the same cavity or in different ones, is a signature of the quantum character of those states. This multimode Wigner function can be directly measured by a simple extension of a method previously proposed by two of the present authors. A single measurement on an atom which crosses the three modes is enough to distinguish between quantum mechanics and local hidden variable theories.

Keywords: Wigner function, entanglement, GHZ, cavity QED, quantum measurement

1. Introduction

Phase-space representations of quantum states have been important tools for exploring the connections between quantum and classical physics. Even though it is not possible to define true phase-space probability distributions in quantum mechanics, quasi-probabilities allow one to write down quantum averages of products of operators, written in some previously specified ordering, as classical-like integrals of the quasi-probability distributions multiplied by c -numbers which replace the operators. Different distributions are obtained depending on the ordering adopted for the operator products. Among the several possible quantum phase-space distributions, the Wigner function, originally proposed by Wigner in 1932 [1], and corresponding to symmetric ordering [2], has distinguished itself as a very useful tool for the representation of quantum states, and for the discussion of the classical limit of quantum mechanics. Indeed, the Wigner function is the only distribution which leads to the correct marginal distributions, for any direction of integration in phase space [3], a property shared by any bona fide classical probability distribution. In addition, it is a bounded distribution. It is thus ideal for analysing the classical limit of quantum mechanics; in particular, it has been successfully applied to the discussion of the quantum–classical transition for classically chaotic systems [4]. Furthermore, it has been realized in recent years that the Wigner function of the electromagnetic field can be actually measured, either through tomographic methods [5, 6], or through convolutions obtained

by photon counting [7], or even through direct measurements in cavity QED [8]. The measurement of the Wigner functions of trapped ions [9] and of atomic beams [10] has also been experimentally demonstrated. In the former case, the Wigner function of the vibrational Fock state with $n = 1$ was obtained, and the negative values of this distribution around the origin of phase space, which are a signature of the quantum nature of this state, were clearly exhibited.

The Wigner function is also useful for studying another intriguing quantum feature, namely entanglement. Banaszek and Wódkiewicz [11] showed that we can build a Bell-type inequality using four points of a Wigner distribution corresponding to two modes of the electromagnetic field. This result has been recently extended to the n -mode case [12] using the inequalities proposed in [13]. More recently, Kim and Lee have also related a test of quantum nonlocality for cavity fields to the measurement of a two-mode Wigner function [14]. Methods for measuring this distribution were proposed in [14, 15].

Striking nonlocality tests can be obtained with the GHZ states discovered by Greenberger, Horne and Zeilinger [16]. These three-qubit entangled states allow the distinction between quantum mechanics and local realism with just a single realization of a particular experiment. In this sense it is a rare case in quantum mechanics where we do not need to explore its probabilistic properties to test it (even though in practice one would have to repeat the experiment in order to build up statistics, due to experimental imperfections, such as the inefficiency of detection). Experimental realizations

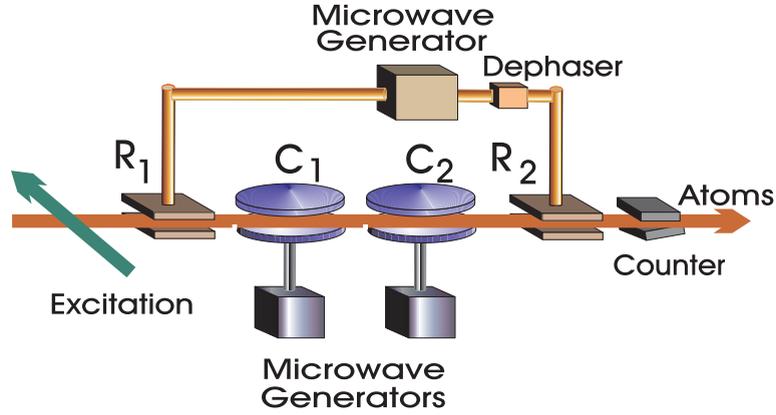


Figure 1. Scheme for measuring a multimode Wigner function. The figure illustrates the two-cavity case.
(This figure is in colour only in the electronic version, see www.iop.org)

of these states have been made for spin- $\frac{1}{2}$ particles [17], photon polarization states [18] or even two-level atoms [19]. The entangled qubits may also be realized by modes of the electromagnetic field in one or more cavities, which can be in the vacuum or the one-photon state. Entangled states of the electromagnetic field in two modes of a single cavity have already been realized experimentally [20].

In this paper, we show that the value at the origin of phase space of the Wigner function corresponding to an entangled state of three modes of the electromagnetic field carries a unique signature of the quantum character of the state. That value is equal to the lower bound of the Wigner function for a quantum state, while local realism would lead to a value equal to the upper bound of the same function. The relevance of this result is enhanced by the fact that this Wigner function can be experimentally measured in a very simple way. Furthermore, we show that it is possible to characterize the quantum entanglement with a single realization. This result extends therefore to three-particle entanglement the ideas presented in [11, 14].

In section 2 we show how to extend the idea of direct measurement of the Wigner function of one-mode cavity fields presented in [8] to the case when N modes are involved either in the same cavity or in different ones. In section 3 we show how to build GHZ states of cavity modes in the same or in different cavities, and relate the probing of these states with the scheme discussed in the previous section. We show then that a single value of the three-mode Wigner function, namely the one at the origin of phase space, is enough to characterize the quantum nature of the state. Our conclusions are summarized in section 4.

2. Measuring a multimode Wigner function in cavity QED

The Wigner function corresponding to one mode of the electromagnetic field in a state described by the density operator $\hat{\rho}$ can be written as [21]

$$W(\alpha, \alpha^*) = 2 \text{Tr}[\hat{\rho} \hat{D}(\alpha, \alpha^*) e^{i\pi \hat{a}^\dagger \hat{a}} \hat{D}^{-1}(\alpha, \alpha^*)]. \quad (1)$$

Here, \hat{a} and \hat{a}^\dagger are the field mode annihilation and creation operators (which satisfy the commutation relation $[a, a^\dagger] = 1$),

α is a complex number and $\hat{D}(\alpha, \alpha^*) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$ is the displacement operator: it represents the action of a classical current on the electromagnetic field. When applied to the vacuum, it yields a coherent state with amplitude α . For microwave cavities, this operator describes the action of a microwave generator. The operator $\exp(i\pi \hat{a}^\dagger \hat{a})$ is the parity operator: note that $\exp(i\pi \hat{a}^\dagger \hat{a})|n\rangle = (-1)^n |n\rangle$, where $|n\rangle$ is a Fock state.

The Wigner function corresponding to N orthogonal modes such that $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$, is given by

$$W(\{\alpha_i, \alpha_i^*\}) = 2^N \text{Tr} \left[\hat{\rho} \prod_{i=1}^N \hat{D}_i(\alpha_i, \alpha_i^*) e^{i\pi \hat{a}_i^\dagger \hat{a}_i} \hat{D}_i^{-1}(\alpha_i, \alpha_i^*) \right], \quad (2)$$

where $\hat{D}_i(\alpha_i, \alpha_i^*) = \exp(\alpha_i \hat{a}_i^\dagger - \alpha_i^* \hat{a}_i)$. In both (1) and (2) the normalization is chosen so that $\int \{d^2\alpha_i/\pi\} W(\alpha_i, \alpha_i^*) = 1$, where $\{d^2\alpha_i/\pi\} = \prod_i (d^2\alpha_i/\pi)$ and $d^2\alpha_i \equiv d(\text{Re } \alpha_i) d(\text{Im } \alpha_i)$.

Equations (1) and (2) show that the Wigner function at a given point of the phase space is given by the quantum average of the displaced parity of the quantum state. In other words, the value of the Wigner function at any point $\alpha = \{\alpha_1, \alpha_2, \dots\}$ in phase space can be measured by first displacing each mode i with a complex amplitude α_i , and then measuring the sum of the parities for the several modes of the resulting field. These modes may be in the same or in different cavities. As shown below, the method once proposed in [8] can be easily generalized to contemplate these situations.

We consider first the measurement of the Wigner function of modes with the same frequency in different cavities. These high- Q cavities (C_1, C_2, \dots in figure 1), containing the fields to be measured, are connected to microwave generators and placed between two low- Q cavities (R_1 and R_2 in figure 1), which are fed by the same microwave generator (in recent experiments, the two zones R_1 and R_2 correspond to two low- Q modes of the same cavity, which contains the field to be probed, see for instance [23]). A dephaser between this generator and R_2 allows one to change the relative phase η between the fields in R_1 and R_2 . The high- Q cavities are made of superconducting mirrors facing each other. This system is crossed by a velocity-selected atomic beam, such that an atomic transition $e \leftrightarrow g$ is resonant with the fields in R_1 and

R_2 . Even though the field in these two cavities is very weak in typical experimental realizations (average photon number equal to unity), it may nevertheless be treated as a classical field, due to the large dissipation rate [22]. Under the action of this field, the atom undergoes a $\pi/2$ rotation in the subspace spanned by the states e and g : $|e\rangle \rightarrow [|e\rangle + e^{i\eta}|g\rangle]/\sqrt{2}$, and $|g\rangle \rightarrow [-e^{-i\eta}|e\rangle + |g\rangle]/\sqrt{2}$, with η replaced by 0 in R_1 . On the other hand, the atom interacts dispersively with the fields in C_i , with a detuning δ , during a time t_{int} (the detuning and the interaction time are assumed here to be the same for all cavities). Static electric fields applied across the mirrors of these cavities control the atomic transition frequency through the Stark effect, and allow one to change both δ and t_{int} for each cavity.

The dephasings corresponding to the states $|e\rangle$ and $|g\rangle$ are implemented respectively, in each cavity i , by the unitary operators $\hat{T}_e^{(i)}(\phi) = \exp[i\phi(\hat{a}_i^\dagger \hat{a}_i + 1)]$ and $\hat{T}_g^{(i)}(\phi) = \exp(-i\phi \hat{a}_i^\dagger \hat{a}_i)$, where $\phi = (\Omega^2/\delta)t_{\text{int}}$ is the one-photon phase shift, and the Rabi frequency Ω measures the coupling between the atom and the cavity modes (assumed to be the same for all cavities). Before the atom crosses the system, the fields in the cavities C_i are displaced by the complex amplitudes α_i , by turning on the corresponding microwave generators for some specified time intervals. The overall density operator is transformed then into $\hat{\rho}' = \left[\prod_i \hat{D}_i(\alpha_i, \alpha_i^*) \right] \hat{\rho} \left[\prod_i \hat{D}_i^{-1}(\alpha_i, \alpha_i^*) \right]$. Finally the internal state of the atom is detected by a field ionization device (the ‘counter’ in figure 1). This experiment is repeated many times, starting each run with the same field in the several cavities, and the probabilities P_e and P_g of detecting the probe atom in states e or g are determined. It is easy to show, following the same procedure as in [8], that

$$\Delta P = \text{Re} \left\{ e^{i(\eta+N\phi)} \times \text{Tr} \left[\hat{\rho} \prod_{i=1}^N \hat{D}_i(\alpha_i, \alpha_i^*) e^{2i\phi \hat{a}_i^\dagger \hat{a}_i} \hat{D}_i^{-1}(\alpha_i, \alpha_i^*) \right] \right\}, \quad (3)$$

where $\Delta P = P_g - P_e$. This leads, with the choice $\phi = \pi/2$ and $\eta = -N\pi/2$, to the desired multimode Wigner function:

$$\Delta P = W(\{-\alpha_i, -\alpha_i^*\})/2^N. \quad (4)$$

A similar scheme can be applied to the measurement of the Wigner function of modes with different frequencies, in the same or in different cavities, by properly tuning the atomic transition to the modes involved, through the Stark shift induced by the static field applied to the mirrors. This procedure, especially suited to quasi-degenerate modes, has been adopted recently in order to build entangled states of two modes in the same cavity [20]. In order to measure the Wigner function, a single microwave generator connected to the cavity would be sequentially tuned to the modes to be displaced, before the probing atom crosses the cavity. Alternatively, for a couple of quasi-degenerate modes and a third mode farther away, one could use a three-level scheme, so that the transition $e \leftrightarrow i$ would be coupled dispersively with two of the modes (by Stark shifting the transition frequency), while the transition $g \leftrightarrow i$ would be coupled with the other one. Again, when the dispersive shifts of levels e and g become equal to $\pi/2$ per photon, the measurement of the probabilities of finding the

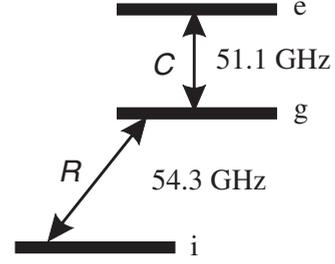


Figure 2. Level configuration for measuring the Wigner function in the subspace $\{|0\rangle, |1\rangle\}$. The frequency values correspond to the experiment described in [19].

atom in levels e and g leads to the Wigner function of the probed modes.

For our purposes, another procedure can be adopted, which is easier to implement experimentally, since we will be interested in the measurement of the Wigner function at the origin of phase space for states that contain either one or zero photons. It is possible then to replace the dispersive interaction by a 2π resonant one, as done in [23]. This is advantageous because the $\pi/2$ phase shift per photon, needed in the dispersive case, requires a relatively long interaction time, as compared with the available cavity damping times. The relevant atomic levels (e , g and i) are shown in figure 2. The cavity field is resonant with the $e \leftrightarrow g$ transition, while i is a reference level (the $g \leftrightarrow i$ transition is non-resonant with the fields in the high- Q cavities). Stark shifting may put the transition $e \leftrightarrow g$ successively into resonance with the modes involved. The atom is now prepared in a superposition state $(|g\rangle + |i\rangle)/\sqrt{2}$. If there is one photon in the mode, the state $|g\rangle$ undergoes a full 2π Rabi cycle, thus acquiring a π phase shift, exactly as if the interaction were dispersive. The difference from the previous approach is that here the π phase shift is associated with just one of the atomic levels, while before it was distributed between levels e and g ($\pi/2$ for each). It is easy to show that (4) also applies here, with the choice $\eta = 0$, and with ΔP now equal to $P_g - P_i$.

3. Three-mode Wigner function and GHZ states

We show now that the value of the Wigner function at the origin of phase space distinguishes between quantum mechanics and local hidden variable theories. We start by recalling the argument given by David Mermin in [24], for spin- $\frac{1}{2}$ particles.

Let $|+_z\rangle$ and $|-_z\rangle$ be the eigenstates of the Pauli operator $\hat{\sigma}_z$, with eigenvalues $+1$ and -1 respectively, and consider the GHZ state:

$$|\text{GHZ}\rangle = \frac{|+_z, +_z, +_z\rangle - |-_z, -_z, -_z\rangle}{\sqrt{2}}, \quad (5)$$

which implies that if we measure the three spins in the z direction, we find them all either up or down. This state is an eigenstate of the operators $\hat{\sigma}_{x_1} \hat{\sigma}_{y_2} \hat{\sigma}_{y_3}$, $\hat{\sigma}_{y_1} \hat{\sigma}_{x_2} \hat{\sigma}_{y_3}$ and $\hat{\sigma}_{y_1} \hat{\sigma}_{y_2} \hat{\sigma}_{x_3}$, with eigenvalue $+1$. Therefore, if one measures the spins of any two particles in the y direction, the spin of the third one in the x direction must be well defined, and such that the product of the results must yield $+1$. Also, if one measures the spins of two of the particles in the x and y direction, the y

component of the spin of the third one is well defined. Since this determination of the x and y components of the spin of the third particle is performed without any local interaction with it (only the spin components of the other two particles are measured), a local realistic theory would consider these two spin components ‘elements of reality’, that is, it would assume that the complete set of values $m_{y_1}, m_{x_1}, m_{y_2}, m_{x_2}, m_{y_3}, m_{x_3}$ is well defined for the three particles. Since

$$(m_{x_1} m_{y_2} m_{y_3}) = 1,$$

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one must have

$$(m_{x_1} m_{y_2} m_{y_3})(m_{y_1} m_{x_2} m_{y_3})(m_{y_1} m_{y_2} m_{x_3}) = 1.$$

The values $m_{y_1}, m_{y_2}, m_{y_3}$ appear twice in the last product, which means that, if a local hidden variable theory holds, the equality $m_{x_1} m_{x_2} m_{x_3} = 1$ must be true. However, the state we are considering is also an eigenstate of the operator $\hat{\sigma}_{x_1} \hat{\sigma}_{x_2} \hat{\sigma}_{x_3}$ with eigenvalue -1 . Therefore, in contradiction with the hidden variable result, quantum mechanics predicts that for this GHZ state the product of the measured values for the three spins along the x direction should be equal to -1 . This shows that a single measurement should be enough to discard one of the two theories, making the GHZ state a powerful tool to test quantum nonlocality.

In order to apply these considerations to modes of the field in one or more cavities, we remark first that the parity of the corresponding states is defined in a two-dimensional space (odd or even). Thus, it can be used to test quantum features of GHZ-type states. It is interesting to notice that, as shown in section 2, the mean parity of a multimode state is exactly the quantity measured at the origin of phase space for the associated Wigner function.

We define now, for the three-mode system, the corresponding $\hat{\sigma}_x$ operators in the basis of the field states $|0_i\rangle$ and $|1_i\rangle$:

$$\hat{\sigma}_{x_i} = |0_i\rangle\langle 0_i| - |1_i\rangle\langle 1_i|. \quad (6)$$

The eigenvectors of the corresponding $\hat{\sigma}_{z_i}$ operators will be $|+z\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-z\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. The GHZ state is given by

$$\begin{aligned} |\text{GHZ}\rangle &= (|+z, +z, +z\rangle - |-z, -z, -z\rangle)/\sqrt{2} \\ &= \frac{1}{2}(|-x, -x, -x\rangle + |-x, +x, +x\rangle \\ &\quad + |+x, -x, +x\rangle + |+x, +x, -x\rangle) \\ &= \frac{1}{2}(|1, 1, 1\rangle + |1, 0, 0\rangle + |0, 1, 0\rangle + |0, 0, 1\rangle). \end{aligned} \quad (7)$$

This symmetrical state is an eigenstate of the product of operators $\hat{\sigma}_{x_1} \hat{\sigma}_{x_2} \hat{\sigma}_{x_3}$ with eigenvalue -1 . On the other hand, in the subspace spanned by the states $|0\rangle, |1\rangle$, the parity operator coincides exactly with $\hat{\sigma}_{x_i}$ defined by (6). Therefore, the Wigner function may be written in the following way, in the subspace spanned by the states $\{|0_i\rangle, |1_i\rangle\}$:

$$W(\{\alpha_i = 0\}) = 2^3 \text{Tr}\{\hat{\rho} \hat{\sigma}_{x_1} \hat{\sigma}_{x_2} \hat{\sigma}_{x_3}\}. \quad (8)$$

For the state (7), this yields -8 , which is the minimum possible value for the Wigner function. This result should be contrasted

with that predicted by a local hidden variable theory, which would lead to the value $+1$ for the measurement of $\hat{\sigma}_{x_1} \hat{\sigma}_{x_2} \hat{\sigma}_{x_3}$, and therefore to the value $+8$ for $W(\{\alpha_i = 0\})$.

In general, to obtain the value of the Wigner function at the origin one needs to repeat the experiment many times. However, if the goal is just to distinguish between the two possible extremes (± 8) of the Wigner function, each one corresponding to one of the possible eigenvalues of $\hat{\sigma}_{x_1} \hat{\sigma}_{x_2} \hat{\sigma}_{x_3}$ (± 1), then only one realization is enough. Each single measurement will have the same result, as these possibilities represent mutually exclusive events. In other words, if the state is an odd (or even) state, the mean value of the parity operator (the average of $\hat{\sigma}_{x_1} \hat{\sigma}_{x_2} \hat{\sigma}_{x_3}$) coincides with any single measurement of this observable (since the state is an eigenvalue of the measured operator). If $W(0, 0, 0)$ is equal to $+8$ (-8), the atoms are always measured in state $|g\rangle$ ($|e\rangle$). Therefore, quantum mechanics predicts that the final state of the probing atom in a single realization is $|e\rangle$, while a local hidden variable theory would predict that the atom should exit the system in state $|g\rangle$.

We conclude this section by presenting a possible way to build the GHZ state discussed above. One starts with the three modes in the vacuum state, and sends two atoms through the cavities. The first atom is prepared in the state $(|e\rangle + |g\rangle)/\sqrt{2}$. One assumes that, if the atom is in the state $|e\rangle$, it undergoes a π rotation, so that $(|e\rangle + |g\rangle)|0\rangle/\sqrt{2} \rightarrow (|1\rangle + |0\rangle)|g\rangle/\sqrt{2}$. The atom thus transfers its coherence to the field in the first mode. The atom is then reexcited to the state $(|e\rangle + |g\rangle)/\sqrt{2}$ ($\pi/2$ pulse) and interacts in the same way with the second mode, going through the third mode without interacting and exiting the system in the state $|g\rangle$. The first two modes are thus left in the state $(|0\rangle + |1\rangle)_1(|0\rangle + |1\rangle)_2/2$. The second atom is also sent in the state $(|e\rangle + |g\rangle)/\sqrt{2}$, and, as in the experiment described at the end of the last section, undergoes a 2π rotation if it is in state $|e\rangle$ and there is one photon in the mode, for the first two modes. It is then subjected to a $\pi/2$ classical pulse, with phase chosen so that now $|g\rangle \rightarrow (|e\rangle + |g\rangle)/\sqrt{2}$, $|e\rangle \rightarrow (|e\rangle - |g\rangle)/\sqrt{2}$. The resulting entangled state is then $(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|e\rangle + (|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2)|g\rangle$. Finally, the atom leaves one photon in mode 3 if it is in state $|e\rangle$ (π rotation). The second atom thus exits the system also in the state $|g\rangle$, and leaves the three modes in the state (7).

4. Conclusion

It has been recently remarked by some authors that the Wigner function can be used to characterize the entanglement of two or more subsystems. We have shown here that the quantum character of a GHZ state can be recognized in the Wigner function corresponding to this state in a very simple way: it suffices to check the value of this distribution at the origin of phase space. Quantum mechanics and local hidden variable theories predict opposite signs for this value.

It is interesting to observe that the proposed method for measuring the multimode Wigner function does not change the number of photons in the modes. It is therefore very much related to a quantum non-demolition measurement of the photon number [23, 25]. It is easy to show [8] that an undetected atom does not change the trace on the right-hand side of (3). This implies that the proposed scheme is quite insensitive to the detection efficiency of the atomic counters

(of the order of $40 \pm 10\%$ in recent experiments [19]), as long as this efficiency is the same for both states. The measurement accuracy does depend however on the detector's selectivity, that is, the ability to distinguish between the two atomic states.

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