

Decoherence, Pointer Engineering, and Quantum State Protection

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(Received 4 September 2000)

We present a proposal for protecting states against decoherence, based on the engineering of pointer states. We apply this procedure to the vibrational motion of a trapped ion, and show how to protect qubits, squeezed states, approximate phase eigenstates, and superpositions of coherent states.

DOI: 10.1103/PhysRevLett.86.4988

PACS numbers: 03.65.Yz, 03.67.Lx, 42.50.Dv, 42.50.Vk

It is well known that the interaction of a quantum system with its surrounding environment may lead to quantum entanglement between system and environment, and to an irreversible loss of information on the system. Which set of states is less sensitive to entanglement (“pointer states”) depends on the concrete form of the interaction between system and environment [1]. This interaction rapidly entangles any superposition of these states with unobservable states of the environment in such a way that the reduced density matrix of the system is indistinguishable from a statistical mixture. On the other hand, pointer states are more stable, minimizing the rate at which the system loses purity. The decoherence process by which coherent superpositions of pointer states get transformed into statistical mixtures is at the heart of the quantum theory of measurement [2], and plays an essential role in the classical limit of quantum mechanics [3].

Fighting decoherence has become a major challenge in the last years, motivated by recent progress in the theory of quantum information processing, which relies on the possibility of preserving quantum coherence [4,5]. It is also of interest to high-precision frequency measurements in ion traps [6]. Several strategies have been devised. They include quantum error correction schemes [7], feedback implementations [8,9], the realization of qubits in symmetric subspaces decoupled from the environment [10], and dynamical decoupling techniques [11].

In linear ion traps, by far the most important decoherence effect is the one associated with the motional state [12,13]. In the present paper, we show that decoherence in the vibrational motion of a trapped ion can be suppressed by generating, through the techniques of “reservoir engineering” [14], artificial reservoirs associated with properly chosen pointer observables, which have the states to be preserved as their eigenstates, and which dominate over other dissipation processes. We exemplify this procedure by showing how to protect from decoherence several kinds of nonclassical states.

Under the hypotheses of Markovian dynamics and complete positivity [15], a master equation describing the reduced dynamics of a system interacting with its environment can be written in the Lindblad form

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}\hat{\rho} \equiv \sum_i (\gamma_i/2) (2\hat{c}_i\hat{\rho}\hat{c}_i^\dagger - \hat{c}_i^\dagger\hat{c}_i\hat{\rho} - \hat{\rho}\hat{c}_i^\dagger\hat{c}_i), \quad (1)$$

where $\hat{\rho}$ is the reduced density operator of the system in the interaction picture, and we have neglected the unitary evolution term $-(i/\hbar)[\hat{H}, \hat{\rho}]$. The operators \hat{c}_i are closely related to the system operators present in the interaction Hamiltonian and γ_i measures the strength of the system-environment coupling. In this case, the pointer basis is given by the set of all the eigenstates of the operator \hat{c}_i . If all \hat{c}_i are Hermitian, their eigenstates are steady states of the master equation (1), which are not affected by the environment [10]. If instead the \hat{c}_i 's are not Hermitian, the states of the pointer basis will not necessarily be steady states of (1). Our strategy for the protection of a specific quantum state against the environment consists in engineering, via external driving of the system, a system-environment coupling, so that the net effect is to add to the master equation (1) an extra term, yielding

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}\hat{\rho} + (\Gamma_{\text{eng}}/2) (2\hat{d}\hat{\rho}\hat{d}^\dagger - \hat{d}^\dagger\hat{d}\hat{\rho} - \hat{\rho}\hat{d}^\dagger\hat{d}). \quad (2)$$

The operator \hat{d} is chosen so that the state one wants to protect is the *only* steady state of Eq. (2) without the environment term $\mathcal{L}\hat{\rho}$. For $\Gamma_{\text{eng}} \gg \gamma$, the steady state of (2) will be very close to the state to be protected (if the state is not unique, the term $\mathcal{L}\hat{\rho}$ could still induce transitions between the steady states). Besides, *any* state of the system will decay into the state chosen to be protected. Therefore, this is also a procedure for preparing quantum states in the presence of decoherence.

In the following we apply this method to the one-dimensional motion of the center-of-mass of an ion confined in an electromagnetic trap. The reservoir engineering process will be implemented by letting the ion interact with several laser beams of adequate frequencies and intensities, which are quasis resonant to an electronic transition $|1\rangle \leftrightarrow |2\rangle$ of frequency ω_{21} .

For our purposes it is important to consider the ion to be in the regime of resolved sidebands, given by $\nu_u \gg \Gamma, |\Omega_n|$, where Γ is the electronic energy decay

rate, ν_u is the ion vibrational frequency along axis \mathbf{u} , and Ω_n is the (complex) Rabi frequency corresponding to laser n , tuned to the k th red vibrational sideband of the ion. Under these conditions the interaction term corresponding to laser n , can be described, in the interaction picture, by [16]

$$\begin{aligned}\hat{H}_{\text{int}}(t) &= \hbar g(\hat{A}_{21}\hat{d} + \hat{A}_{12}\hat{d}^\dagger) \\ &= \frac{1}{2} \hbar \Omega_n (i\eta_n)^k \hat{A}_{21} \hat{f}_k(\hat{a}^\dagger \hat{a}) \hat{a}^k + \text{H.c.},\end{aligned}\quad (3)$$

where g is taken to be real and

$$\hat{f}_k(\hat{a}^\dagger \hat{a}) = e^{-\eta_n^2/2} \sum_{l=0}^{\infty} \frac{(-1)^l \eta_n^{2l}}{l!(l+k)!} (\hat{a}^\dagger)^l \hat{a}^l. \quad (4)$$

Here the operators \hat{a} and \hat{A}_{21} are the annihilation operator of a quantum of the ionic vibrational motion and the electronic flip operator, respectively. The quantity $\eta_n = \sqrt{\hbar(\mathbf{k}_n \cdot \mathbf{u})^2/2M\nu_u}$ is the Lamb-Dicke parameter with respect to the direction of vibration, fixed by the unit vector \mathbf{u} . M is the ion's mass and \mathbf{k}_n is the wave vector of laser n . It is assumed that the Lamb-Dicke parameters corresponding to directions orthogonal to \mathbf{u} are much smaller than one, as it is the case in linear traps.

In the Born-Markov limit, the time evolution of the vibronic density operator $\hat{\rho}$ for the direction \mathbf{u} (say x) is

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= -\frac{i}{\hbar} [\hat{H}_{\text{int}}, \hat{\rho}] + \frac{\Gamma}{2} (2\hat{A}_{12}\tilde{\rho}\hat{A}_{21} - \hat{A}_{22}\hat{\rho} - \hat{\rho}\hat{A}_{22}) \\ &\quad + \mathcal{L}\hat{\rho},\end{aligned}\quad (5)$$

where the second term corresponds to spontaneous emission with energy relaxation rate Γ , and

$$\tilde{\rho} = \frac{1}{2} \int_{-1}^1 ds W(s) e^{i\eta(\hat{a} + \hat{a}^\dagger)s} \hat{\rho} e^{-i\eta(\hat{a} + \hat{a}^\dagger)s} \quad (6)$$

accounts for changes of the vibrational energy along the x direction due to spontaneous emission with angular distribution $W(s)$ (tilde over operators indicates interaction picture). Here $\eta = (\omega_{21}/c)\sqrt{\hbar/2M\nu_x}$. The last term of (5) describes the coupling of the center-of-mass motion to the environment, and has the general form (1). However, the precise form of this dissipation term is not important for our purposes.

The matrix elements of (5) with respect to the electronic basis yield the equations

$$\dot{\hat{\rho}}_{11} = -ig(\hat{d}^\dagger \hat{\rho}_{21} - \hat{\rho}_{12} \hat{d}) + \Gamma \tilde{\rho}_{22} + \mathcal{L}\hat{\rho}_{11}, \quad (7)$$

$$\dot{\hat{\rho}}_{22} = -ig(\hat{d} \hat{\rho}_{12} - \hat{\rho}_{21} \hat{d}^\dagger) - \Gamma \hat{\rho}_{22} + \mathcal{L}\hat{\rho}_{22}, \quad (8)$$

$$\dot{\hat{\rho}}_{12} = -ig(\hat{d}^\dagger \hat{\rho}_{22} - \hat{\rho}_{11} \hat{d}^\dagger) - \frac{\Gamma}{2} \hat{\rho}_{12} + \mathcal{L}\hat{\rho}_{12}. \quad (9)$$

We assume now that the decay rate Γ is by far the largest rate in the system. One can then eliminate $\hat{\rho}_{12}$ and $\hat{\rho}_{22}$ adiabatically. Since $\mathcal{L} \propto \gamma$, one gets

$$\hat{\rho}_{12} = -(2ig/\Gamma) (\hat{d}^\dagger \hat{\rho}_{22} - \hat{\rho}_{11} \hat{d}^\dagger) [1 + O(\gamma/\Gamma)]. \quad (10)$$

Replacing (10) into (8), we see that $\hat{\rho}_{22} \approx O[(g/\Gamma)^2] \hat{\rho}_{11}$. We can therefore approximate the reduced density operator for the vibrational mode, $\hat{\rho}_v = \hat{\rho}_{11} + \hat{\rho}_{22}$, by $\hat{\rho}_{11}$. We get then, adding up (7) and (8), after replacing $\hat{\rho}_{12}$ by (10) and neglecting terms proportional to $\hat{\rho}_{22}$ (except those proportional also to Γ) and γ/Γ (we verified numerically that these are indeed excellent approximations),

$$\begin{aligned}\dot{\hat{\rho}}_v &= \frac{2g^2}{\Gamma} [2\hat{d}\hat{\rho}_v\hat{d}^\dagger - \hat{d}^\dagger\hat{d}\hat{\rho}_v - \hat{\rho}_v\hat{d}^\dagger\hat{d}] \\ &\quad - \Gamma(\hat{\rho}_{22} - \tilde{\rho}_{22}) + \mathcal{L}\hat{\rho}_v.\end{aligned}\quad (11)$$

We will base our considerations on this equation. The first term on the right-hand side (rhs) has the form (1). This is the ‘‘engineered reservoir,’’ with a decay constant $\Gamma_{\text{eng}} = 4g^2/\Gamma$.

Neglecting terms of $O(\eta^4)$ in the expansion of the second term on the rhs of Eq. (11), one can show that its contribution is $\sim(\eta^2/5)(4g^2/\Gamma)\hat{\rho}_v$, that is, $(2\eta^2/5)$ multiplied by the engineered-reservoir term. For $\eta \approx 0.25$, this yields a factor of $\sim 1/40$, a small correction, which is, however, fully taken into account in our numerical simulations. Therefore, the action of the engineered reservoir will be the dominant one as long as $\Gamma_{\text{eng}} \gg \gamma$.

In recent experiments with trapped ions, random fields seem to play an important role in the decoherence process [13]. Their effect may also be described by (1). We write the random field as $E(t) = \mathcal{E}^{(+)}(t) \times \exp(-i\nu t) + \mathcal{E}^{(-)}(t) \exp(i\nu t)$, and the interaction Hamiltonian in the rotating wave approximation as [13]: $\hat{H} = -\mu[\mathcal{E}^{(+)}(t)\hat{a}^\dagger + \mathcal{E}^{(-)}(t)\hat{a}]$. Iterating the equation of motion for the density operator, and using that for stationary fields [17] $\langle \mathcal{E}^{(-)}(t)\mathcal{E}^{(-)}(t') \rangle = \langle \mathcal{E}^{(+)}(t)\mathcal{E}^{(+)}(t') \rangle = 0$, we get, in the Markovian limit $\langle \mathcal{E}^{(+)}(t)\mathcal{E}^{(-)}(t') \rangle = 2D\delta(t-t')$,

$$\begin{aligned}\dot{\hat{\rho}} &= (\mu^2 D/\hbar^2) (2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a} \\ &\quad + 2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger),\end{aligned}\quad (12)$$

which corresponds to an infinite temperature thermal reservoir (thermal photon number $N_T \rightarrow \infty$, dissipation rate $\gamma \rightarrow 0$, γN_T constant). Both random fields and thermal reservoirs will be considered in our simulations.

In order to protect a state $|\psi\rangle$, we look for \hat{d} such that $\hat{d}|\psi\rangle = \lambda|\psi\rangle$ with $\lambda = 0$, and make sure that there is no further eigenstate $|\phi\rangle$ of \hat{d} such that $[\hat{d}, \hat{d}^\dagger]|\phi\rangle = 0$. This implies that $|\psi\rangle$ is the only pure steady state of (2) without $\mathcal{L}\hat{\rho}$. Since $\Gamma_{\text{eng}} \gg \gamma$, this yields a good approximation of the corresponding steady state of (2).

We consider first the protection of the class of states $|\psi\rangle = \sum_{n=0}^N c_n |n\rangle$, where $|n\rangle$ is an energy eigenstate of the vibrational motion and $c_n \neq 0$. The operator $\hat{d} = \hat{g}(\hat{n})\hat{a} + \hat{h}(\hat{n})$ has $|\psi\rangle$ as its only eigenstate with eigenvalue $\lambda = 0$ provided the eigenvalues of $\hat{g}(\hat{n})$ and $\hat{h}(\hat{n})$ fulfill the constraints $g(m) = -[h(m)/\sqrt{m+1}]c_m/c_{m+1}$ ($m = 0, \dots, N-1$) and N is the first zero of $h(m)$. For this case, $|\psi\rangle$ is the only steady state of (2).

Inspection of Eqs. (3) and (4) shows that the operator $\hat{g}(\hat{n})\hat{a}$ can be engineered by driving the ion with N laser fields, tuned to the first vibrational sideband of the ion. The values of the Rabi frequencies Ω_n of the N lasers are given by the following system of N linear equations ($m = 0, \dots, N - 1$):

$$\sum_{n=1}^N e^{-\eta_n^2/2} \eta_n \Omega_n \sum_{l=0}^m \frac{(-1)^l \eta_n^{2l}}{l!(l+1)!} \frac{m!}{(m-l)!} = \frac{ih(m)}{\sqrt{m+1}} \frac{c_m}{c_{m+1}}, \quad (13)$$

where the Lamb-Dicke parameters η_n depend on the orientation of the laser fields with respect to the x direction. The operator $\hat{h}(\hat{n})$ is constructed by driving the ion with two laser fields resonant with the electronic transition, one of them propagating orthogonal to the x axis (say y direction, with $\eta_y \ll 1$). The Rabi frequencies of these two lasers are related by $\Omega_y = -\Omega_x L_N(\eta_x^2)$, where $L_N(x)$ is a Laguerre polynomial of order N (η_x should not be too large for the first zero of $h(m)$ to occur at $m = N$). From (13) it follows that our method is sensitive only to relative intensity and phase fluctuations, since state selection is determined by ratios of Rabi frequencies. These quantities are much easier to stabilize than the absolute intensities and phases of the laser beams.

An important representative of the class of states presented above is the ‘‘qubit’’ state $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$. This state can be generated and protected against the action of an external reservoir with just three lasers, two of them, with Rabi frequencies Ω_x, Ω_y , used to build up $h(m)$, and the third one with Rabi frequency Ω_1 , so as to satisfy Eq. (13). Choosing $\eta_1 = \eta_x = \eta$, these Rabi frequencies must satisfy the following condition:

$$\frac{\eta \Omega_x}{i \Omega_1} = -\frac{c_1}{c_0} \quad \text{and} \quad \frac{\Omega_y}{i \Omega_1} = e^{-\eta^2/2} \frac{c_1}{c_0} \frac{1 - \eta^2}{\eta}. \quad (14)$$

In this case, $\Gamma_{\text{eng}} = \eta^2 \Omega_1^2 / \Gamma$ in Eq. (2). In order for the corresponding reservoir to win over the environment reservoir, one needs $\eta^2 \Omega_1^2 / \Gamma \gg \gamma$, but at the same time $\Gamma, \Omega_1 \ll \nu$ and $\Gamma \gg \eta \Omega_1$. These requirements are satisfied if $\Gamma \approx 4$ MHz, $\Omega_1 \approx 2$ MHz, $\eta = 0.2$, $\nu \approx 20$ – 30 MHz, as long as $\gamma \ll \eta^2 \Omega_1^2 / \Gamma \approx 40$ kHz. Figure 1a displays the fidelity $F(t) = \text{Tr}\{\hat{\rho}(0)\hat{\rho}(t)\}$, with the ion initially in the vibrational state $|\psi\rangle = \frac{1}{2}(|0\rangle + |1\rangle)$ [all our numerical simulations are obtained from Eq. (5), and we always assume the ion to be initially in the electronic ground state]. Both a thermal and a random field reservoir have been considered. As can be seen, the system rapidly reaches a steady state with fidelity very close to unity.

One should remark that feedback procedures [9] do not protect states involving superpositions of $|0\rangle$ and $|1\rangle$, since the loss of one photon by the state $|1\rangle$ completely erases any phase information about the original state. Our procedure, however, works very well in this case.

Another interesting example is the approximate phase state [18] $|\psi\rangle = (1/\sqrt{N+1}) \sum_{n=0}^N e^{in\phi} |n\rangle$, which can be generated and protected by $N+2$ lasers. Figure 1b displays the fidelity $F(t)$, for $N=3$, and η 's in the range 0.1–0.2.

A class of states which is specially fragile against the action of decoherence is the one formed by mesoscopic

superpositions of coherent states. Under action of an external reservoir, these states decay to a mixture of coherent states in an extremely short time, inversely proportional to the distance between the two states in phase space [3]. Our technique can also be applied to the Schrödinger-cat-like state [19] $|\phi_+\rangle = (|\alpha\rangle + i|-\alpha\rangle)/\sqrt{2}$. Since $|\phi_+\rangle$ has no ‘‘hole’’ in its number distribution, it can be approximated by one of the states $|\psi\rangle$ discussed above. Consequently, one could, with the use of $N+2$ lasers, generate and protect the state $|\psi\rangle = \sum_{n=0}^N c_n |n\rangle$, with the first N coefficients c_n equal to the corresponding coefficients of $|\phi_+\rangle$. One should notice, however, that it is possible in this case to find directly a Lindblad operator \hat{d} , which has the state $|\phi_+\rangle$ as its only eigenstate with zero eigenvalue: $\hat{d} = \mathcal{T} \hat{a} \mathcal{T}^\dagger = e^{i\pi \hat{n}} \hat{a} + i\alpha$. Here \mathcal{T} is the unitary operator $\exp[i\pi \hat{n}(\hat{n}-1)/2] \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$, which yields $|\phi_+\rangle$ when applied to the vacuum. For this choice of \hat{d} , we plot in Fig. 1c the fidelity $F(t)$, for the initial state $|\phi_+\rangle$. An open problem is how to engineer this operator with a finite number of laser beams.

Finally, we describe the protection of a squeezed state. We set $\hat{d} = \hat{a} + \chi \hat{a}^\dagger$, where $\chi = \tanh r$ and r is the squeezing factor. The corresponding setup consists of two lasers along the direction of squeezing, resonant with the first red (laser 1) and the first blue (laser 2) sidebands, and with Rabi frequencies satisfying $\Omega_2/\Omega_1 = \chi$ [14]. The numerical simulation is shown for $r = 0.6$ in Fig. 1d, for a realistic set of parameters ($\eta_1 = \eta_2 = \eta = 0.05$). Higher values of squeezing render our method less effective, since the presence of higher photon numbers would lead to stronger dissipation by the ‘‘natural’’ reservoir.

In conclusion, we have suggested a method for protecting quantum states of the vibrational motion of a trapped ion against decoherence by generating artificial reservoirs which have the states to be protected as pointer states. It differs from dynamical decoupling techniques [11] (which require pulses with durations smaller than the reservoir correlation time) and feedback techniques [9] (which have been proposed for the protection of ‘‘cat-like’’ states). More general pointer states can be generated by applying unitary transformations to the states and operators discussed here. Indeed, the transformed states would still be the sole steady solutions of the master equation with the transformed operators. This is precisely the mechanism which leads to the protection of the states $|\phi_+\rangle$ above, and also of squeezed states, since they are related by unitary transformations to the vacuum, which is the only steady state for a zero-temperature reservoir (for which $\hat{d} = \hat{a}$). Our method is experimentally robust, since it depends only on the stabilization of relative intensities and phases, and leads to the

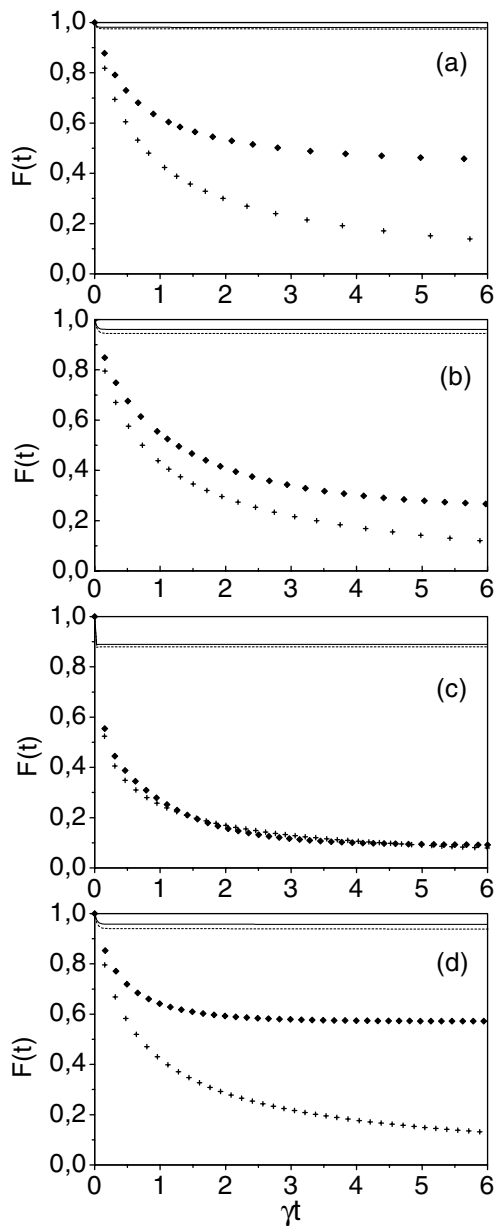


FIG. 1. Time evolution of the fidelity $F(t)$ for several initial nonclassical states, in the presence of a thermal reservoir with $\langle n_{\text{th}} \rangle = 0.5$ (full curves and lozenges) and a random field with $\Gamma_{\text{eng}} = 0$. The full and dashed curves correspond to $\Gamma_{\text{eng}} = 40\gamma$ for (a), (b), and (d), and $\Gamma_{\text{eng}} = 150\gamma$ for (c). The nonclassical states are the following: (a) $|\psi\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$; (b) $|\psi\rangle = \frac{1}{2}\sum_{n=0}^3 |n\rangle$; (c) $|\psi\rangle = (1/\sqrt{2})(|\alpha\rangle + i|-\alpha\rangle)$, with $\alpha^2 = 3$; and (d) vacuum squeezed state with $r = 0.6$.

successful generation and protection of qubits, squeezed states, and approximate phase and Schrödinger-cat-like states, even in the deleterious presence of random electric fields.

We acknowledge the partial support of Conselho Nacional de Desenvolvimento Científico e Tecnológico

(CNPq), Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ), Fundação Universitária José Bonifácio (FUJB), and Programa de Apoio a Núcleos de Excelência (PRONEX).

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