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Teleportation of an atomic state between two cavities using nonlocal microwave fields

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Implementing the ideas of Bennett *et al.* [Phys. Rev. Lett. **70**, 1895 (1993)], we present an experimentally feasible scheme for the teleportation of an unknown atomic state between two high-Q cavities containing a nonlocal quantum superposition of microwave field states. This experiment provides alternative tests of quantum nonlocality involving high-order atomic correlations.

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Quantum nonlocality is one of the most striking predictions of modern physics [2,3]. Two quantum-correlated systems cannot be considered to be independent even if they are far apart. Local hidden variable theories lead to results concerning correlation measurements in contradiction with the quantum-mechanical predictions [3] verified in several experiments [4].

Possible implications of quantum nonlocality have ranged from cryptography [5] to computers [6]. More recently, Bennett *et al.* [1] have shown that an entangled pair of spin-$\frac{1}{2}$ particles could be used, with the addition of information transmitted through a classical channel, to teleport an unknown quantum state from one observer to another. Teleportation, according to their scheme, would involve measurements made by one of the observers on four possible independent entangled states, consisting of the particle to be replicated and one of the spins of the correlated pair. Information on these measurements, transmitted through classical channels to the other observer, would allow him to reconstruct the original state on the second spin of the correlated pair, even though the original state remains necessarily unknown to the first observer, since he disposes of only one particle.

Cavity quantum electrodynamics provides new methods to build and measure nonclassical coherent superpositions of states of the electromagnetic field [8]. It is in particular possible to prepare nonlocal field states simultaneously occupying two cavities [9,10]. We show here that such nonlocal fields can be used to build a "teleportation machine": an atom, sent across the first cavity, has its quantum state replicated on another atom sent across the second cavity. As opposed to previous discussions of this question, we describe in a concrete way the sequence of measurements necessary to teleport the state. We also estimate the efficiency of detection necessary for teleporting a state with a certain precision. Our discussion makes clear the fact that the proposed scheme constitutes a new generation of high-order atomic correlation experiments. Bell's inequalities, as well as the experiments discussed in [9], refer to measurements on correlated pairs of particles. Teleportation involves on the other hand at least three-atom correlations (the scheme proposed in this paper involves actually a four-atom correlation).

A sketch of the teleportation experiment is displayed in Fig. 1. The setup consists of two identical and initially empty high-Q cavities (C_1 and C_2), and three atomic beams (C, A, and B) made of identical two-level atoms (levels $|e\rangle$ and $|g\rangle$). The $e\rightarrow g$ transition is close to resonance with the cavity mode frequency. After switching on beam C, the first atom c of this beam that crosses the two cavities establishes the nonlocal correlation between them. The atom a to be duplicated belongs to beam A, which crosses only C_1. Its state is reconstructed on an atom b of beam B, which crosses only C_2. In practice, e and g must be circular Rydberg levels with adjacent principal quantum numbers. Due to their

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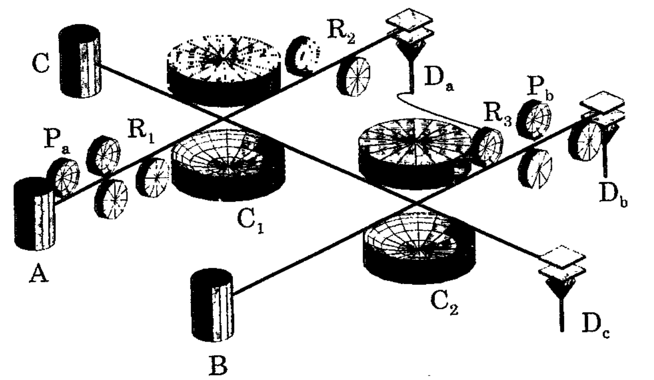

FIG. 1. Sketch of the two-cavity teleportation experiment.

strong coupling to microwaves and their very long radiative decay times, circular levels are ideally suited [11] for preparing and detecting long-lived correlations between atom and field states. Atoms in circular Rydberg states, which can be prepared on each beam at a given time with a well-defined velocity, are counted with high efficiency by state-selective field ionization detectors D_a, D_b, and D_c. By applying timed sequences of pulsed electric fields on the cavity mirrors, and taking advantage of the Stark effect of e and g, the atoms can be tuned in and out of resonance, making the atom-cavity interactions resonant or dispersive during a preset time interval. Auxiliary microwave zones (R_1,R_2 on beam A, R_3 on beam B) play the role of atomic state "polarizers" and "analyzers" and are used to perform the manipulations required by the teleportation scheme. Finally, two other microwave zones P_a on beam A and P_b on beam B are employed to prepare the state to be teleported and to analyze the fidelity of the teleportation process.

The teleportation machine is first prepared by sending across both cavities an atom c in state $|e\rangle$. This atom is made resonant with the cavities and undergoes, on the $e \to g$ transition, a $\pi/2$ pulse in C_1 and a π pulse in C_2. This can be easily achieved by properly setting, through Stark field adjustments, the times during which the atom is resonant with each cavity. The atomic transitions are accompanied by corresponding photon number changes. When c has undergone the first $\pi/2$ pulse, the second cavity is still empty and the "atom $c+C_1$" system is in a state which corresponds to a linear superposition with equal weights of the e and g atomic states correlated to zero and one photon, respectively, in C_1. If c is still in level e after leaving C_1 in its vacuum state, it will, with unit probability, be transferred to g by the π pulse in C_2 and leave a photon in the second cavity. If it emits a photon in C_1 and exits it in level g, it will be unaffected by its coupling with the vacuum in C_2 and will leave the second cavity empty. It is thus easily seen that the atom always exits C_2 in state g, while the field is left in the entangled state

$$|\Psi_c\rangle=(|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2)/\sqrt{2}\ , \tag{1}$$

where the index 1 or 2 refers to the first or second cavity, respectively. The presence of one photon ($|1\rangle$) in either one of the cavities implies that the other is in the vacuum state ($|0\rangle$), with a maximal quantum entanglement between the two possibilities. Once atom c is detected in D_c the "teleportation machine" is ready and one can send across C_1 the atom a to be teleported.

This atom is prepared by the microwave zone P_a in an arbitrary e,g superposition $|\phi_a\rangle=c_e|e_a\rangle+c_g|g_a\rangle$ (supposedly unknown to the observers). The combined "atom a+field" state is then the tensor product of $|\Psi_c\rangle$ and $|\phi_a\rangle$. This product can be conveniently expanded as

$$|\Psi\rangle=\frac{1}{2}[|\Psi^{(+)}\rangle(c_e|1\rangle_2+c_g|0\rangle_2)+|\Psi^{(-)}\rangle(c_e|1\rangle_2-c_g|0\rangle_2)$$
$$+|\Phi^{(+)}\rangle(c_e|0\rangle_2+c_g|1\rangle_2)+|\Phi^{(-)}\rangle(c_e|0\rangle_2-c_g|1\rangle_2)]\ , \tag{2}$$

where we have introduced Bell's basis [13] of the "atom $a+C_1$" states:

$$|\Psi^{(\pm)}\rangle=\frac{1}{\sqrt{2}}\,(|e_a\rangle|0\rangle_1\pm|g_a\rangle|1\rangle_1)\ , \tag{3a}$$

$$|\Phi^{(\pm)}\rangle=\frac{1}{\sqrt{2}}\,(|e_a\rangle|1\rangle_1\pm|g_a\rangle|0\rangle_1)\ . \tag{3b}$$

Each Bell state of "$a+C_1$" is correlated with a certain superposition of one- and zero-photon field states in C_2, which contains information on the state of the atom to be teleported. Bennett *et al.*'s idea [1] transposed to our situation is to perform a measurement on "$a+C_1$" which collapses this system in one of the Bell states, automatically projecting the C_2 field in one of the four combinations appearing in Eq. (2). These combinations are obtained from the initial state $|\phi_a\rangle$ by known unitary transformations of a two-level system in which $|0\rangle_2$ and $|1\rangle_2$ have replaced $|e_a\rangle$ and $|g_a\rangle$. Our teleportation problem is thus twofold: (i) How to perform on "$a+C_1$" a measurement whose eigenstates are given by Eq. 3? This important point was not addressed in [1] and only partially solved in [12]. (ii) How to replicate on an atom b the information contained in the C_2 field state ?

Let us start with the first question, which requires a two-step approach and involves appropriate atomic manipulations in zones R_1 and R_2. These zones are first set so that a undergoes $\pi/2$ pulses in each of them. Furthermore, a is tuned to have a dispersive interaction with the field in C_1. This setup is equivalent to a recently demonstrated Ramsey atomic interferometer [11]. For a given setting of the microwave in R_1 and R_2, the probability for an atom to undergo an $e \to g$ transition exhibits fringes versus the photon number in C_1. Changing this photon number does indeed shift the atomic transition frequency (light shift effect), which translates into a periodic change of the $e \to g$ transition probability induced in the two separated oscillatory field zones R_1 and R_2. By choosing properly the detuning between a and C_1, the phase of the fringes can be shifted by π when the photon number varies by one unit. Moreover, the Ramsey interferometer can be adjusted so that the $e \to g$ transfer probability is one when C_1 is empty (and thus zero when C_1 contains one photon). Since the atom-field interaction is dispersive, the photon number in the cavity always remains unchanged. When a crosses the interferometer, the "$a+C_1$" system thus undergoes the transformations: $|e_a\rangle|0\rangle_1 \to -|g_a\rangle|0\rangle_1$, $|e_a\rangle|1\rangle_1 \to -|e_a\rangle|1\rangle_1$, $|g_a\rangle|0\rangle_1 \to |e_a\rangle|0\rangle_1$, $|g_a\rangle|1\rangle_1 \to |g_a\rangle|1\rangle_1$, which can be derived from the formulas given in Ref. [8]. Applying these transformations to the Bell states of Eq. (3a), one gets

$$|\Psi^{(\pm)}\rangle \to -|g_a\rangle \frac{1}{\sqrt{2}}(|0\rangle_1 \mp |1\rangle_1) , \tag{4a}$$

$$|\Phi^{(\pm)}\rangle \to -|e_a\rangle \frac{1}{\sqrt{2}}(|1\rangle_1 \mp |0\rangle_1) . \tag{4b}$$

State selective detection of atom a by D_a thus indicates whether the $a+C_1$ system is in a "Ψ" or a "Φ" Bell state.

In order to completely determine the $a+C_1$ state, one must now decide between the alternatives left for each possible outcome of the measurement on atom a. This can be done by sending through the same system a second reference atom a', prepared in the $|g\rangle$ state. Fields in P_a and R_1 are now switched off and a' is tuned to interact resonantly with C_1, undergoing a π pulse and leaving the cavity empty if it initially contains one photon. The second zone R_2 still produces a $\pi/2$ pulse. The joint state of the system atom $a'+C_1$ thus evolves in the following way:

$$|g_{a'}\rangle \frac{1}{\sqrt{2}}(|0\rangle_1 \pm |1\rangle_1) \to \frac{1}{\sqrt{2}}(|g_{a'}\rangle \pm |e_{a'}\rangle)|0\rangle_1 \to \begin{cases} |e_{a'}\rangle|0\rangle_1 \\ |g_{a'}\rangle|0\rangle_1 . \end{cases} \tag{5}$$

The first arrow in this equation refers to the transformation undergone when a' crosses C_1 and the second to the evolution in R_2.

Therefore, after measuring atoms a and a', one gets complete information on the Bell state characterizing the $a+C_1$ system, with the following correspondences: $g_a, g_{a'} \to |\Psi^{(+)}\rangle$, $g_a, e_{a'} \to |\Psi^{(-)}\rangle$, $e_a, g_{a'} \to |\Phi^{(+)}\rangle$, $e_a, e_{a'} \to |\Phi^{(-)}\rangle$.

We have then fulfilled the requirement (i), and, after the measurement on the a,a' atom pair, C_2 contains a field that, within a known unitary transformation, replicates the unknown state ϕ_a. If we want to replicate this state on an atom, we must now address question (ii). Atom b, prepared in state $|g\rangle$, is then sent across C_2 and tuned to resonance in order to produce a π pulse if there is one photon in C_2. In this case, it leaves the cavity in the vacuum state: $|g_b\rangle|1\rangle_2 \to |e_b\rangle|0\rangle_2$. On the other hand, nothing happens to the system if C_2 is in the vacuum state before atom b crosses it. In this way, the information stored in the field state is completely transferred to atom b:

$$(\alpha|1\rangle_2 + \beta|0\rangle_2) \otimes |g_b\rangle \to (\alpha|e_b\rangle + \beta|g_b\rangle) \otimes |0\rangle_2 , \tag{6}$$

with $\alpha,\beta = \pm c_e, \pm c_g$. Atom b thus leaves C_2 in a state which differs from ϕ_a by a known unitary transformation. Applying the inverse transformation in R_3, one can thus reconstruct the initial state, completing the teleportation scheme. Note that the setting of the R_3 microwave zone requires the knowledge of the a,a' measurement outcomes, which has to be transferred from D_a to R_3 by a classical information channel ("wire" in Fig. 1). The teleportation scheme is thus completed, in a realistic way, on atoms crossing the two cavities separately.

We have assumed so far perfect atomic detection efficiency, which is certainly not achieved in a real experiment. Inefficient detection will affect the average success of the teleportation scheme. In order to measure the fidelity of the teleportation process, a stream of atoms a could be prepared in a well defined state by P_a, and the state of a beam of atoms b could then be analyzed, with the help of the microwave zone P_b followed by the detector D_b. Each measurement would imply the sampling of a large number of detection events (which would involve preparing repetitively the correlated two-cavity system and the measurements of atoms a, a', and b on a large ensemble of particles, with at least two different settings of zone P_b). If the detection efficiency is not unity, the result of these measurements will yield a two-by-two density matrix ρ_b which describes the state of atom b, statistically averaged over all kinds of partially inefficient atomic detections. One could then compare the replica with the initial state by defining a teleportation fidelity coefficient for a given state ϕ_a as the matrix element $I = \langle \phi_a|\rho_b|\phi_a\rangle$. One can also define an "average" fidelity $\bar{I}$ by averaging I over all possible states ϕ_a.

For perfect teleportation, we should have $I=1$. Easy estimates of the possible values of this quantity in several situations are obtained by representing $|\phi_a\rangle$ in terms of spherical coordinates, which amounts to writing quite generally an atomic state as $|\phi\rangle = \cos(\theta/2)|e\rangle + \sin(\theta/2)e^{i\phi}|g\rangle$, and

$$I = \cos^2\frac{\theta_a}{2}\,\rho_{b,ee} + \sin^2\frac{\theta_a}{2}\,\rho_{b,gg} + \frac{1}{2}\sin\,\theta_a(e^{i\phi}\rho_{b,eg} + e^{-i\phi}\rho_{b,ge}). \tag{7}$$

Suppose we reconstruct the state of particle b at random, without having received any information from the first cavity. This corresponds to ρ_b equal to half the unit matrix and $I=1/2$. Suppose now that the detection efficiency is unity, but that only the first bit of information is used to reconstruct the state (atom a' is not detected). In this case, only the Φ/Ψ character of the Bell state is determined, and it is easy to see from Eqs. (2) and (6) that the probabilities of finding the atom in levels e and g are well reproduced, but the phases of the corresponding amplitudes are not. We should thus set $\rho_{b,ee} = \cos^2(\theta_a/2)$ and $\rho_{b,gg} = \sin^2(\theta_a/2)$ in Eq. (7), with a random phase for $\rho_{b,eg}$. We get then $I = (1/2)(1+\cos^2\theta_a)$. For $\theta_a = 0$, this is equal to 1, as one should expect, since then no atomic coherence is initially present. On the other hand, for $\theta_a = \pi/2$ we have $I=1/2$, equivalent therefore to complete absence of information (the original populations are then equal, and therefore transmitting just the atomic population information is equivalent to having a complete statistical mixture). The average fidelity coefficient $\bar{I}$ when only population information is transferred is equal to 2/3.

Finally, let us consider the case in which there is no quantum coherence between the two cavities. This would correspond to having a classical alternative for the location of the single photon: if it is not found in one of the cavities, one can then say that it was in the other cavity even before the measurement was made. This situation may be mimicked by introducing a random phase φ in (1), which becomes then $|\Psi'_c\rangle=(|0\rangle_1|1\rangle_2+e^{i\varphi}|1\rangle_1|0\rangle_2)/\sqrt{2}$. It is easy to see that this phase, carried over to (2), is equivalent to transferring no information at all about the relative phase of c_e and c_g, thus yielding the value 2/3 for $\bar{I}$. This means that, under this condition, the second bit of information (that is, the measurement of atom a') becomes superfluous.

The above discussion makes it clear that $\bar{I}=2/3$ corresponds to the fidelity of "classical teleportation" [7]. In order to test quantum mechanical nonlocality, one needs therefore to have $\bar{I}>2/3$. We examine in the following the requirements on detection efficiency imposed by this constraint.

The triggering atom c must be the first one to cross the initially empty C_1 and C_2 cavities. If D_c fails to detect it, the experiment is triggered by a subsequent atom and fields having more than one photon are generated, which reduces the quantum correlation between the two cavities required for teleportation. In the worst possible case, I is then reduced to 1/2. If η_c is the probability of detecting atom c, this yields: $\bar{I}\geqslant\eta_c\times\overline{I_{aa'b}}+(1-\eta_c)\times\frac{1}{2}$, where $\overline{I_{aa'b}}$ is the average fidelity coefficient now taking into account the detection efficiencies of atoms a, a', and b and assuming that atom c is detected. A similar argument can be applied to the other atoms. If we do not detect the first atom in beam A, this deteriorates phase information, but does not change population information, which can be retrieved by detecting another atom in the beam, since the interaction of these atoms is dispersive. If atom a is detected, but atom a' is not, this would have no consequence on the population information, which depends only on the first bit. Finally, if atom b is not detected, complete information would be lost, since it would absorb the single photon present in C_2, and the following atoms would exit in a superposition of e and g with equal weights. Denoting by η_a and η_b the detection efficiencies of D_a and D_b respectively, one can write therefore

$$\overline{I_{aa'b}}\geqslant\tfrac{1}{2}(1-\eta_b)+\eta_b[\eta_a^2+\tfrac{2}{3}\eta_a(1-\eta_a)+\tfrac{2}{3}(1-\eta_a)]\,. \tag{8}$$

Assuming finally $\eta_a=\eta_b=\eta_c=\eta$, we get

$$\bar{I}\geqslant\frac{1}{2}+\frac{\eta^2}{6}+\frac{\eta^4}{3}\,. \tag{9}$$

The condition $\bar{I}>2/3$ is satisfied for $\eta>0.7$, a quite mild requirement, as compared to other tests of quantum nonlocality. The dispersion in the velocities of the atomic beams can also be easily accounted for. It would imply a departure from the ideal pulse area in each of the cavities and the Ramsey zones, thus leading to wrong bits of information. It can also be assimilated therefore to an effective efficiency. Detailed calculations show that a 10% velocity dispersion would increase the lower bound in the detection efficiency by about 5%, for $\bar{I}>2/3$. Of course, the total time involved in the preparation of the correlated state and the subsequent detection of the atoms should be much smaller than the dissipation time of the cavities.

We have shown that coherent nonlocal superpositions of fields in cavity quantum electrodynamics can be used for teleportation of an atomic state between two cavities. Such an experiment would provide tests of quantum nonlocality. These phenomena correspond to a new generation of experiments based on atomic correlations of higher order, such as "entanglement swapping," [14] as opposed to the second-order correlations involved in the demonstrations of Bell's inequalities.

Note added. We just learned that a similar scheme of cavity QED teleportation is reported by T. Sleator and H. Weinfurter (unpublished).

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