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Darkness at night

Henning Knutsen

Stavanger College, Box 2557, Ullandhaug, N-4004 Stavanger, Norway

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Abstract. A simple derivation of the formula for the flux of light at a point from cosmic light sources is given. Both the obscuration of distant sources by nearby ones and the expansion of the universe are taken into account. The explanation of the dark night sky is mainly that the distance travelled by a light particle is much smaller than the mean free path for a photon. Both the obscuration and the expansion are relatively unimportant. We live in a young universe with little energy.

Sammendrag. En enkel utledning av formelen for lysfluks i et punkt fra de kosmiske lyskilder blir gitt. Be at de nærliggende lyskilder vil skygge for de fjerne lyskilder og universets ekspansjon blir tatt i betraktning. Forklaringen på den mørke nattehimmelen er i hovedsak at avstanden som en lyspartikkel har beveget seg, er mye mindre enn midlere fri veilengde for et foton. Både de skygge-effekten og ekspansjonen er nokså lite viktige. Vi lever i et ungt univers med lite energi.

1. Introduction and history

A good scientist must certainly be intelligent, well informed and have a large capacity for work. However, what really separates the genius from the ordinary human being is the capability to wonder. The genius is astonished by the darkness in the dead of night. For him the darkness represents an important and strange observation. The common human being experiences only what is well known, simple and obvious.

The dark night-sky riddle was possibly first formulated by Thomas Digges more than four centuries ago. The story of this so-called Olbers' paradox which so long resisted solution has been properly told in the excellent book by Edward Harrison [1]. Today the dark night serves as a cornerstone for modern relativistic cosmology and is one of the few basic facts telling us that big bang cosmology is on the right track.

The motivation for this paper is mostly *educational*. There exist many well written texts explaining different aspects of the riddle. However, most of these papers are written for the educated layman [1–19]. There is, of course, no shortage of research papers treating the problem in full detail [20–31]. What we miss is a proper text which is self-contained and easily comprehensible at an *undergraduate* level. In this paper we want to present those aspects of the problem which are most important from a physical point of view. We want to avoid being lost in subtle details which are not essential and necessary for a proper understanding of the physics behind the solution of the riddle. However, even quite recent texts exist which are misleading or plainly wrong [32, 33].

2. Statement of the riddle

A philosopher once started to think about which of the two heavenly bodies, the sun or the moon, is most important for us. Thinking deeply for a long time he came up with the answer: the moon is most important since it shines during the night when it is dark. The sun shines during the day, but then there is broad daylight in any case!

Now, the obvious and common answer to the question of why the sky is dark at night is that our sun is then down and is shining on the other side of the earth. But our sun is an ordinary star, and there are many stars in the sky. Hence, the proper question is why the light from the stars is so feeble. Certainly, the light from a distant star is more feeble than the light from a star close to us. But there are many distant stars.

To be more specific, let us calculate the amount of energy received by a unit area of the surface of the earth from the light sources within a spherical shell with thickness dD at a distance D from the earth (see figure 1). We take the z -axis to be normal to our unit area. We denote the density of light sources by n_0 and the luminosity of a light source by L_0 . The energy emitted per time by the volume element with volume $D^2 \sin \theta d\theta d\varphi dD$ is given by $L_0 n_0 D^2 \sin \theta d\theta d\varphi dD$.

The energy dl_e received per time and area from the volume element is given by

$$dl_e = L_0 n_0 D^2 \sin \theta d\theta d\varphi dD \frac{\cos \theta}{4\pi D^2}. \quad (1)$$

The factor $\cos \theta$ here stems from the fact that generally the bundle of light rays from the volume element is not normal to the unit area. Surprisingly, this factor has been dropped in the nice paper by Wesson *et al* [28].

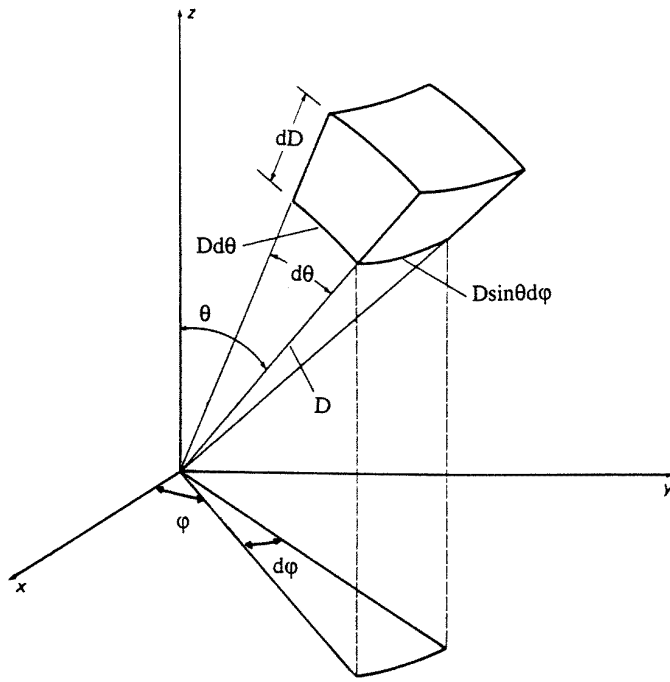


Figure 1. Light is emitted from the volume element and received in a unit area at the origin and normal to the z-axis.

Moreover, these authors have not cared about the fact that an observer on the earth can only observe the light sources above the horizon. The amount of energy dl_s received per time and area from the spherical shell is now given by

$$dl_s = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} L_0 n_0 D^2 \sin \theta \cos \theta \times \frac{1}{4\pi D^2} d\theta d\varphi dD. \quad (2)$$

Hence, the energy received from every shell with thickness dD is the same

$$dl_s = \frac{1}{4} L_0 n_0 dD, \quad (3)$$

since the number of light sources increases in the same way as the energy from a light source decreases. However, expression (3) cannot be correct if our universe is infinite. We would be fried immediately by the infinite amount of energy from all the shells.

3. The obscuration effect

Anyone who has been inside a large wood knows that we cannot look out of the wood if it is large enough. The trees which are far away are partly covered by closer trees (see figure 2). Looking into the vast universe, in every direction we should see a light source. The sky

should accordingly be covered by light sources. Let us calculate this obscuration effect properly.

How many collisions will N objects moving a distance dD through a distribution of obstacles experience? This number must be the same as the number of collisions one object experiences moving a distance $N dD$ through the obstacles. But this number is $N dD/\gamma$ where γ is the mean distance an object travels between collisions. Hence, the probability that a photon is absorbed travelling a small distance dD is just dD/γ .

However, this number is also the probability that a photon travelling outwards from a light source and reaching a shell with thickness dD a distance D from the light source will be absorbed in this shell (see figure 3). But this number is the ratio of the cross sectional area of the obstacles in the shell to the area of the shell. Taking the light sources (obstacles) to be spherical objects with radius a we have

$$\frac{dD}{\gamma} = \frac{n_0 4\pi D^2 dD \pi a^2}{4\pi D^2}. \quad (4)$$

The mean distance a photon travels before being absorbed by a light source, i.e. the mean free path γ , is thus given by

$$\gamma = \frac{1/n_0}{\pi a^2}, \quad (5)$$

i.e. the volume of space per light source divided by the cross section of a light source.



Figure 2. In a forest our distant view is obstructed by a background of tree trunks.

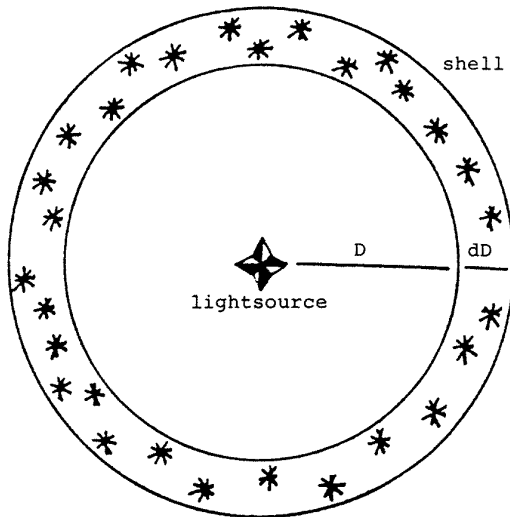


Figure 3. Some of the light rays reaching the shell will be absorbed by the obstacles within the shell.

Hence, the ratio of the amount of energy $-dE$ absorbed in a shell and the energy E reaching the shell is

$$-\frac{dE}{E} = \frac{dD}{\gamma}, \tag{6}$$

where dE is considered negative since the energy from the light source is reduced travelling through the shell.

Integrating equation (6) we obtain

$$E = E_0 e^{-D/\gamma}, \tag{7}$$

where E_0 is the amount of energy emitted by the central light source. Hence, the obscuration factor, i.e. the probability P that an emitted photon reaches the shell at a distance D from the light source, is given by

$$P(D) = e^{-D/\gamma}. \tag{8}$$

Taking this obscuration effect into account, equation (3) takes the form

$$dl_s = \frac{1}{4} L_0 n_0 e^{-D/\gamma} dD. \tag{9}$$

The total amount of energy l received per time and area from the whole universe is found by integrating equation (9), and we obtain

$$l = \int_0^\infty \frac{1}{4} L_0 n_0 e^{-D/\gamma} dD = \frac{L_0}{4\pi a^2}, \tag{10}$$

i.e. the energy received per unit time and unit area should be equal to the energy emitted per unit time and unit area from the surface of a light source! This is the classical form of Olbers' paradox.

With an infinite speed of light one can of course explain the dark night sky assuming our universe to be an island of light sources surrounded by an empty black space.

However, we now know that the speed of light is finite. Hence, looking out in space, we are also looking back in time. The riddle can accordingly be immediately solved by assuming that the light sources have been shining for a finite time only or by assuming that the universe itself has not existed throughout eternity.

Moreover, modern physics, i.e. Einstein's special relativity, tells that the mass m and energy E_{source} of a light source are related by

$$E_{\text{source}} = mc^2, \quad (11)$$

where c is the speed of light. The energy received per unit time and area cannot equal the energy emitted per unit time and area, since space would then be filled with radiation in equilibrium with the matter of the light sources. The energy density u of the radiation would be given by

$$u = \frac{4}{c} \frac{L_0}{4\pi a^2}, \quad (12)$$

and the energy E_{space} contained in the volume $1/n_0$ of space per light source would be

$$E_{\text{space}} = \frac{L_0}{n_0 c \pi a^2}. \quad (13)$$

Inserting numbers, it is seen that

$$E_{\text{source}} \ll E_{\text{space}}. \quad (14)$$

The mean free path γ of the photons can also easily be found if the universe was filled with radiation in equilibrium with the matter of the light sources. We then just ask how long it would take an energy source to fill its volume $1/n_0$ of space. That time T is given by

$$L_0 T = \frac{1}{n_0} u, \quad (15)$$

and remembering equation (12) we obtain

$$T = \frac{1}{n_0 \pi a^2 c}. \quad (16)$$

The distance d this radiation has travelled is accordingly given by

$$d = \frac{1/n_0}{\pi a^2}, \quad (17)$$

in accordance with expression (5).

4. Expansion of the universe

To construct a non-static universe model we must take Einstein's theory of general relativity into account. We assume that the cosmological principle is valid for our universe, i.e. the universe is unchanging from point to point. Space is accordingly homogeneous and isotropic (the same in any direction) about each point. The line element ds which relates events in space and time is then given by the Robertson–Walker metric:

$$ds^2 = dt^2 - \frac{R^2(t)}{c^2} \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right), \quad (18)$$

k being $+1, 0$ or -1 according to whether the spatial geometry of the model has positive, zero or negative curvature. The coordinate t here is the cosmic time,

i.e. the proper time for every observer moving with the substratum (light sources). The space coordinate r is co-moving i.e. the coordinates r and t are so adjusted that the spherical surface $r = \text{constant}$ moves with the material lying on its surface. The scale factor $R(t)$ has the dimension of time while r is dimensionless.

The thickness dD of the shell is now given by

$$dD = \frac{R(t) dr}{\sqrt{1 - kr^2}}. \quad (19)$$

The mean free path γ of a photon is no longer a constant since the universe is expanding. Inserting into equation (6) and integrating we find that the obscuration factor now reads

$$P(r) = \exp \left[-\pi a^2 \int_0^r n(\bar{r}) R(\bar{r}) \frac{d\bar{r}}{\sqrt{1 - k\bar{r}^2}} \right]. \quad (20)$$

5. Redshift factor

However, since the universe is expanding the energy of a photon received by an observer is not the same as the energy of the photon emitted by a light source. Moreover, the time interval for reception of photons is not the same as the time interval for emission of the same photons. But the number of waves emitted must be the same as the number of waves observed. Hence, we have the relation

$$\nu_{\text{em}} dt_{\text{em}} = \nu_{\text{obs}} dt_{\text{obs}}, \quad (21)$$

where ν is the frequency and dt is the proper time.

The emission of a photon can be defined as two events $E_{\text{em}}(t_{\text{em}}, r_{\text{em}}, \theta, \varphi)$ and $E'_{\text{em}}(t_{\text{em}} + dt_{\text{em}}, r_{\text{em}}, \theta, \varphi)$, and the reception of this photon moving radially towards the observer as the two events $E_{\text{obs}}(t_{\text{obs}}, 0, \theta, \varphi)$ and $E'_{\text{obs}}(t_{\text{obs}} + dt_{\text{obs}}, 0, \theta, \varphi)$. The equation of motion for the photon is found from equation (18) by putting $ds = 0$. We obtain

$$\frac{c}{R(t)} dt = - \frac{dr}{\sqrt{1 - kr^2}} \quad (22)$$

where the minus denotes that the photon is travelling towards the observer at the origin of the radial coordinate r .

Integrating equation (22) we obtain

$$\int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{c}{R} dt = - \int_{r_{\text{em}}}^0 \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_{\text{em}} + dt_{\text{em}}}^{t_{\text{obs}} + dt_{\text{obs}}} \frac{c}{R} dt \quad (23)$$

which yields

$$\int_{t_{\text{em}}}^{t_{\text{em}} + dt_{\text{em}}} \frac{c}{R} dt = \int_{t_{\text{obs}}}^{t_{\text{obs}} + dt_{\text{obs}}} \frac{c}{R} dt. \quad (24)$$

Taking the scale factor R to be constant during the short time interval dt , we obtain

$$\frac{dt_{\text{em}}}{R_{\text{em}}} = \frac{dt_{\text{obs}}}{R_{\text{obs}}}. \quad (25)$$

Remembering Planck's relation for the energy e of electromagnetic radiation with frequency ν , i.e.

$$e = h\nu, \quad (26)$$

where h is Planck's constant, using equations (21), (25) and (26) we now obtain that the energy L_{obs} received per time reads

$$L_{\text{obs}} = L_0 \left(\frac{R_{\text{em}}}{R_{\text{obs}}} \right)^2. \quad (27)$$

To obtain the energy received by the observer per unit time and area, the distance D in the numerator in equation (1) must be replaced by $R(t_{\text{em}})r$ and the distance D in the denominator must be replaced by $R(t_{\text{obs}})r$.

Hence, we find that the energy l received per time and area is given by

$$l = \int_0^{r^*} \frac{1}{4} L_0 \left[\frac{R(t_{\text{em}})}{R(t_{\text{obs}})} \right]^2 n(t_{\text{em}}) \left[\frac{R(t_{\text{em}})}{R(t_{\text{obs}})} \right]^2 \times \exp \left[-\pi a^2 \int_0^r n(t) R(t) \frac{d\bar{r}}{\sqrt{1-k\bar{r}^2}} \right] \frac{R(t_{\text{em}}) dr}{\sqrt{1-kr^2}} \quad (28)$$

where r^* is the radial coordinate of the most distant light source from which we receive light at the observing moment t_{obs} .

6. Change of variables

Equation (28) can be simplified using relation (22). Remembering that $\bar{r} = 0$ corresponds to $\bar{t} = t_{\text{obs}}$ and $\bar{r} = r$ corresponds to $\bar{t} = t_{\text{em}}$, the exponential factor takes the simple form

$$\exp \left[\pi a^2 c \int_{t_{\text{obs}}}^{t_{\text{em}}} n(t) dt \right] \equiv P(t_{\text{em}}, t_{\text{obs}}). \quad (29)$$

We further have that $r = 0$ corresponds to $t_{\text{em}} = t_{\text{obs}}$ and $r = r^*$ corresponds to the time t^* when the radiation from the most distant light source was emitted.

Equation (28) now reads

$$l = \int_{r^*}^0 \frac{c}{4} L_0 \left[\frac{R(t)}{R(t_0)} \right]^4 n(t) \exp \left[\pi a^2 c \int_{t_0}^t n(\bar{t}) d\bar{t} \right] dt \quad (30)$$

where we have written $t_{\text{em}} = t$ and $t_{\text{obs}} = t_0$.

Equation (30) is the formula we will use to discuss the dark night sky. We emphasize that to arrive at equation (30) we have assumed that the luminosity of the light sources is not changing with time. Furthermore, we have assumed that the energy of a light ray incident upon a light source is totally absorbed.

7. Einstein–de Sitter model

Einstein's field equations for a perfect fluid read

$$8\pi G\rho = \frac{3}{R^2}(kc^2 + \dot{R}^2) - \Lambda, \quad (31)$$

and

$$8\pi Gp/c^2 = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{kc^2}{R^2} + \Lambda \quad (32)$$

where G is the gravitational constant, ρ is the energy density, p is the pressure and Λ is the so-called cosmological constant. A dot here means differentiation with respect to cosmic time t .

However, equations (31) and (32) yield the following relation

$$\frac{d(\rho R^3)}{dt} + p/c^2 \frac{dR^3}{dt} = 0, \quad (33)$$

which is the energy conservation equation. We further have

$$\rho = nm, \quad (34)$$

where m is the mass of a light source.

Equation (33) then yields immediately: *the number of light sources within a co-moving shell of matter is constant if and only if the pressure vanishes.*

For these models we thus have

$$n[R(t)]^3 = n_0 R_0^3, \quad (35)$$

where the suffix 0 denotes values for the observing moment, i.e. t_0 .

What really interests us is to compare the energy received per area and time and the energy emitted by the light source per area and time. Hence, we calculate the ratio

$$f \equiv \frac{l}{L_0/4\pi a^2}. \quad (36)$$

From equations (30) and (35) we now obtain

$$f = \frac{c}{\gamma_0} \int_{t^*}^{t_0} \frac{R(t)}{R_0} \exp \left\{ \frac{c}{\gamma_0} \int_{t_0}^t \frac{R_0^3}{[R(\bar{t})]^3} d\bar{t} \right\} dt. \quad (37)$$

For the Einstein–de Sitter model which has $p = 0$, $\Lambda = 0$ and $k = 0$, the solution for the scale factor is

$$R(t) = bt^{2/3}, \quad (38)$$

where b is a constant. t^* is taken to be the moment when the stars started to radiate.

With the substitution $y = (t/t_0)^{1/3}$, equation (37) takes the following form:

$$f = 3\beta e^\beta \int_\varepsilon^1 y^4 e^{-\beta y^{-3}} dy, \quad (39)$$

where $\varepsilon = (t^*/t_0)^{1/3}$,

$$\beta \equiv \frac{ct_0}{\gamma_0} = ct_0 \pi a^2 n_0 = \frac{a^2 c}{6mGt_0} = \sqrt{\frac{\pi \rho_0}{6G}} \frac{a^2 c}{m}. \quad (40)$$

The last equation is obtained using equations (31) and (38).

It is immediately seen that the sky is dark if the stars started to shine only recently ($\varepsilon \approx 1$).

We now take our light sources to be stars and compare the light received with the energy emitted from the surface of a star. To simplify the calculations we take $t^* = 0$.

The result is quite impressive. With $a =$ radius of the sun $= 7 \times 10^8$ m, $m =$ mass of the sun $= 2 \times 10^{30}$ kg and the age of the universe $t_0 \approx 15 \times 10^9$ years, we obtain $\beta \approx 0.39 \times 10^{-12}$.

With this value for β , we find

$$f \approx 0.234 \times 10^{-12}. \quad (41)$$

The energy received from all the stars is really extremely feeble, and there is no wonder that the sky is dark at night.

The main reason for the dark night sky is the small energy density in the universe ($\beta \approx 0$). Moreover, it should not be said without reservation that the night sky is dark just because the universe is still young [17]. The night sky will also be dark when the universe grows very old ($t_0 \rightarrow \infty$). To see this, it is not even necessary to take into account that the stars will radiate for the time mc^2/L_0 only.

However, the inverse value of the Hubble constant $H_0^{-1} = (R/\dot{R})_0$ given by the redshift and the distance of the distant galaxies yields an upper limit for the age of our universe. Hence, we conclude that the sky is not dark because we live in an old universe.

To examine if the expansion of the universe is important for the dark night sky phenomenon for the Einstein-de Sitter model, we compare this model with the corresponding static model for which we put $R =$ constant. Using again the substitution $y = (t/t_0)^{1/3}$, we obtain

$$\frac{l(\text{expanding})}{l(\text{static})} = 3\beta e^\beta (1 - e^{-\beta})^{-1} \int_0^1 y^4 e^{-\beta y^{-3}} dy. \quad (42)$$

With $\beta = 0.39 \times 10^{-12}$ this ratio takes the value 0.6.

Hence, we conclude that the expansion is *not* the reason why the sky is dark at night.

Next we examine if absorption is important for the dark night sky phenomenon. We suppress absorption when P in equation (29) is taken to be unity. This would be the case if the light sources were point sources, i.e. if a vanishes. We now obtain

$$\frac{l(\text{absorption})}{l(\text{no absorption})} = 5e^\beta \int_0^1 y^4 e^{-\beta y^{-3}} dy. \quad (43)$$

With our previous value for β this ratio is very close to unity.

Hence, we conclude that the absorption is not the reason why the sky is dark at night.

To examine the importance of a finite speed of light we consider expression (28). With an infinite speed of light the photons are observed at the very moment they are emitted. Hence, we take R and n to be constants in equation (28). The radial coordinate of the most distant

light source we observe is then infinite, and with $k = 0$, we obtain

$$f = \pi a^2 n R \int_0^\infty \exp[-\pi a^2 n R r] dr = 1. \quad (44)$$

With an infinite speed of light we are back at the classical form of Olbers' paradox.

Our conclusion is that what determines how much light we receive from the light sources in the universe is the following two independent conditions:

- (1) t^* should be close to t_0 , i.e. the stars have been radiating for a short time;
- (2) the constant β should be small, i.e. the speed of light must be finite, and the light sources must be small and contain little energy.

8. Steady-state model

We now examine the model for our universe which has been Big Bang cosmology's most important rival. For the steady-state model the solution of the scale factor R reads

$$R(t) = e^{Ht}, \quad (45)$$

where the Hubble 'constant' H is now really a constant.

Inserting into equation (32) we now obtain with $k = 0$

$$8\pi p/c^2 = -3H^2. \quad (46)$$

Hence, the pressure does not vanish, and the number of light sources within a co-moving shell is not a constant. Equation (35) is thus not valid for the steady-state model. Here we have the main mistake committed in [33] which leaves the discussion of the steady-state model in that paper valueless.

The main motivation of the steady-state model was the perfect cosmological principle. This principle states that the universe is not only spatially isotropic and homogeneous, but the universe at large should be unchanging with time. The density n of the light sources must then be constant and the age of the universe must be infinite, i.e. $t^* = -\infty$.

For a static model ($H = 0$) without absorption, we now find that the amount of light received is infinite. When absorption is taken into account, we find $f = 1$ for the static model. Hence, for the steady-state model, the expansion of the universe is *necessary* to have a dark night sky.

For an expanding model we find

$$f = \begin{cases} (1 + B)^{-1} & \text{with absorption} \\ B^{-1} & \text{without absorption} \end{cases} \quad (47)$$

where

$$B = \frac{4H}{\pi a^2 n c}. \quad (48)$$

To have a dark night sky ($f \ll 1$) for an expanding steady-state model it is thus necessary and sufficient to have a large value for the parameter B , i.e. we must have small light sources which are far away from one

another. The speed of light must also be finite, and, in addition, the expansion of the universe should not be too slow.

9. Newtonian model

For these simplified models with Euclidean geometry the universe is considered as an uniformly expanding sphere of particles (light sources). The observer is at the origin of the sphere, and Hubbles law is valid, i.e.

$$v_s = H_0 D, \quad (49)$$

where v_s is the speed away from the observer of a light source. Accepting Newton's corpuscle theory of light, i.e. light particles obey the ordinary classical law of velocity addition, it is seen that there exists a horizon in this universe. A photon emitted towards the observer at the origin from a light source with speed $v_s = c$, will never reach the observer. Hence, the observer does not receive energy from the light sources beyond this horizon. Dropping the obscuration factor, equation (3) is integrated from the origin to the horizon $D_H = c/H$, and we obtain

$$f = \frac{\pi a^2 n c}{H}. \quad (50)$$

The reason why the sky is dark at night in this kind of expanding universe is thus exactly the same as in the steady-state universe. It should not be said that 'the dark night is a result of the expansion of the universe' [32]. The expansion is again necessary, but not sufficient.

If we take obscuration into account we find from equation (9)

$$f = 1 - \exp(-\pi a^2 n c / H). \quad (51)$$

Consider now a universe which contains enough matter to start a recollapse in the far future. At the moment τ when the recollapse sets in, the Hubble constant would vanish. If the stars are still shining at this moment, the whole sky would according to equation (51) be as bright as the surface of the sun. However, this is just a warning that the horizon argument should not be used for this kind of model. Equation (9) should be integrated from the origin out to a distance $D = c\tau$ and equation (51) take the form

$$f = 1 - \exp(-\pi a^2 n c \tau). \quad (52)$$

Hence, the universe is dark even at the moment when the recollapse sets in.

It is, however, quite amusing to calculate the amount of light lost if the horizon argument is used for an Einstein-de Sitter model. We must then find the moment when the photons from the light sources which now have speed c away from us were emitted.

This moment must replace the birth of the universe $t = 0$ in equation (30). We find the radial coordinate r^* for these light sources from

$$c/H_0 = R_0 \int_0^{r^*} \frac{dr}{\sqrt{1 - kr^2}}. \quad (53)$$

Equation (22) now yields

$$-c \int_{t_0}^T \frac{dt}{R(t)} = \int_0^{r^*} \frac{dr}{\sqrt{1 - kr^2}} = \frac{c}{H_0 R_0}. \quad (54)$$

For the Einstein-de Sitter model, we find

$$T = \frac{1}{8} t_0. \quad (55)$$

Using our previous substitution we obtain

$$\frac{l(\text{horizon})}{l} = \frac{\int_{1/2}^1 y^4 e^{-\beta y^{-3}} dy}{\int_0^1 y^4 e^{-\beta y^{-3}} dy} \approx \frac{31}{32}. \quad (56)$$

The Newtonian model is thus in reasonable agreement with the Einstein-de Sitter model.

10. Conclusion

We have given an educational discussion of the dark night sky phenomenon aimed for an undergraduate level. Starting with the simplest form of Olbers' paradox, we have later taken both obscuration and expansion (Einstein-de Sitter model) into account. For historical reasons we have also discussed the steady-state model and a Newtonian model. The result of the examination is as follows.

It is dark at night because:

- (1) the speed of light is finite;
- (2) the universe is still young, and the stars only started to shine rather recently;
- (3) the light sources in the universe are small;
- (4) the light sources in the universe are far apart, so the energy density in the universe is very small.

Both obscuration and expansion are quite unimportant.

'Do not complain below the stars because of lack of bright moments in your life. Alas, they are twinkling as if they wanted to talk to you.'

H Wergeland, Norway's greatest poet (1841).

Later he had really grasped their message:

'The world must still be young.' (1844).

Acknowledgments

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