arXiv:1205.3309v1 [astro-ph.CO] 15 May 2012

The Scale of Cosmic Isotropy

C. Marinoni[1,2], **J. Bel**[1] **and A. Buzzi**[1]

[1] Centre de Physique Théorique, Aix-Marseille Université, CNRS UMR 7332, case 907, F-13288 Marseille, France

[2] Institut Universitaire de France, 103, bd. Saint-Michel, F-75005 Paris, France

E-mail: christian.marinoni@cpt.univ-mrs.fr

Abstract. The most fundamental premise to the standard model of the universe, the Cosmological Principle (CP), states that the large-scale properties of the universe are the same in all directions and at all comoving positions. Demonstrating this theoretical hypothesis has proven to be a formidable challenge. The cross-over scale R_{iso} above which the galaxy distribution becomes statistically isotropic is vaguely defined and poorly (if not at all) quantified. Here we report on a formalism that allows us to provide an unambiguous operational definition and an estimate of R_{iso}. We apply the method to galaxies in the Sloan Digital Sky Survey (SDSS) Data Release 7, finding that $R_{iso} \sim 150h^{-1}$Mpc. Besides providing a consistency test of the Copernican principle, this result is in agreement with predictions based on numerical simulations of the spatial distribution of galaxies in cold dark matter dominated cosmological models.

1. Introduction

The assertion that the cosmic mass distribution appears homogeneous and isotropic, that is uniform, to a family of typical observers that move with the same average velocity of the surrounding matter (fundamental or comoving observers) has far reaching consequences in cosmology [1]. It entails that the geometry of space-time is highly symmetric and completely described by the simple Robertson & Walker metric [2, 3]. Furthermore, it implies that space expands at a rate that is set by the equations of Friedman & Lemaitre [4, 5].

The very first surveys of the three-dimensional distribution of optical galaxies showed that the topology of the large-scale structure is very complex and irregular [6, 7]. Because of this departure from exact uniformity, the CP is regarded as a coarse-grained model of the universe, a statistical description of the mass distribution that applies only on sufficiently large scales where the finest details of the galaxy clustering pattern become irrelevant.

More recently, two-dimensional observations of the Cosmic Microwave Background (CMB) [8] at high redshift, and, more locally, of radio [9] and infrared [10] sources, confirmed that the universe is nearly isotropic about us. What is challenging is to show that it is isotropic also about distant observers, namely, that it is uniform. Therefore, despite its founding importance, the case for the CP rests more on philosophical rather than on empirical evidences; it is enough to postulate that we are not privileged observers (the so called Copernican principle) to deduce that if the universe appear isotropic about our position, it must also appear isotropic to observers in other galaxies ([11]).

The tremendous explanatory power of the standard model of cosmology cannot be advocated as an indirect demonstration of the CP, since they are not the only solutions of the Einstein equations which are able to fit cosmological observations. In particular, many authors have speculated that some effects of the accelerated expansion of the universe [12, 13, 14, 15, 16], which remains fundamentally unexplained in terms of microscopic physics, could be mimicked by allowing CP violations (see a review in [17]). This intriguing possibility has motivated recent attempts of rooting the CP on a more solid basis. Interestingly, there are some encouraging proposals in this direction which are based on the analysis of the large-scale maps of CMB anisotropies [18, 19, 20, 21], of galaxies [22, 23, 24, 25] and of supernovae [26].

Even if we postulate the CP, the picture is not complete unless we identify the averaging scale that is implicit in this assumption, i.e. the scale on which the FLRW model provides an effective, coarse-grained description of the universe [27]. It is generically asserted that the CP holds on domains that are large enough to encompass the biggest gravitational structures of the universe. Yet, few studies have attempted to narrow in on the length value above which clumpiness gives way to uniformity [28].

Past efforts were mostly based on the analysis of the two-point correlation properties of galaxy samples [29, 30]. This approach, however, suffers from severe theoretical drawbacks. Since the average number density of the sample is needed as input, the method presupposes the premise to be tested, i.e. a constant density distribution of matter [31]. Moreover, it does not provide an unambiguous definition of the cross-over scale [32]. As a consequence, the inferred homogeneity length-scales depend on the size of the analyzed sample and range from values as low as $30h^{-1}$Mpc up to $200h^{-1}$Mpc [33, 34, 35, 36, 37, 38]. More recently, orthogonal techniques have been explored which are based on the count-in-cells analysis of observations confined to a spatial hyper-surface of constant time (e.g. [39]). These methods are insensitive to light cone effects, i.e. possible biases arising from comparing galaxy fluctuations at different cosmic epochs, and seem to indicate a transition to homogeneity at a scale of 70^{-1}Mpc[40, 41] (but see [43, 44] for an opposite conclusion).

It is widely believed that, since we cannot point telescopes from any other place but the solar system, it is not possible to establish if also distant observers see an isotropic universe. While this argument is certainly true for apparent 2D quantities such as, for example, the CMB temperature (but see [18]), we show here that it does not apply to 3D maps of the spatial distribution of galaxies. Specifically, we quantify the typical dimension above which independent observers see an isotropic 'bath' of galaxies. Besides establishing an operational definition of the isotropy scale, our approach also provides an overall consistency test of a fundamental facet of the CP, i.e. that we are not privileged observers of the universe.

2. The Method

We identify paths of extremal length irradiating from a given arbitrary target galaxy to every other n^{th} closest neighbors (see Fig. 1). The amplitude of the angle t between these directions and the observer line-of-sight (los) to the target is computed by assuming that the local properties of a homogeneous and isotropic universe are described in terms of the infinitesimal Robertson & Walker [2, 3] line element

$$ds^2 = (cdt)^2 - a^2(t)[d\chi^2 + \Sigma_k^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)]$$

where, c is the light speed, k is the scalar spatial curvature, χ is the radial geodesic comoving distance, $a(t)$ is the cosmic expansion factor, and where, using the Kronecker symbol, $\Sigma_k(\chi) = \delta_{k,1}\sin\chi + \delta_{k,0}\chi + \delta_{k,-1}\sinh\chi$. The spatial part of the metric is invariant under a *quasi-translation* transformation of its coordinates [1]. We can translate the reference frame from the terrestrial observer O to the target A and express the coordinates $\vec{x}_{n/A}$ of its n^{th} neighbor as

$$\vec{x}_{n/A} = \vec{x}_{n/O} - \vec{x}_{A/O}\Big\{ \left[1 - kx^2_{n/O}\right]^{1/2} + \\ + \left[1-(1-kx^2_{A/O})^{1/2}\right]\frac{\vec{x}_{n/O}\cdot\vec{x}_{A/O}}{x^2_{A/O}}\Big\}. \quad (1)$$

By orienting the axes in such a way to minimize the number of non-zero components (we choose $\vec{x}_{n/A} = (\Sigma_k(\chi_{An})\ ,\ 0,\ 0)$, $\vec{x}_{n/O} = (\Sigma_k(\chi_n)\sin\Delta\theta\ ,\ 0,\ \Sigma_k(\chi_n)\cos\Delta\theta)$ and

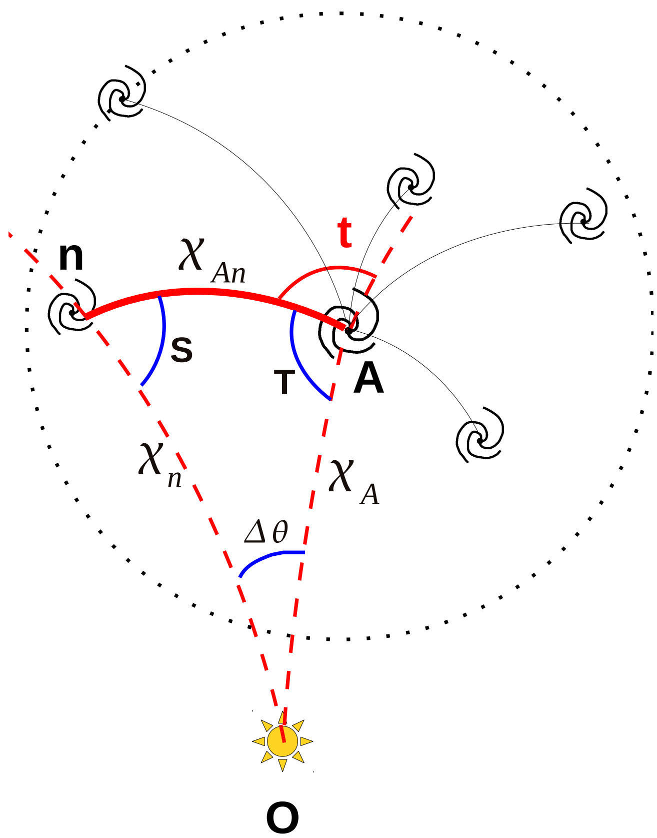

Figure 1. We determine the geodesic connections between a given target galaxy A and all the surrounding galaxies that lie inside a sphere of radius R centered on A. The target galaxy A and its n^{th} closest neighbor subtend an angle $\Delta\theta$ at the observer position O. The tilting angle t measures the inclination of the geodesic separation χ_{An} between A and n with respect to the observer line-of-sight (dashed line). If the CP holds on the scale R we expect this '*spaghetti* to be isotropically oriented about any given target. In other terms we expect the los angle t to be isotropically distributed, i.e. its PDF is $\varphi(t) = (\sin t)/2$.

$\vec{x}_{A/O} = (0\ ,\ 0,\ -\Sigma_k(\chi_A)))$, and by exploiting the identity

$$C_k^2(\chi) + k\Sigma_k^2(\chi) = 1 \tag{2}$$

we obtain

$$\Sigma_k^2(\chi_{An}) = \Sigma_k^2(\chi_n)\sin^2\Delta\theta + \\ + \Big[\Sigma_k(\chi_n)C_k(\chi_A)\cos\Delta\theta - \Sigma_k(\chi_A)C_k(\chi_n)\Big]^2. \tag{3}$$

Further, by repeatedly applying this relation to the 3 edges of the triangle $(\hat{S}, \hat{T}, \hat{\Delta\theta})$ shown in Fig. 1, and by isolating, after some algebra, identical terms in the resulting expressions one obtains the generalized law of sines

$$\frac{\sin\Delta\theta}{\Sigma_k(\chi_{An})} = \frac{\sin T}{\Sigma_k(\chi_n)} = \frac{\sin S}{\Sigma_k(\chi_A)}. \tag{4}$$

Since $t = \pi - T$, it finally follows from eqs (3) and (4) that

$$\sin^2 t = \frac{1}{1 + \left[C_k(\chi_A)\ \cot\Delta\theta - \frac{\Sigma_k(\chi_A)}{\Sigma_k(\chi_n)}\frac{C_k(\chi_n)}{\sin\Delta\theta}\right]^2}. \tag{5}$$

If the CP holds, the *los* angle t has a comoving space Probability Distribution Function (PDF) of a characteristic type ($\varphi(t) = (\sin t)/2$), namely, it is a random variable isotropically distributed with respect to any fundamental observer. Therefore, the expectation value $\mu = \langle \sin^2 t \rangle$ is cosmology independent and equal to 2/3. We define the indicator of galaxy isotropy (IGI) as the estimator m_R constructed by averaging equation (5) over n galaxies inside a sphere of comoving radius R that is centered around any given observer in the universe. On scales R where the CP applies, we expect the measure of m_R to converge to the predicted value $\mu = 2/3$.

The testing protocol is as follows: we assume that the CP holds and we implement standard statistical inference methods to try to falsify it and reject its validity. In detail, we assume the existence of a length-scale R above which the empirical IGI estimates (m_R) are statistically identical to the theoretical prediction (μ). We thus formulate a null hypothesis h_0 according to which the two quantities are not different. We quantify the goodness of the agreement by means of a χ^2 statistics, and, following standard convention, we fix the rejection threshold of h_0 at the 95% level. This means that the hypothesis that the universe is isotropic above a scale R cannot be rejected by data if the probability P of obtaining a worst (larger) χ^2 value is greater than 5%. On the contrary, an eventual failure in identifying the scale of isotropy would unambiguously point at the incoherence of the FLRW model.

Homogeneity and isotropy are properties that characterize the large-scale distribution of *matter* on a 3D spatial hyper-surface at a given instant of time. Since light propagates at a finite speed, the most distant regions of the 3D volume directly accessible to observations are also the furthest in time. As a consequence, the number density of galaxies, an observable that is modulated by local physical processes with their own specific time-scales, is expected to vary as a function of distance. This is a known issue that hampers most of the tests of the CP [45]. In the following we show that, by focusing our attention on the angular distribution of galaxies, instead of their number density fluctuations, we can tackle the past light cone issue.

If the CP holds true, as we assume here, the galaxy spatial number density $\rho_s(r)$ within spherical shells of radius R centered on the terrestrial observer must be independent from r. Note that *shell-homogeneity*, that is $\rho_s = const$, does not imply homogeneity, i.e. invariance under general spatial translations, while the opposite is true. More importantly, the radial constancy of ρ_s does not imply everywhere isotropy (isotropy about arbitrary comoving observers) that is the fundamental facet of the CP that we want to test. As a matter of fact, the distribution of galaxies that surrounds us can be characterized by a constant ρ_s and yet be anisotropic. We therefore remove

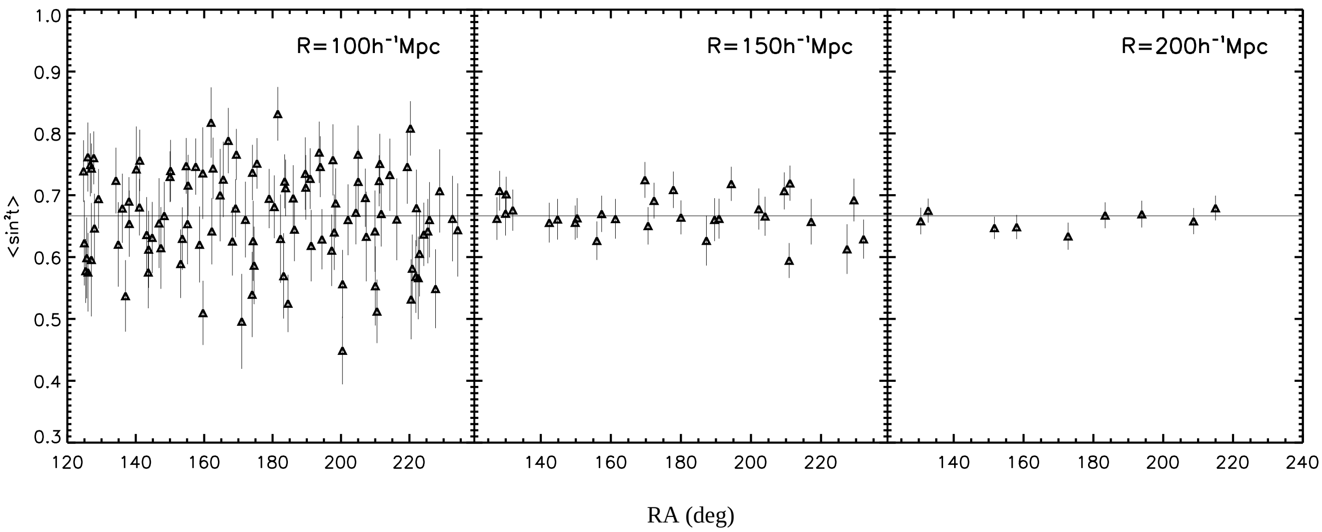

Figure 2. The IGI value measured by different observers, labeled by their right ascension coordinate, is plotted. Each estimate is performed within non-overlapping spheres of comoving radius R (shown in the inset) randomly thrown in the volume covered by the SDSS galaxy survey. Errors are computed as the standard deviation of the mean, and well trace the theoretically expected figure $\sigma = \sqrt{4/(45n)}$. The average IGI value ($m_R = \langle \sin^2 t \rangle$) is $0.678 \pm 0.005, 0.672 \pm 0.006$, and 0.660 ± 0.007 from left to right. The solid line shows the expectation value predicted under the assumption that the CP holds (i.e. $\mu = 2/3$). A goodness of fit statistical analysis yields $\chi^2/dof = (2.1, 1.12, 0.65)$ from the the smaller to the larger scale.

past-light cone artifacts, by imposing that the comoving number density of galaxies be strictly constant within concentric shells centered on us. In practice, we analyze a volume limited catalog of galaxies, that is a sample whose clustering properties are independent of the details of the galaxy luminosity distribution, and we additionally remove, with a random rejection process, any residual radial gradient in the distribution of galaxies.

3. Data

We apply the method to the seventh release of the Sloan Digital Sky Survey [46] which is comprised of $\sim 930,000$ galaxies over a field of view of 9380 deg^2. Our analysis is limited to luminous red galaxies (LRG, [47]) distributed in the North Galactic contiguous area defined by $120 < RA < 240$, $7 < \delta < 56$. A sample with a nearly constant density of galaxies is obtained by volume limiting the SDSS dr7 catalog in the redshift range $0.22 < z < 0.5$. This sample extends on a comoving radial size $\Delta r \sim 700h^{-1}$Mpc (in what follows, we consider a cosmological model characterized by the reduced density of matter $\Omega_m = 0.27$ and dark energy $\Omega_\Lambda = 0.73$, and we assume that the value of the Hubble constant is $H_0 = 72$ km s^{-1}Mpc^{-1}.) The upper redshift limit is fixed by the requirement of measuring the IGI with approximately the same average precision of nearly 1% over all the interval $100 < R < 200Mpc$.

The strict equality $\rho_s(r) = const$ is then imposed by interpolating the observed number density ρ_s of objects in spherical shells centered on us (and with thickness a hundred times smaller than the effective depth of the sample), and by randomly rejecting galaxies using a Monte Carlo process with selection function $\phi(r) = \min(\rho_s)/\rho_s(r)$. The final LRG sample contains a total of ~ 6500 objects, has a mean number density $\rho = 6.14 \cdot 10^{-6} h^3 \mathrm{Mpc}^{-3}$ and covers an effective field of view of 4860 deg^2.

4. Analysis of Data and Comparison to Theoretical Models

For meaningful error interpretation, it is imperative to acquire independent estimations of the observable m_R. Consequently, we do not apply our scheme to every galaxy in the sample, i.e. we do not carve spheres of comoving radius R around each 'extraterrestrial' observer to determine whether they see the same degree of isotropy. Instead, we only select as observers, those target galaxies that are at the center of non-overlapping spheres. As an example, given the geometry of the largest contiguous volume in the SDSS survey, we can place a maximum number $N = 107, 30, 9, 4, 3$ of independent observers. They explore the isotropic distribution of galaxies on length-scales $R = 100, 150, 200, 250$ and $300 h^{-1}$Mpc respectively, and each of these observers are geodesically connected, on average, to $n = 26, 87, 206, 401, 695$ galaxies respectively.

Before analyzing real data, we have first applied our method to synthetic samples simulating spatially random (statistically uniform) galaxy distributions. The point here is to detect the minimum radius R below which our technique is noise-limited and the scale of everywhere isotropy cannot be resolved. Using Montecarlo techniques, we have generated various uniform mock catalogues with galaxy number densities in the range $10^{-4} - 10^{-6} h^3 \mathrm{Mpc}^{-3}$. We have found that, as expected, when the scale R is larger than the mean inter-particle separation $\lambda = \rho^{-1/3}$, the distribution of the t angle statistically converges towards an isotropic PDF. Quantitatively, as soon as $R > 1.5\lambda$, that is when on average $\sim 4 \cdot (1.5)^3$ galaxies are geodesically connected to the observer, we cannot reject the isotropy hypothesis h_0 with a confidence greater 5%. In particular, the everywhere isotropy of a random (uniform Poissonian) distribution of galaxies with the same density of the LRG sample investigated in this study can be unambiguously detected on scales larger than $\sim 85 h^{-1}$ Mpc.

We have then analyzed the LRG galaxy sample extracted from the SDSS survey. The IGI value estimated by distant observers on a scale $R = 100, 150$ and $200 h^{-1}$Mpc is graphically shown in Fig. 2. In accordance with standard theoretical expectations, as the R-scale increases, all the observers are equally likely to observe isotropy, i.e. they loose their specificity and progressively become the 'typical' observer of the universe. The upper panel of Fig. 3 confirms that the galaxy pattern observed from different positions in the universes approaches an isotropic distribution.

The precise scale of transition to isotropy R_{iso} is quantitatively determined as follows. First, by randomly rejecting galaxies from the main LRG sample, we have constructed 1000 subsamples that satisfies to the requirement $\rho_s = const.$ This bootstrapping process, allows us to estimate the central moments and the dispersion of the P statistics. We have then positioned the centers of the maximum number of non-overlapping spheres of radius R that fit inside the survey volume. In particular, we require that the position of the extraterrestrial observers change randomly from sample to sample. For each length-scale R probed, we have finally computed the risk of erroneously rejecting the null hypothesis as the median of P over the 1000 realizations. The lower panel of Fig 3. shows that the median risk is larger than 5% for scales larger than $150h^{-1}$ Mpc. Despite the observed spread in the P-values for large R, essentially due to the low density of the LRG sample, a statistically significant sharp transition towards isotropy at a scale $R_{iso} \sim 150h^{-1}$ is unambiguously detected.

Our conclusions are, within error bars, independent from the density cut we artificially impose to guarantee shell homogeneity. We have verified that samples that are everywhere isotropic on a scale R continue to be everywhere isotropic on that scale if the density threshold imposed by the requirements of shell homogeneity is enhanced. This conclusion follows from the analysis of subsamples obtained by volume-limiting the SDSS catalogue at a lower redshift z_{max}. Interestingly, and in the opposite sense, if the sample is isotropic on a scale R then it continues to be isotropic on that scale, also even when its density is artificially lowered (by randomly rejecting galaxy members). The isotropic length proves robust up until the investigated scale R becomes smaller than ~ 1.5 times the mean inter-particle separation. Below that threshold, as the analysis of random samples already suggested, the predictive power of our indicator breaks down.

We have compared our measurements to predictions of N-body simulations of the large-scale structure of the universe. To this end, we have analyzed with the same technique 50 independent mock catalogs simulating the distribution of LRG galaxies in an SDSS-like survey. They were constructed by the LasDamas project [48] using Λ cold dark matter simulations (with characteristic parameters ΛCDM) simulations $(\Omega_M = 0.25, \Omega_\Lambda = 0.75, h = 0.7, \sigma_8 = 0.8, n_s = 1)$.

Fig. 3 quantifies the confidence level with which the hypothesis h_0 cannot be rejected on a scale R, and compares it to what is expected in the mock catalogs. Not only is a sharp transition towards isotropy at a scale $R \sim 150h^{-1}$ detected in real SDSS data, it is observed in synthetic galaxy catalogs too. This excellent agreement implies that the scale of isotropy $R \sim 150h^{-1}$ is a length that characterizes not only luminous galaxies, i.e. the visible component of the universe, but also of the most massive dark matter halos. The significance of this conclusion is best understood by considering that the everywhere isotropy inferred from real data alone, does not give insight into the corresponding arrangement of the underlying mass component.

5. Conclusion

An acritical acceptance of the Copernican principle might result in what Haynes [49] called the "Verrazzano bias". As in the case of this explorer who, off the coast of the outer banks of North Carolina, mistakenly believed that he had discovered the Pacific Ocean, it is dangerous to draw definite cosmological conclusions on the basis of limited

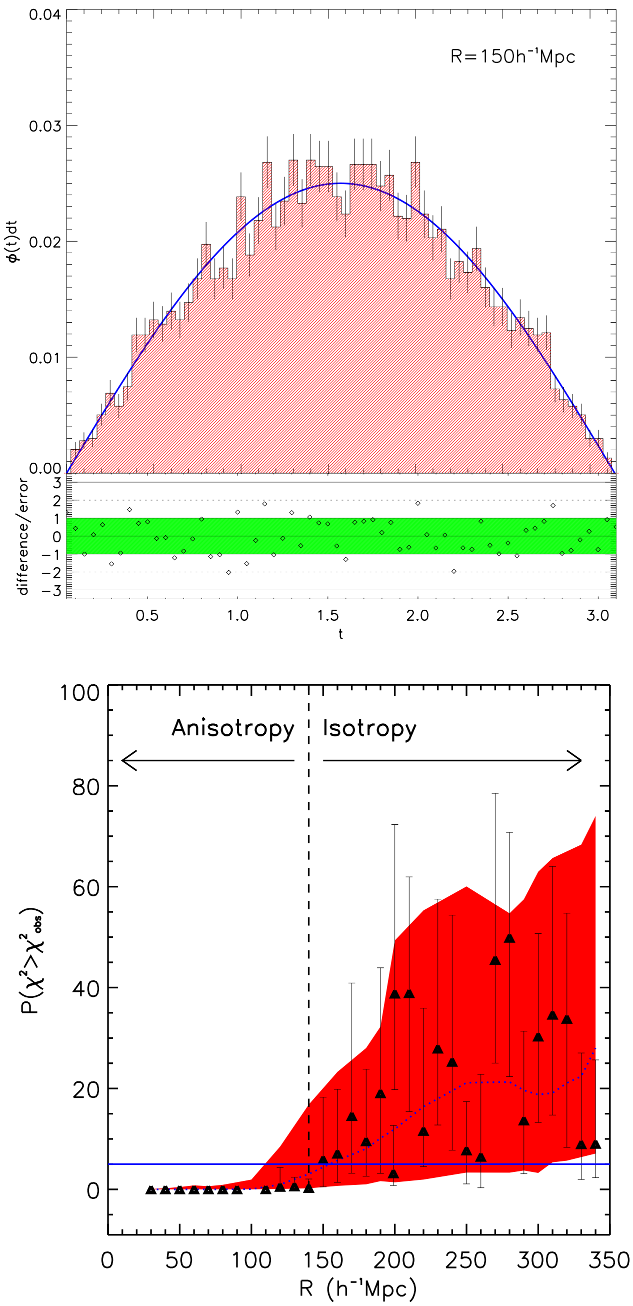

Figure 3. *Upper:* the observed PDF of the *los* angle t (histogram) is compared to the isotropic prediction ($\varphi(t) = (\sin t)/2$) on a scale $R = 150h^{-1}$Mpc. The ratio between model deviations and data errors is also plotted (together with the lines indicating 1σ and 2σ deviations). *Lower:* The confidence with which the hypothesis h_0 cannot be rejected on a given scale, i.e. probability that the assumption of everywhere isotropy is compatible with observations. This is computed as the median of the probability P inferred from 1000 resamplings of the SDSS LRG sample that are shell homogeneous. Errorbars represents the first and third quartile of the distribution of P. The blue line shows the average expectation extracted from the analysis of 50 mocks catalogs simulating the sample. The red envelope shows the confidence threshold bracketing 1σ fluctuations around the average expectation.

data collected from a special spatial position.

In this paper we have presented a new geometrical tool that allows us to assess whether or not, from the view point of a distant galaxy, the large-scale structure of the universe appear almost identical to its aspect from earth. Virtually all of the previous attempts to identify the coarse graining scale above which the visible distribution of matter comply with the requirements of the CP have focused on the analysis of the so called *homogeneity scale*. In this paper we have addressed this same issue from a different angle. We propose to identify this fundamental length with the scale of everywhere isotropy R_{iso}, the scale above which the distribution of galaxies appears isotropic to every comoving observer, that we define as the smallest scale at which the probability of wrongly rejecting the CP is smaller than 5%.

By analyzing state-of-the-art data, we have found that the galaxy distribution, as traced by luminous red galaxies, appears isotropic to every comoving observer in the universe once the averaging scale is larger than $R_{iso} \sim 150h^{-1}$ Mpc. This figure is in excellent agreement with predictions of the spatial clustering of galaxies in ΛCDM simulations.

The advantage of the method is twofold. It does not require any correction for the radial selection function of the redshift sample analyzed nor any radial modeling of the expected number of galaxies as a function of redshift. Instead, it is straightforward to subtract look-back time issues once the focus is shifted from counting objects (the standard methodology of the homogeneity tests) to measuring angles (as implemented by our strategy). Additionally, the procedure used to extract the scale of everywhere isotropy is less ambiguous and questionable than other thresholding strategies proposed in literature. Since the matter distribution converges continuously towards homogeneity/isotropy, it is in fact quite arbitrary to decide which criterium must be adopted to single out an exact scale of transition. In this work we adopt the point of view that the most natural way to test the CP is to assign a probability to the hypothesis that this model is wrong. The goal is to frame the analysis of its coherence within the domain of probability theory, as the intrinsically statistical nature of this cosmological statement explicitly demands. This helps elucidating the meaning of such generic sentences as "....the CP holds on a scales larger than XXX Mpc" and will ease the quantitative comparison of the results obtained with different and independent methods.

Deeper redshift surveys of the universe (such as, for example, BOSS or EUCLID) are currently ongoing and expected to be soon completed. It would be interesting to understand if the scale of everywhere isotropy does scale as a function of cosmic time as predicted by numerical simulations of the gravitational clustering in the universe. This will confirm that the CP is not some temporary assertion about the present day appearance of the universe but a fundamental property of matter distribution at all cosmic epochs. Even more importantly, it will help us to shed light on the physics behind the large-scale uniformity of the universe by answering the question: where does this scale come from?

Acknowldgements. We would like to thank A. Blanchard, P.S. Corasaniti, L. Moscardini, E. Scannapieco, T. Schucker, P. Taxil and S. Troubetzkoy for useful discussions. C.M. is grateful for support from specific project funding of the *Institut Universitaire de France*.

References

[1] Weinberg, S. 1972, Gravitation & Cosmology, Wiley & Son, New York.
[2] Robertson, H. P. 1929, PNAS, 15, 822
[3] Walker, A. G. 1936, Proc. London Math. Soc., 42, 90
[4] Friedman, A. 1922 Zeitschrift fr Physik, 10, 377
[5] Lemaitre, G. 1931, MNRAS, 91, 483
[6] Geller, M. J., & Huchra, J. P. 1989, Science, 246, 897
[7] Giovannelli, R., & Haynes, M. P. 1991, ARA&A, 29, 499
[8] Bennett, C. L. et al. 1994, ApJ., 436, 423
[9] Gregory, P. C. & Condon, J. J. 1991, Astrophys. J. Suppl. Ser., 75, 1011
[10] Strauss, M. A. et al. 1992, ApJ, 385, 421
[11] Ellis, G. F. R. 1976, QJRAS, 16, 245
[12] Riess, A. et al. 1998, AJ, 116, 1009
[13] Perlmutter, S. et al. 1999, ApJ, 517, 565
[14] de Bernardis P, et al. 2000, Nature, 404, 955
[15] Eisenstein, D. J. et al. 2005, ApJ, 633, 560
[16] Marinoni, C. & Buzzi, A. 2010, Nature, 468, 539
[17] Célérier, M.-N. 2007, New Advances in Physics, 1, 29
[18] Goodman, J. 1995, Phys. Rev. D, 52, 1821
[19] Caldwell, R. R. & Stebbins, A. 2008, Phys. Rev. Lett., 100, 191302.
[20] Jia, J. & Zhang, H. B. 2008, JCAP, 12, 002.
[21] Zhang, P., & Stebbins, A. 2011, Phys. Rev. Lett., 107, 041301
[22] Clarkson, C., Bassett, B., & Lu, T. H.-C. 2008, Phys. Rev. Lett., 101, 011301
[23] Uzan, J. P., Clarkson, C. & Ellis, G. F. R. 2008, Phys. Rev. Lett., 100, 191303.
[24] Romano, A. E. 2007, Phys. Rev. D, 76, 103525
[25] Bolejko, K. & Wyithe, J. S. B. 2009, JCAP, 0902, 020
[26] Clifton, T., Ferreira, P. G., & Land, K. 2008, Phys. Rev. Lett., 101, 131302
[27] Stoeger, W. R., & Maartens, R., Ellis, G. F. R. 1995, ApJ, 443, 1
[28] Wu, K. K. S., Lahav, O., & Rees M. J. 1999, Nature, 397, 225
[29] Davis, M. 1997 in Critical Dialogs in Cosmology, ed Turok N (Singapore: World Scientific) pp 13
[30] Guzzo, L. 1997, New Astronomy, 2, 517
[31] Pietronero, L., Montuori, M. & Sylos-Labini, F. 1997, in Critical Dialogs in Cosmology, ed Turok N (Singapore: World Scientific) pp 24
[32] Gaite, J., Domínguez, A. & Pѐrez-Mercader, J. 1999, ApJ, 522: 5
[33] Borgani, S. 1995, Phys. Rep., 251, 1
[34] Cappi, A., Benoist, C., da Costa, L. N., & Maurogordato, S. 1998 A&A, 335, 779
[35] Amendola, L., Palladino, E. 1999, ApJ, 514, 1
[36] Bharadwaj, S., Gupta, A. K., Seshadri, T. R. 1999, A&A, 351, 405.
[37] Martinez, V. J., Lopez-Mart, B. Pons-Bordera M.-J. 2001, ApJ, 554, 5
[38] Pan, J. & Coles, P. 2001, MNRAS, 318, 51
[39] Bagla, J. S., Yadav, J. & Seshadri, T. R. 2005, MNRAS, 390, 829
[40] Hogg, D. W. 2005, ApJ, 624, 54
[41] Yadav, J., Bharadwaj, S., Pandey, B. & Seshadri, T. R. 2005, MNRAS, 364, 601
[42] Sarkar, P., Yadav, J., Pandey, B. & Bharadwaj, S. 2009, MNRAS, 399, 128

[43] Sylos-Labini, F. et al. 2009, A&A, 508, 17
[44] Sylos-Labini, F. et al. 2009, A&A, 496, 7
[45] Heavens, A. F., Jimenez, R. & Maartens, R. 2011, JCAP, 9, 035.
[46] Abazajian, K. N. et al. 2009, ApJS, 182, 543
[47] Eisenstein, D. J. et al. 2001, AJ , 122, 2267
[48] McBride, C. K. et al. 2009, AAS, 41, 253
[49] Haynes, M. P. 1996, PNAS, 93, 14216