



INSTITUTO DE FÍSICA

UNIVERSIDADE FEDERAL DO RIO DE JANEIRO

Pós-Graduação

Mestrado Profissional em Ensino de Física

Aula 3

Eletromagnetismo, Fótons e Luz

Referência: E. Hecht, óptica, Fundação Calouste Gulbekian, segunda edição portuguesa (2002);

Leis Básicas do eletromagnetismo

☞ Deus disse:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

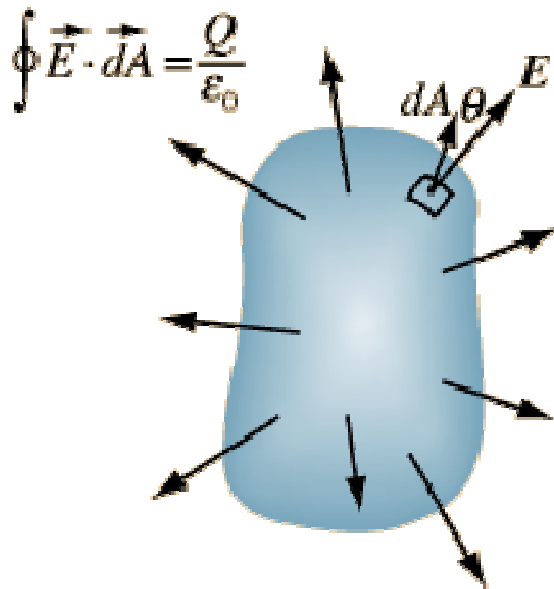
$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \frac{\partial \vec{D}}{\partial t}$$

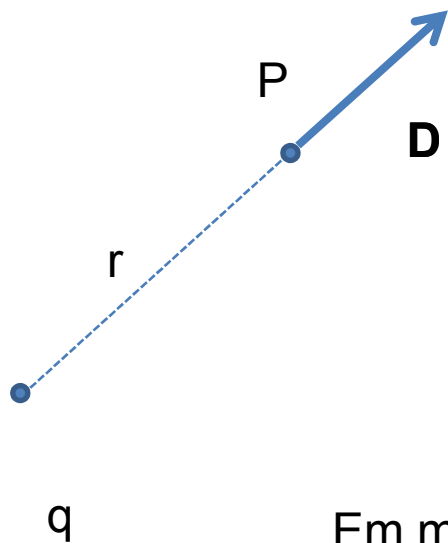
☞ fez-se a luz!

Lei de Gauss da eletricidade

$$\vec{\nabla} \cdot \vec{D} = 0$$



A carga é a fonte do vetor D !



Em meios isotrópicos:

$$\vec{D} = \frac{1}{4\pi} \frac{q}{r^2} \hat{r}$$

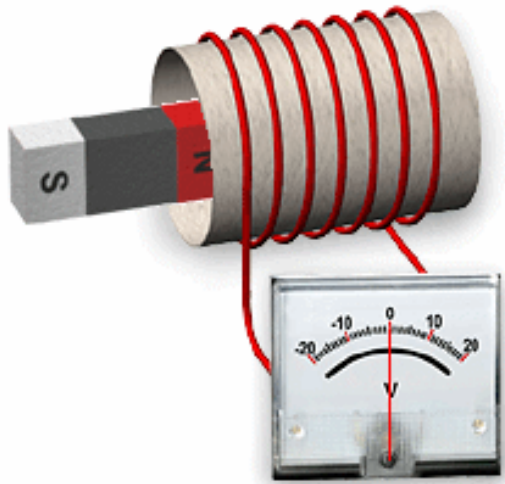
$$\vec{D} = \vec{\epsilon} \vec{E}$$

$$\vec{\epsilon} = \begin{vmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{vmatrix}$$

$$\vec{E} = \frac{1}{4\pi\epsilon} \frac{q}{r^2} \hat{r}$$

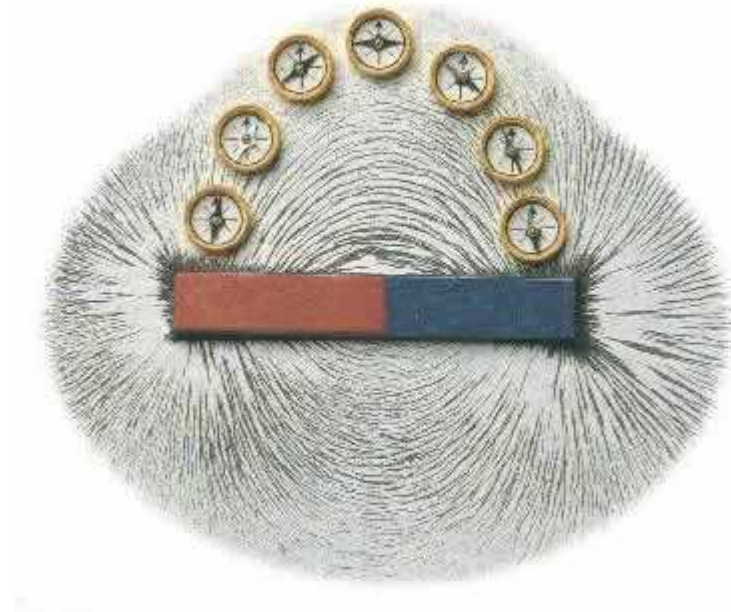
Lei de indução de Faraday

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0}$$



Lei de Gauss (magnética)

$$\vec{\nabla} \cdot \vec{B} = 0$$



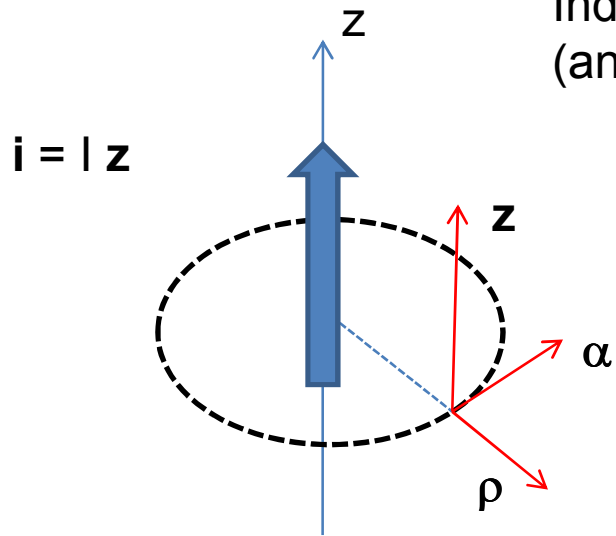
Não existem monopolos magnéticos!

Lei de Ampère

$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$



Lei de Ampère



Campo magnético

$$\vec{H} = \frac{1}{2\pi} \frac{I}{\rho} \hat{\alpha}$$

Indução magnética
(análogo ao E)

$$\vec{B} = \tilde{\mu} \vec{H}$$

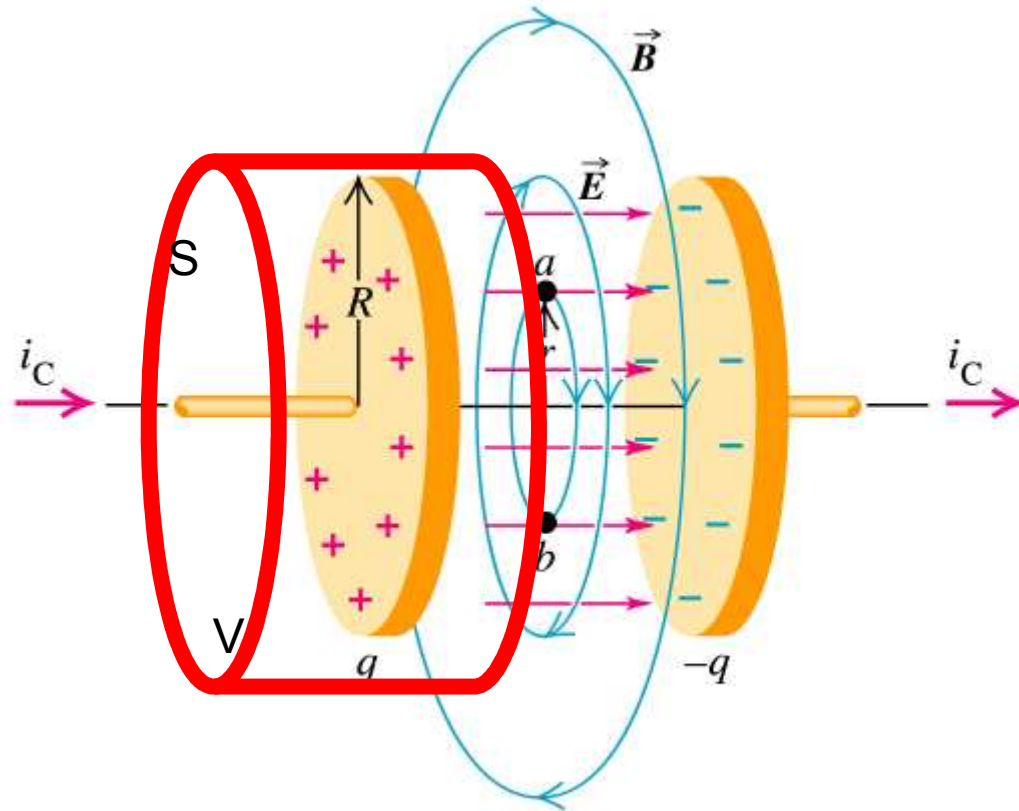
$$\tilde{\mu} = \begin{vmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{vmatrix}$$

$$\vec{B} = \frac{\mu}{2\pi} \frac{I}{\rho} \hat{\alpha}$$

$$\vec{\nabla} \times \vec{H} = \vec{j}$$

As fontes do campo magnético H são as correntes!

Mas falta um termo na lei de Ampère...



$$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

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Seja um volume V que contenha uma das placas do capacitor, mas não a outra. Durante o processo de carga, a corrente atravessa a face esquerda da superfície S . Como não pode haver corrente nenhuma saindo da superfície S , as cargas se acumulam na placa e aumentam com o tempo. Maxwell percebeu que faltava um termo na lei de Ampère.

$$\vec{j} = \lambda \vec{E}$$

Lei de Ohm

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{j} + \mu_o \frac{\partial \vec{D}}{\partial t}$$

Lei de Ampère

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_o \lambda (\vec{\nabla} \times \vec{E}) + \mu_o \epsilon_o \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t}$$

Aplicando o rotacional:

$$\vec{\nabla} \times (\vec{\nabla} \times) = \vec{\nabla}(\vec{\nabla} \cdot) - \vec{\nabla}^2$$

pela Lei de Faraday

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \vec{\nabla}^2 \vec{B} = -\vec{\nabla}^2 \vec{B} = \left(-\mu_o \lambda \frac{\partial \vec{B}}{\partial t} \right) + \mu_o \epsilon_o \frac{\partial}{\partial t} \left(\frac{\partial \vec{B}}{\partial t} \right)$$

$$\vec{\nabla}^2 \vec{B} - \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} - \mu\lambda \frac{\partial \vec{B}}{\partial t} = 0$$

$$\lambda = 0$$

$$\vec{\nabla}^2 \vec{B} - \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

Energia e momentum

Uma onda eletromagnética transporta energia e momentum.

Vetor de Poynting

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{H} = \sqrt{\frac{\epsilon}{\mu}} \hat{k} \times \vec{E}$$

E=ZH (lei de Ohm)

$$Z = \sqrt{\frac{\mu}{\epsilon}} = 377\Omega \quad (\text{no vácuo})$$

Densidade de energia

$$u = \frac{1}{2} [\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}]$$

$$\vec{S} = u\vec{v}$$

Luz e matéria

$$v = \frac{1}{\sqrt{\epsilon\mu}}$$

$$n \equiv \frac{c}{v} = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} = \sqrt{K_e K_m} = \sqrt{K_e}$$

1

Qual o significado físico da dependência de n com o comprimento de onda da luz ?

Por quê a velocidade de fase num meio difere de c ?

Do ponto de vista eletrostático podemos separar os meios em três tipos fundamentalmente diferentes:

$$\text{Vácuo} \rightarrow \varepsilon = \varepsilon_0, \lambda = 0$$

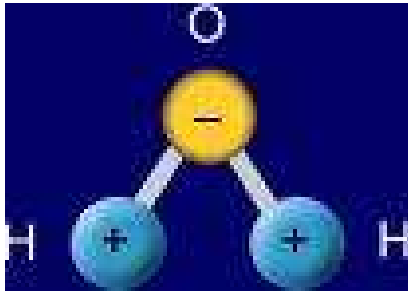
$$\text{Condutores} \rightarrow \varepsilon \approx \varepsilon_0, \lambda > 0$$

$$\text{Dielétricos} \rightarrow \varepsilon > \varepsilon_0, \lambda = 0$$

Dispersão

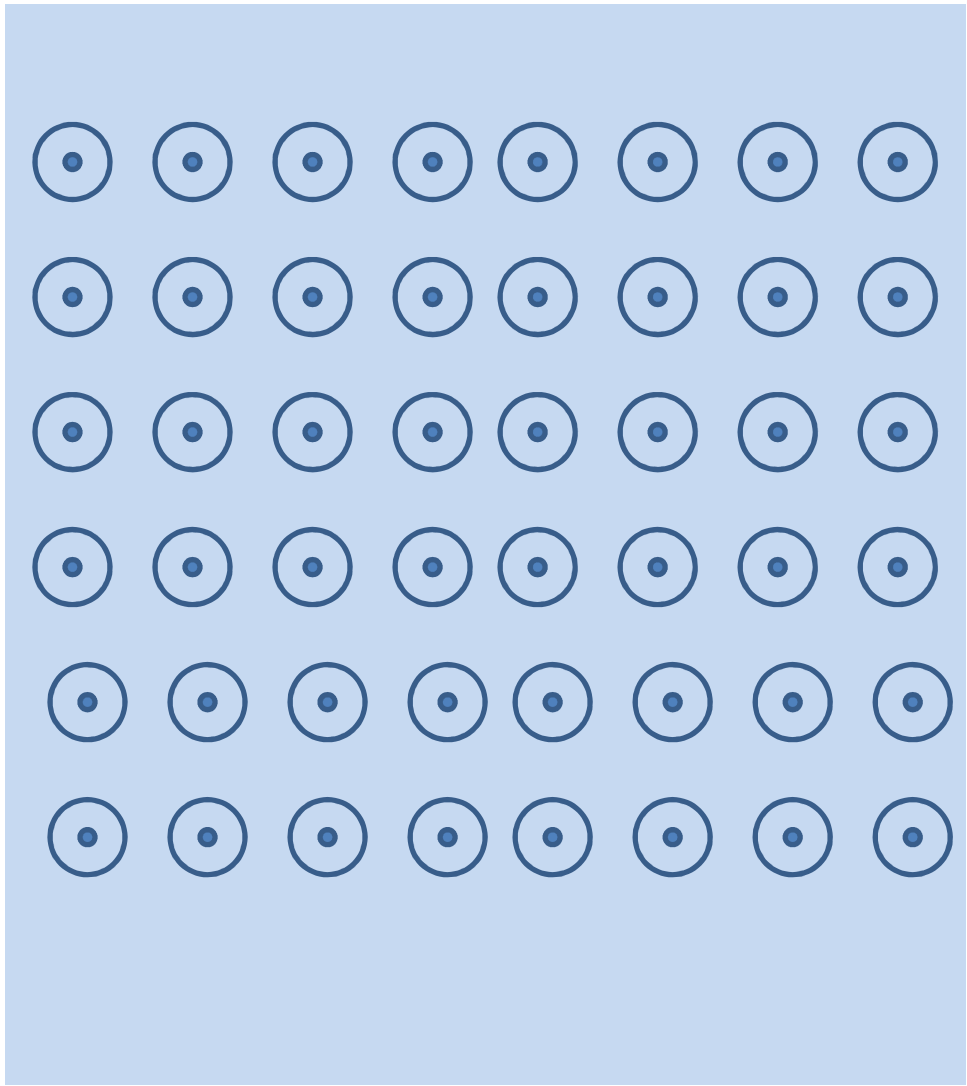
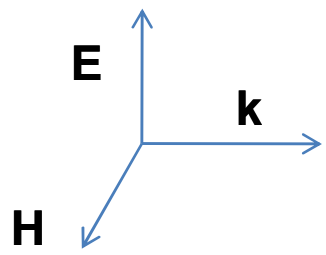
Introduzimos um novo vetor (polarização = momento dipolar) resultante por unidade de volume)

$$\mathbf{P} = d\mathbf{p}/dV$$

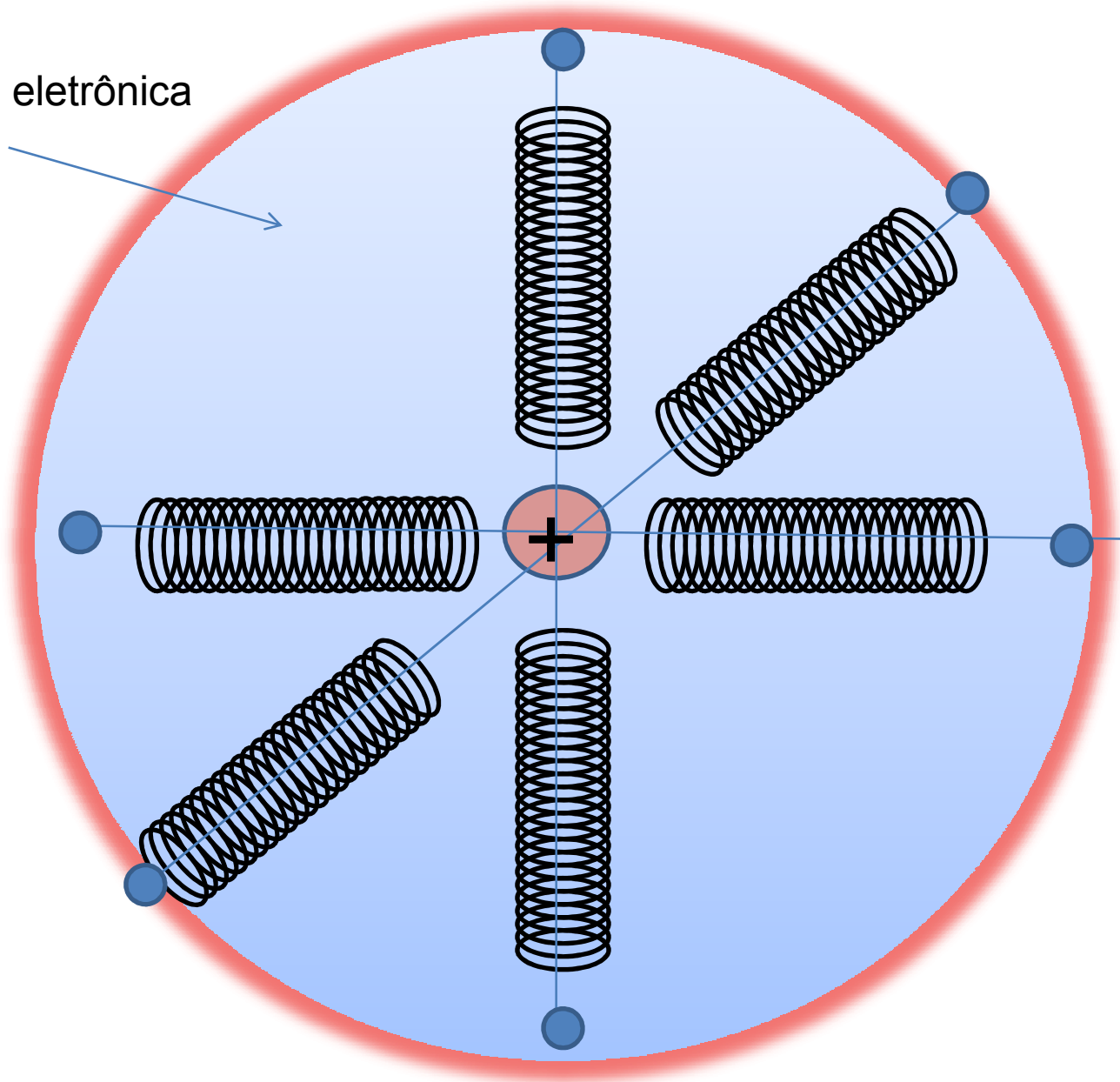


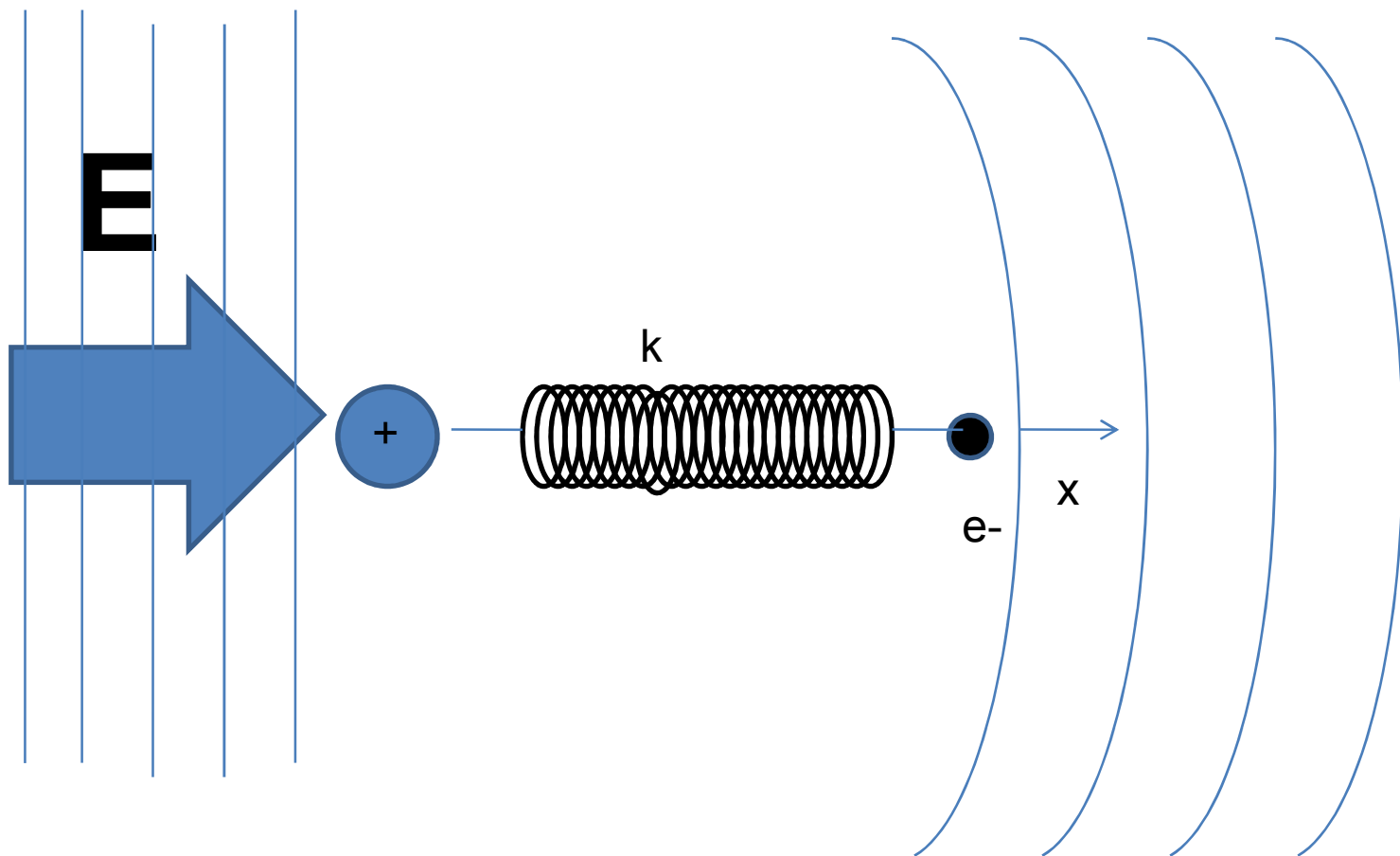
$$\mathbf{P} \equiv \mathbf{D} - \epsilon_0 \mathbf{E}$$

É nulo no vácuo ($\mathbf{D} = \epsilon_0 \mathbf{E}$) e nos condutores ($\mathbf{D} = \mathbf{E} = 0$)



Nuvem eletrônica





$$m \frac{d^2 x}{dt^2} + kx + b \frac{dx}{dt} = E(t)$$

$$m \frac{d^2 x}{dt^2} + kx + b \frac{dx}{dt} = qE_0 e^{i\omega t}$$

Procuramos por soluções do tipo: $x(t) = x_0 e^{i\omega t}$

$$\frac{dx}{dt} = i\omega x_0 e^{i\omega t} = i\omega x(t)$$

$$\frac{d^2 x}{dt^2} = -\omega^2 x(t)$$

$$m \frac{d^2 x}{dt^2} + kx + b \frac{dx}{dt} = qE_o e^{i\omega t}$$

$$- \omega^2 x_o e^{i\omega t} + \frac{k}{m} x_o e^{i\omega t} + \frac{b}{m} i\omega x_o e^{i\omega t} = \frac{qE_o}{m} e^{i\omega t}$$

$$x_o = \frac{qE_o}{m} \left[\frac{1}{\omega_o^2 - \omega^2 + i\gamma\omega} \right] = |x_o| e^{i\phi}$$

$$|x_o|^2 = x_o^* x_o = \left(\frac{qE_o}{m} \right)^2 \left[\frac{1}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2} \right]$$

$$x_o = \frac{qE_o}{m} \left[\frac{1}{\omega_o^2 - \omega^2 + i\gamma\omega} \right] = |x_o| e^{i\phi}$$

$$|x_o|^2 = x_o^* x_o = \left(\frac{qE_o}{m} \right)^2 \left[\frac{1}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2} \right]$$

$$x(t) = \frac{qE}{m} \sqrt{\frac{1}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2}} e^{i[\omega t + \phi]}$$

Em um volume ΔV contém N moléculas, com Z elétrons cada

$$\vec{x}(t) = \left(\frac{q}{m} \right) \left(\frac{\vec{E}(t)}{\omega_o^2 - \omega^2 + i\gamma\omega} \right)$$

Momento de dipolo instantâneo $\vec{p} = q\vec{x}(t) = \left(\frac{q^2}{m} \right) \left(\frac{\vec{E}(t)}{\omega_o^2 - \omega^2 + i\gamma\omega} \right) = \alpha \epsilon_o \vec{E}(t)$

polarizabilidade $\alpha \equiv \left(\frac{q^2}{\epsilon_o m} \right) \left(\frac{1}{\omega_o^2 - \omega^2 + i\gamma\omega} \right)$

Força do oscilador: $\sum_j f_j = 1$ ← Fração, em cada molécula de elétrons cuja frequência seja ω_{oj}

Vetor de polarização $\vec{P} = \frac{\sum_k \vec{p}_k}{\Delta V} = \frac{\sum_k \alpha_k \epsilon_o \vec{E}}{\Delta V} = NZ \left(\sum_j f_j \alpha_j \right) \epsilon_o \vec{E}$

Vetor polarização $\vec{P} = (\varepsilon - \varepsilon_o)\vec{E} = (K_e - 1)\varepsilon_o\vec{E}$

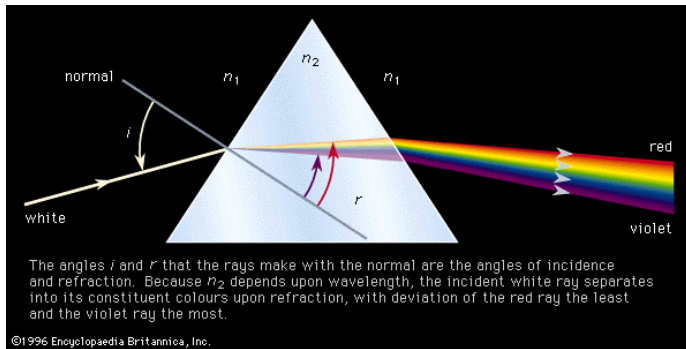
Permissividade relativa $K_e = \frac{\varepsilon}{\varepsilon_o}$

$$\vec{P} = \frac{\sum_k \vec{p}_k}{\Delta V} = \frac{\sum_k \alpha_k \varepsilon_o \vec{E}}{\Delta V} = NZ \left(\sum_j f_j \alpha_j \right) \varepsilon_o \vec{E}$$

$$K_e - 1 = NZ \left(\sum_j f_j \alpha_j \right)$$

$$K_e = 1 + \frac{NZq^2}{\varepsilon_o m} \left(\sum_j \frac{f_j}{\omega_o^2 - \omega^2 + i\gamma\omega} \right)$$

Série de Taylor



$$n = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{K_e}$$

$$n(\omega) = \left(1 + \frac{NZq^2}{\epsilon_0 m} \left(\sum_j \frac{f_j}{\omega_0^2 - \omega^2 + i\gamma\omega} \right) \right)^{1/2}$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2}$$

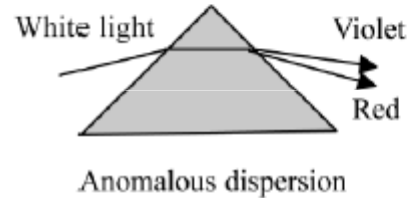
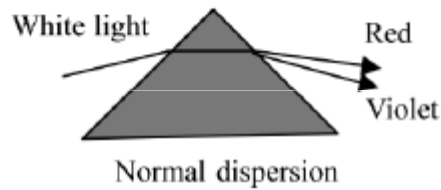
$$n(\omega) \approx 1 + \frac{NZq^2}{2\epsilon_0 m} \left(\sum_j \frac{f_j}{\omega_0^2 - \omega^2 + i\gamma\omega} \right)$$

$$n(\omega) = n - i\Xi$$

$$n = 1 + \frac{NZq^2}{2\epsilon_0 m} \left(\sum_j \frac{f_j}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2} \right)$$

$$\Xi = \frac{NZq^2}{2\epsilon_0 m} \left(\sum_j \frac{f_j}{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2} \right)$$

$$\frac{dn}{d\omega} > 0$$



$$\frac{dn}{d\omega} < 0$$

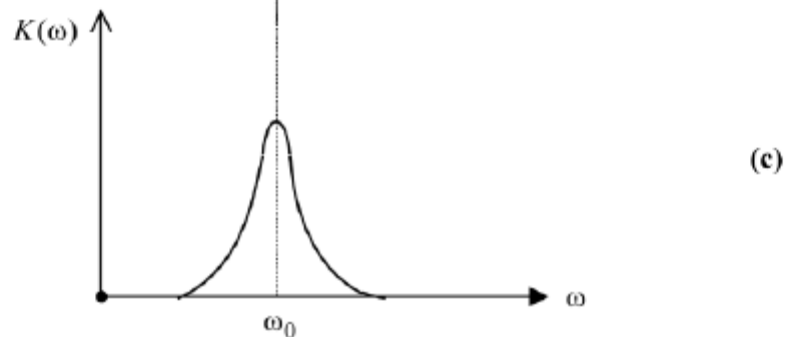
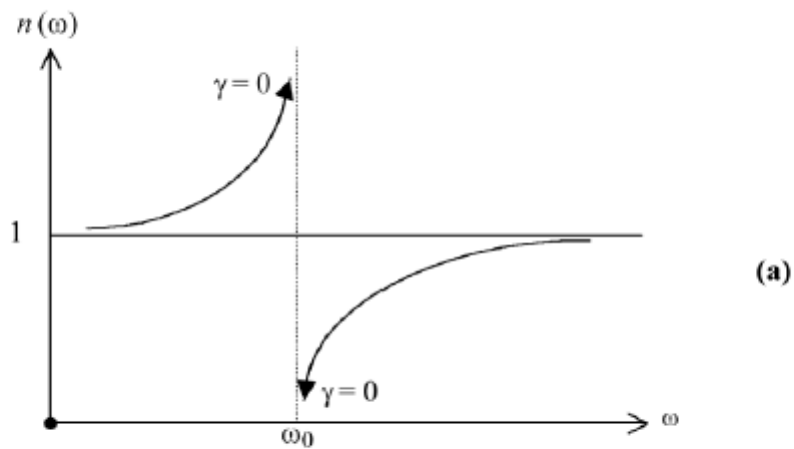


FIGURE 8.1 (a) Dependence of n on frequency. For no damping we have a singularity; (b) normal and anomalous dispersion; (c) dependence of K on frequency. The maximum is not at infinity if $\gamma \neq 0$.

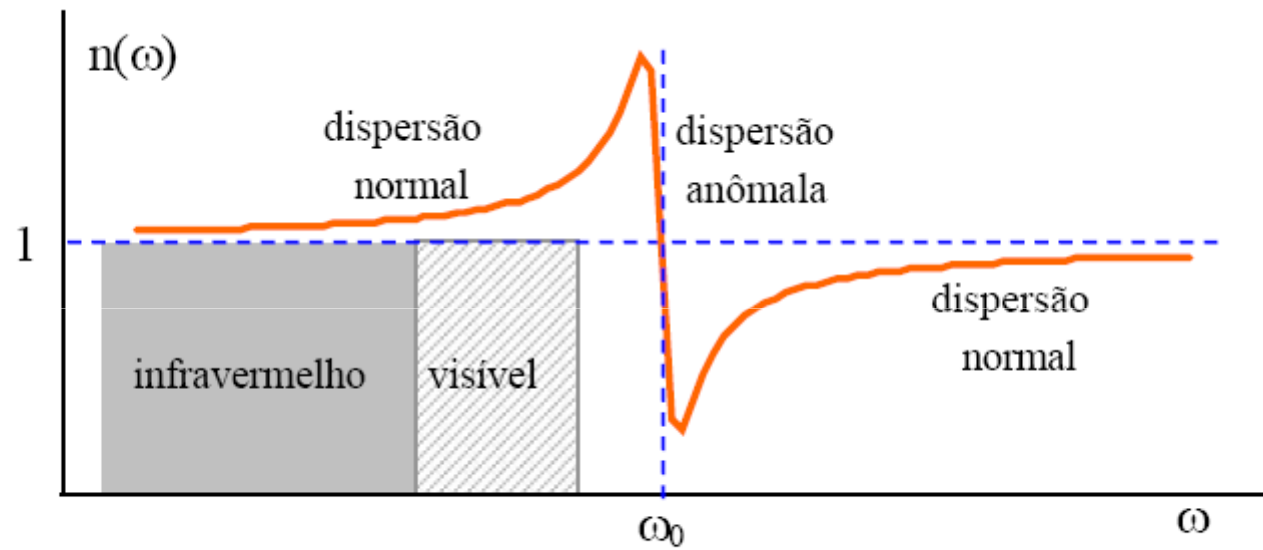


Fig. 9.2 - Dependência do índice de refração com a frequência da luz.

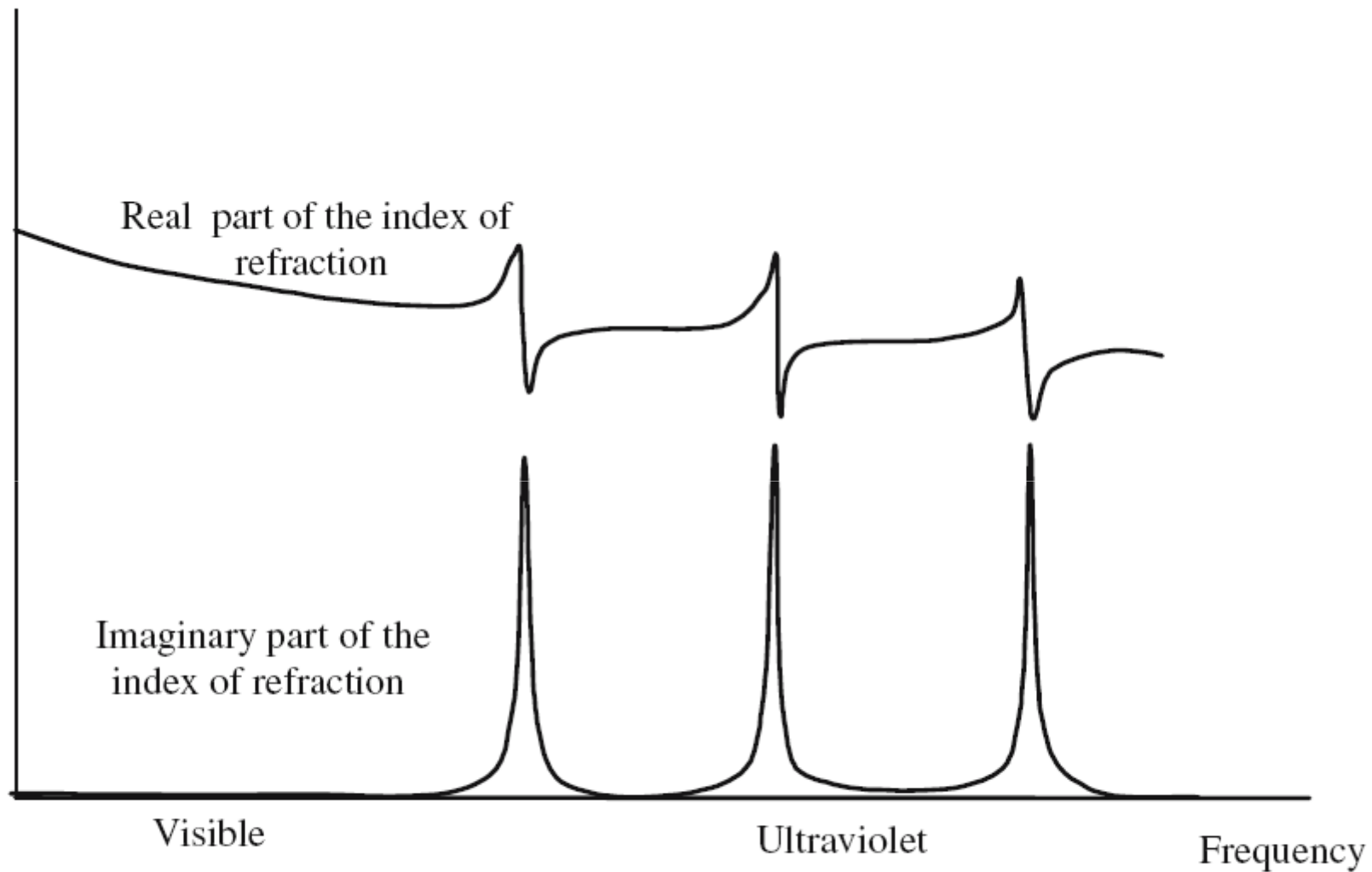


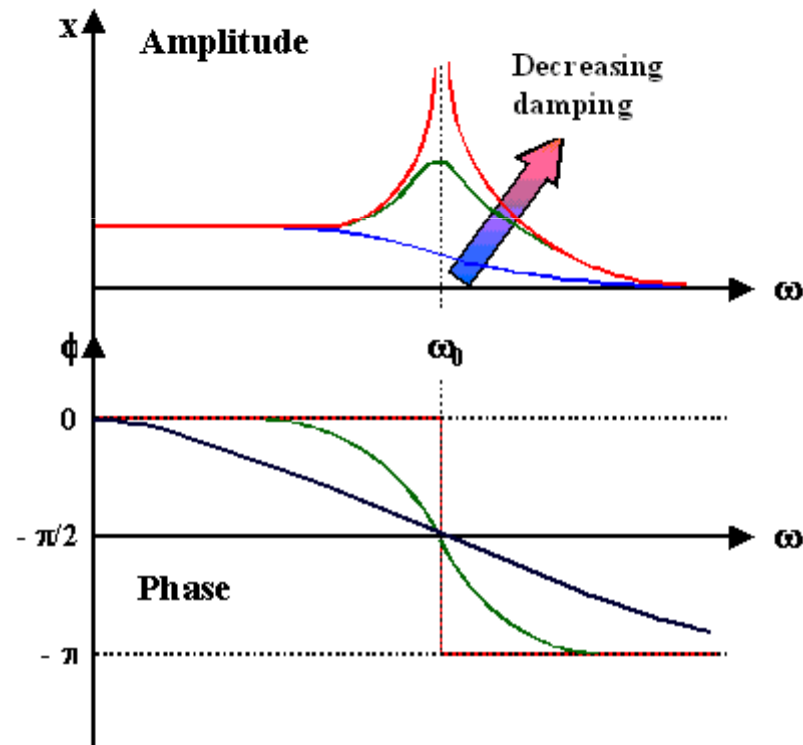
Figure 8.3. Dispersion and absorption curves of a material that is transparent in the visible and has three absorption bands in the ultraviolet.

$$x(t) = \frac{qE_o}{m} \left[\frac{1}{\omega_o^2 - \omega^2 + i\gamma\omega} \right] = |x_o| e^{i(\omega t + \phi)}$$

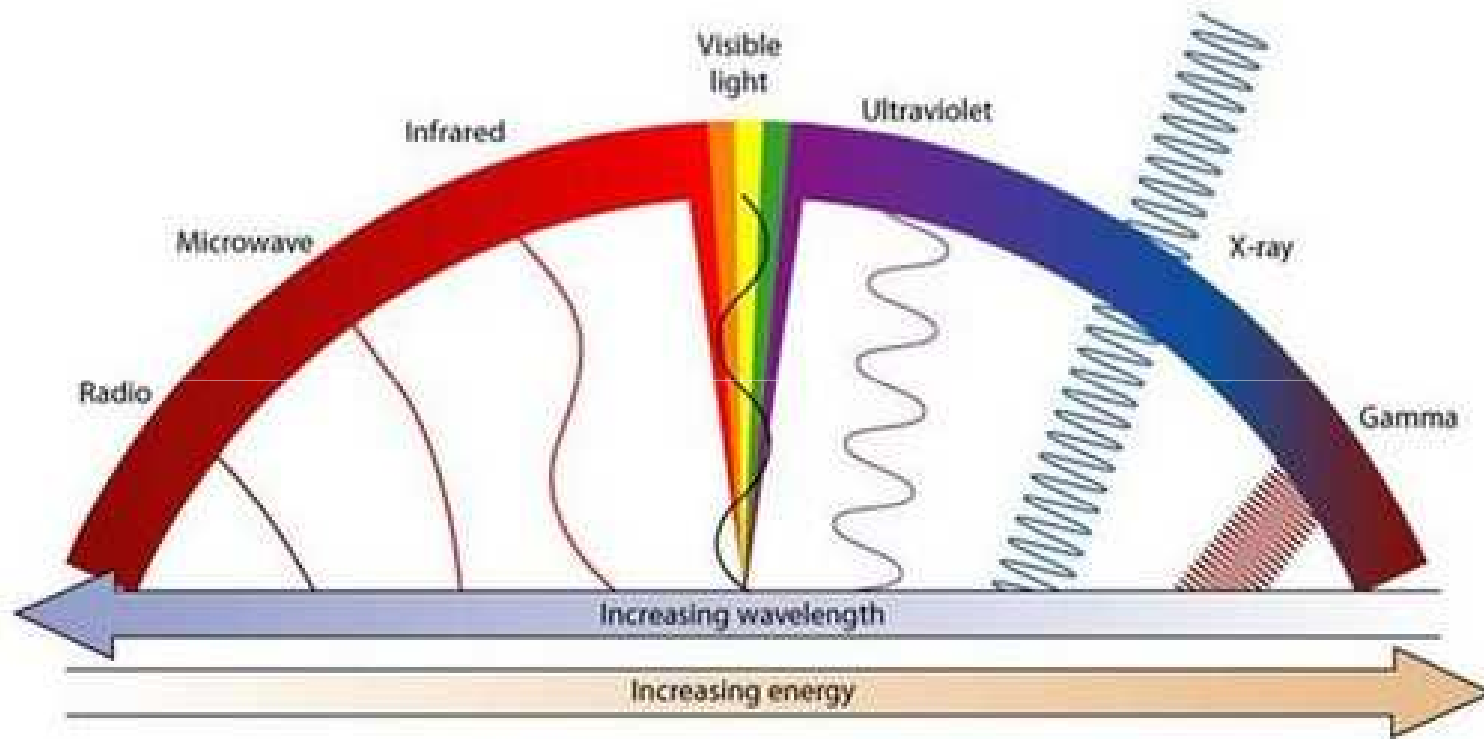
$$|x_o| = \sqrt{x_o^* x_o} = \left(\frac{qE_o}{m} \right) \left[\frac{1}{(\omega_o^2 - \omega^2)^2 + (\gamma\omega)^2} \right]^{1/2}$$

$$\text{tg } \phi = \frac{\text{sen } \phi}{\text{cos } \phi} = \frac{-\gamma\omega}{\omega_o^2 - \omega^2}$$

$$\operatorname{tg} \phi = \frac{\operatorname{sen} \phi}{\operatorname{cos} \phi} = \frac{-\gamma \omega}{\omega_o^2 - \omega^2}$$



O espectro eletromagnético



THE ELECTROMAGNETIC SPECTRUM

Penetrates Earth Atmosphere?

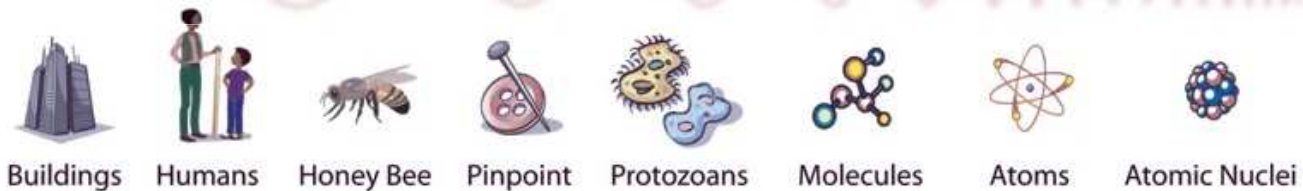


Wavelength (meters)

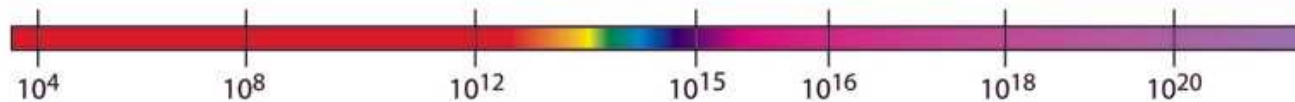


10^3 10^{-2} 10^{-5} $.5 \times 10^{-6}$ 10^{-8} 10^{-10} 10^{-12}

About the size of...



Frequency (Hz)



Temperature of bodies emitting the wavelength (K)

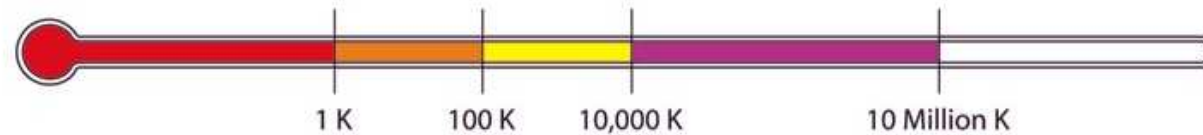
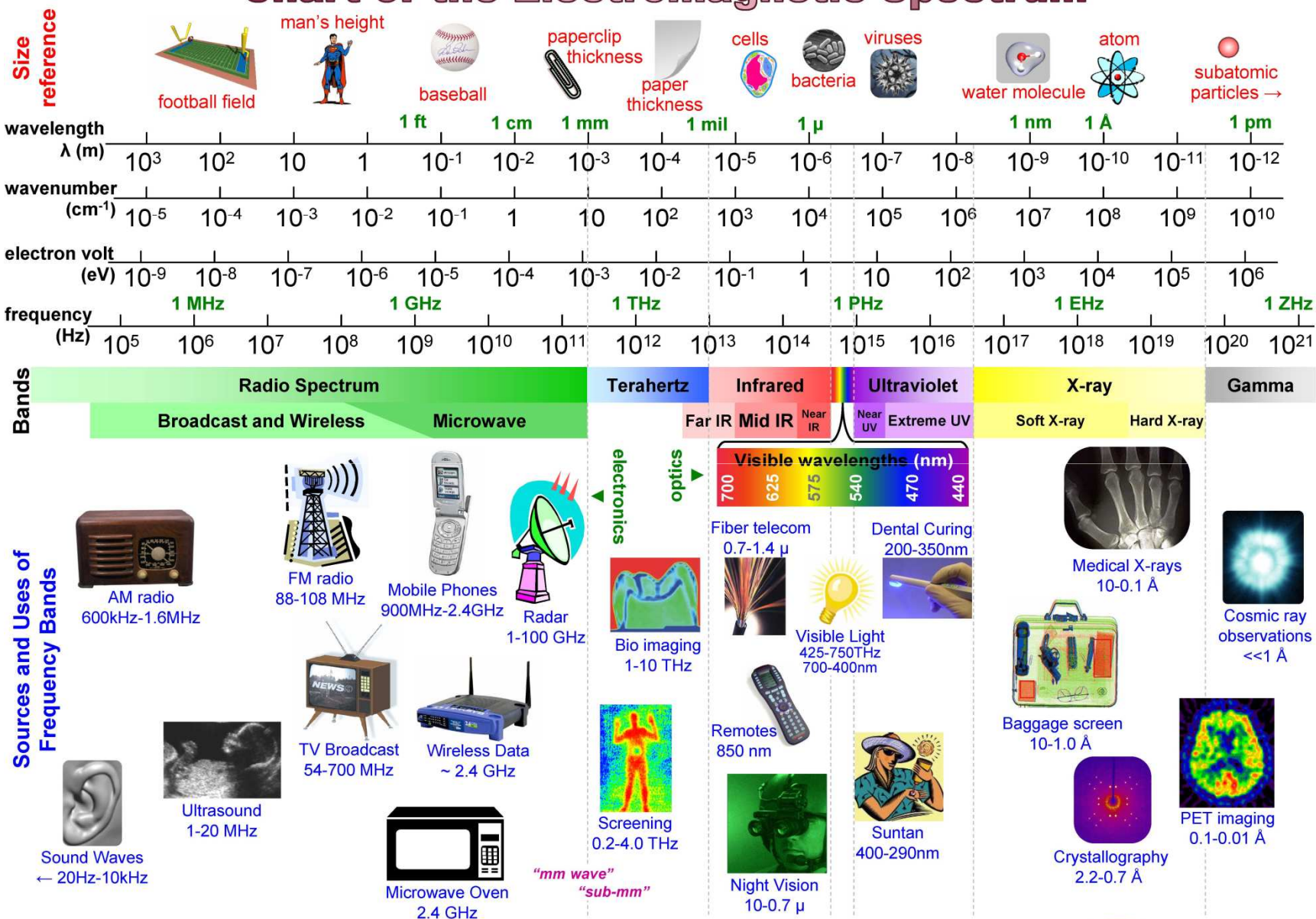
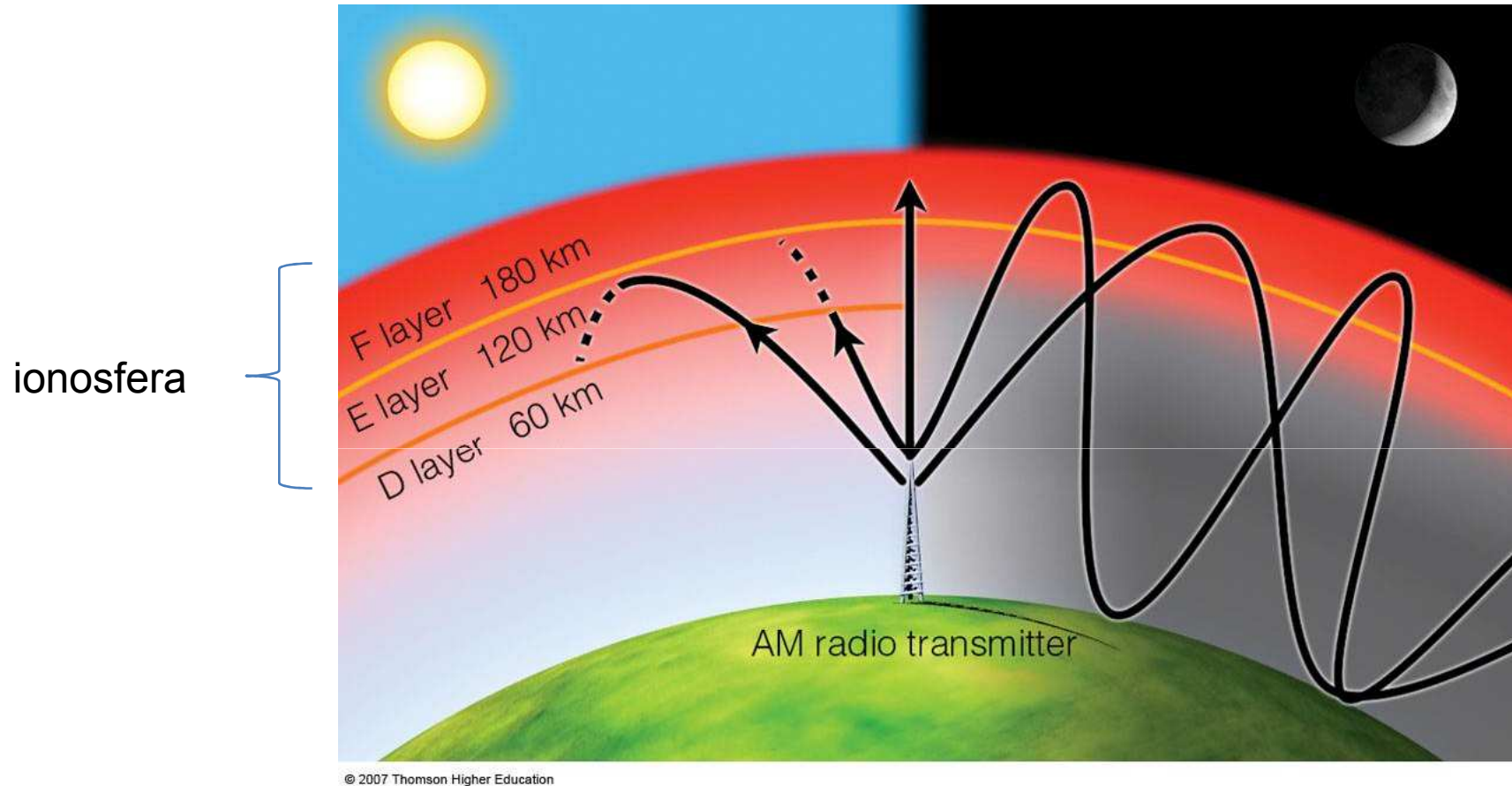


Chart of the Electromagnetic Spectrum



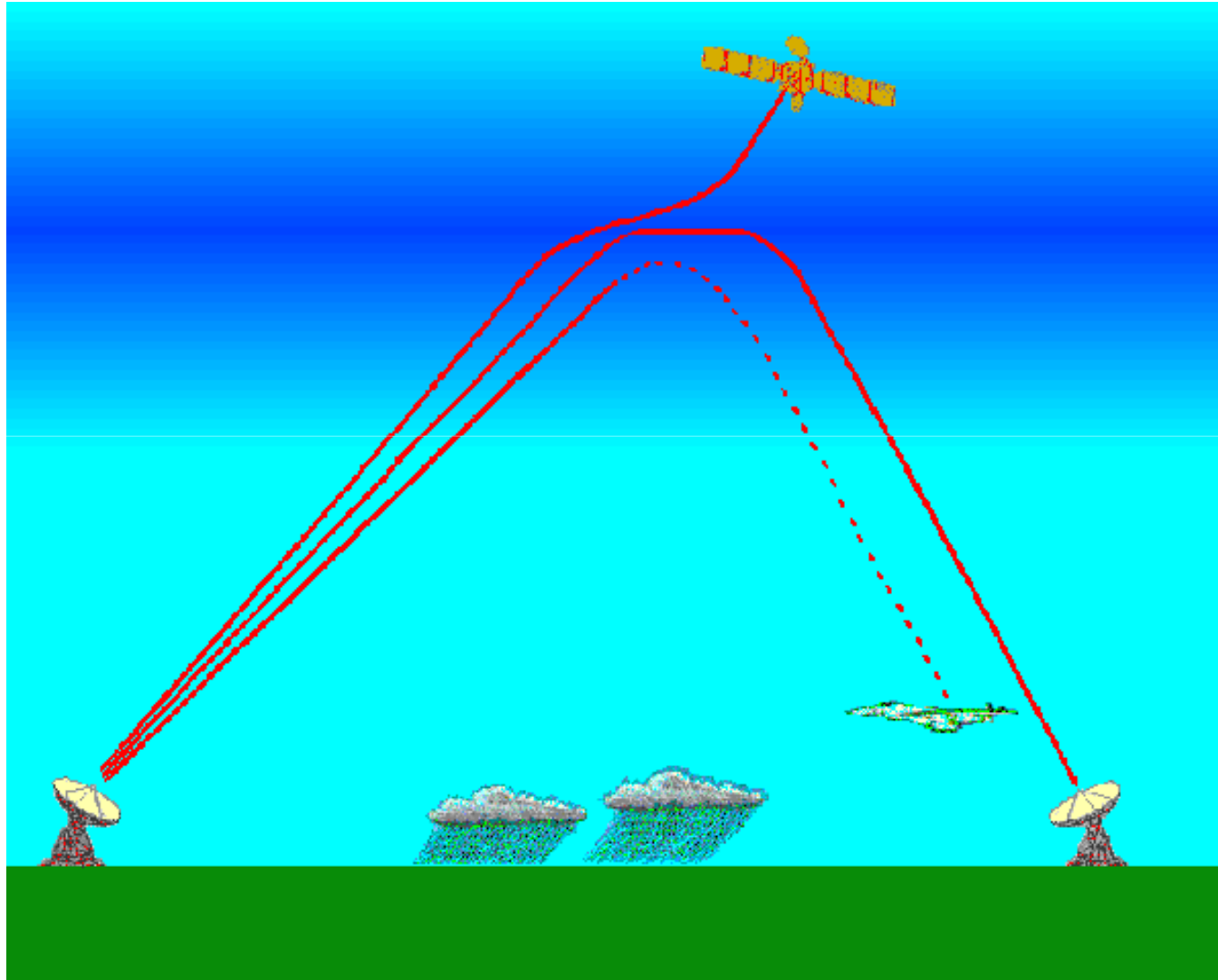
$$\lambda = 3 \times 10^8 / \text{freq} = 1 / (\text{wn} * 100) = 1.24 \times 10^{-6} / \text{eV}$$

Ondas de rádio



Camada D → absorve ondas AM e desaparece à noite;
Camadas E e F → refletem as ondas AM de volta para a Terra

Ondas de rádio



Não há limite superior para o comprimento de onda teóricamente possível. Podemos fazer oscilar um objeto carregado e no processo gerar uma onda eletromagnética de grande comprimento de onda. Já foram detectadas na Terra ondas com comprimento de onda superior a 28 milhões de km vindas do espaço exterior.

Micro-ondas

Medindo a velocidade da luz com chocolate ou mariola e um micro-ondas



infravermelho

Jon Grepstad Photography



ultravioleta



Ultra violeta



UV



visível

Raios x

18X24 ON 22X28IN PAPER 30.00 OR 11X14 ON 12X16IN PAPER 14.00

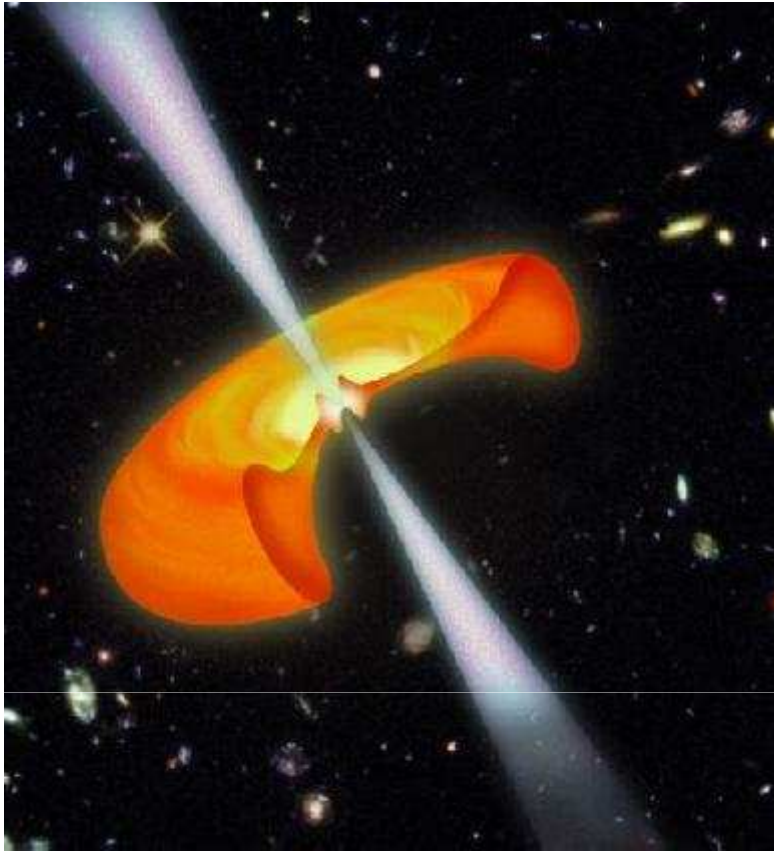


STEVEN S. MEYER'S ICELAND POPPY I

Raios gama

Radiações eletromagnéticas mais energéticas (10^4 eV até 10^{19} eV) e com menor comprimento de onda. São emitidas por partículas que sofrem transições no interior do núcleo atômico.





Gamma-Ray Burst Physics

Gamma-ray bursts (GRB) are sudden, intense flashes of gamma-rays which, for a few blinding seconds, light up in an otherwise fairly dark gamma-ray sky. They are detected at the rate of about once a day, and while they are on, they outshine every other gamma-ray source in the sky, including the sun. Major advances have been made in the last three or four years, including the discovery of slowly fading x-ray, optical and radio afterglows of GRBs, the identification of host galaxies at cosmological distances, and finding evidence for many of them being associated with star forming regions and possibly supernovae.